On Trade and the Stability of (Armed) Peace[†]

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Abstract: We consider an environment in which two sovereign states with overlapping ownership claims on a resource/asset first arm and then choose whether to resolve their dispute violently through war or peacefully through settlement. Both approaches depend on the states' military capacities, but have very different outcomes. War precludes trade between the two states and can be destructive; however, once a winner is declared, arming is unnecessary in future periods. By contrast, a peaceful resolution under the threat of war today avoids destruction and supports mutually advantageous trade; yet, settlements must be renegotiated and the states must arm in future periods to resolve their ongoing dispute. Paying special attention to the importance of trade on arming incentives and payoffs in this context, we explore the conditions under which "armed peace" arises as the perfectly coalition-proof equilibrium over time. Our analysis reveals that, depending on the destructiveness of war, time preferences, and the distribution of initial resource endowments, greater gains from trade (jointly determined by trade costs and the substitutability of traded commodities) can reduce arming and pacify international tensions. Even when the gains from trade are relatively small, peace might be sustainable, but only for more uneven distributions of initial resources.

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If an economically self-sufficient man starts a feud against another autarkic man, no specific problems of "war-economy" arise. But if the tailor goes to war against the baker, he must henceforth produce his bread for himself.

Ludwig von Mises (1949, p. 824)

1 Introduction

To what extent does economic interdependence between countries induce more peaceful relations between them? The time-honored "liberal peace" hypothesis, advanced and extensively tested by scholars of international relations, states that, because war significantly reduces or completely destroys opportunities for trade, the cost of war rises as national economies become more highly integrated.[1] With greater interdependence, therefore, we should expect extended peace (e.g., Polachek, 1980).[2] The logic of the liberal peace hypothesis, however, abstracts entirely from arming decisions.[3] On the one hand, such decisions are influenced by the anticipation of war or peace. On the other hand, by influencing the amount of resources available for production of tradable commodities, they condition the potential gains from trade and, consequently, the relative appeal of peace. As such, taking a general equilibrium approach to capture the interrelated choices of arming and conflict initiation (or peace) would yield a more complete analysis that can deepen our understanding of the links between trade and international relations.

Perhaps surprisingly since the liberal peace hypothesis builds on a very basic principle in economics (namely, that trade is mutually beneficial), theoretical research by economists on trade and conflict is relatively scant. Aside from Polachek's (1980) seminal contribution, Skaperdas and Syropoulos (2001) augment a Heckscher-Ohlin model of trade with conflict over resources between two identical and small countries. In that setting, depending on world prices, trade can amplify arming incentives and thus the associated security costs to such an extent so as to swamp any gains from trade. The analysis, however, considers just the case of two small countries that trade with the rest of the world, and thus does not address the possible importance of interdependence between them.[4] Garfinkel et al. (2018)

[1] While some have found that war has little to no significant effect on trade (e.g., Barbieri and Levy, 1999), Glick and Taylor (2010) present compelling evidence of a significantly negative and persistent effect.

[2] Although the expansion of world trade along with the sharp drop in the frequency of interstate wars witnessed in the decades following the World War period would appear to support the optimism of this hypothesis, the evidence based on formal testing is somewhat mixed. Oneal and Russett (1997, 1999), for example, find that the likelihood of war breaking out between two countries depends negatively on the interdependence between them, whereas Barbieri (2002) presents evidence of no significant relation between trade and war. See Copeland (2015) for a comprehensive survey of alternative views and the empirical evidence regarding trade and war in the international relations literature.

[3] Global military spending as a fraction of GDP has fallen considerably since the end of the Cold War, but remains significant. In 2017, it totaled \$1,739 billion or 2.2 percent of global GDP (Tian et al., 2018).

[4] Also, see Garfinkel et al. (2015) who extend that analysis, examining possible asymmetries in initial

explore the importance of economic interdependence in the context of interactions between two large countries that compete for claims to a resource and subsequently trade with each other. In that setting, where the endogeneity of arming and the accompanying resource costs impact world prices, trade typically induces lower arming and greater payoffs. But, neither this analysis nor Skaperdas and Syropoulos (2001) considers the cost of conflict to disrupt trade. Martin et al. (2008) study this disruptive effect, but abstract from the endogeneity of arming and, thus, the resource costs of conflict (however resolved) that can influence the terms of trade and thus the gains from trade.[5]

In this paper, we combine these approaches to gain further insight into when and how trade openness, among other economic factors, matters for international relations. Our analysis builds on a model of trade where, along the lines of Armington (1969), each country produces a tradable intermediate product that can serve as an input in the production of a consumption good. Diversity of inputs enhances each country's ability to produce that final good, and herein lie the possible gains from trade. We augment that model with conflict over claims of ownership to an asset/resource used to produce the intermediate goods.[6] A key feature of the analysis is its distinction between the mobilization of resources to arm and the deployment of those arms in open conflict, along the lines of Garfinkel and Skaperdas (2000) and McBride and Skaperdas (2014) and others.[7]

The basic setup of the model is as follows. Once the contending countries have decided how much of their initial endowments to devote to arming, they choose how to resolve their dispute over the remaining resources—what we call the "residual" resource. One option involves open conflict or war, modeled as a winner-take-all contest, with both countries deploying their arms to improve their respective chances of victory and some fraction of the residual resource being destroyed as a result. At the same time, open conflict destroys the possibility of trade between the two countries. The other option involves "armed peace," in which the two countries negotiate a division, partly based on their first-stage arming choices, of the residual resource. Since no arms are actually deployed, this option implies no destruction. Furthermore, it leaves open the possibility of subsequent trade.[8] Indeed,

resources as well as more general functional forms for preferences and technologies.

[5]Although Jackson and Nei (2015) take a novel approach to study this issue (one based on a network framework that views alliances as having both military and trade benefits), they too abstract from the endogenous determination of conflict costs.

[6]The assumption of a materialistic motive for arming and war is consistent with the history of empire building, but also relevant in current conflicts over resources—e.g., the ongoing dispute between China, Taiwan, Vietnam, the Philippines, Indonesia, Malaysia, and Brunei over control of the Spratly and Paracel islands in the South China Sea (fueled by the suspected abundance of natural resources tied to those islands and the surrounding sea) that might escalate to war despite their trading relations.

[7]These analyses, however, do not consider trade between adversaries.

[8]While unarmed peace is a possibility in our setting when gains from trade are sufficiently large, there always exists a unique equilibrium under settlement with strictly positive arming by both countries. Skaperdas and Syropoulos (2002) similarly study the interactions between large countries that compete for a productive

whatever arms the contending countries choose, they always have a *short-run* incentive to negotiate a peaceful settlement.

But, when the countries take a *longer-run* perspective, settlement need not emerge in equilibrium. The reason is that settlement in the current period concerns the division of resources in that period only. Without being able to commit today to a division of the contested resource in the future, the countries must continue to arm beyond the current period to settle their ongoing dispute, implying the diversion of additional resources away from the production of goods for consumption. The possible appeal of open conflict is that it gives the victor a strategic advantage in future conflict, so that fighting today reduces future arming costs relative to those under settlement.[9] In fact, despite its effect to rule out trade, open conflict is always a subgame perfect, Nash equilibrium in a multi-period setting. Although peaceful settlement may also emerge as an equilibrium outcome, depending on the possible gains from trade, time preferences, and the degree of conflict's destructiveness, it could be Pareto dominated by open conflict. In such cases, open conflict is a "strong perfect equilibrium" or, equivalently (since we are considering a two-country setting), a "perfectly coalition-proof" equilibrium (Bernheim et al., 1987).[10]

A central component of our analysis involves identifying the conditions under which peaceful settlement is immune to both coalitional and unilateral deviations and, thus, emerges as the stable equilibrium outcome. Consistent with the findings of other analyses that distinguish between mobilization of resources to produce guns and the decision to use those guns in open conflict, we find that peaceful settlement is more likely to be the stable equilibrium outcome, when countries discount the future heavily (or equivalently, the shadow of the future is weak) and the destructive effects of war are large.

The main contribution of this paper, however, is to characterize the importance of trade openness in each country's optimizing choices as they depend on the initial distribution of ownership claims on resources. The key to this link is summarized in the per-period gains from trade, which as in standard trade models depend negatively on the elasticity of substitution between tradable inputs in the production of final consumption goods, the level of trade costs, and the unevenness of the initial distribution of resources.

Our analysis shows that there exists a threshold level of the destructiveness of war above which neither trade nor the initial distribution of resource ownership matters for the

resource via bargaining; but, with a focus on how the anticipation of trade affects their arming choices, they do not consider the choice between settlement and war.

[9]Fearon (1995) and Powell (2006) provide similar arguments based on the notion that negotiated settlements for future divisions are not enforceable, but they are distinct in their emphasis on the importance of exogenous shifts in power. Skaperdas and Syropoulos (1996) do not consider the choice between war and settlement, but similarly emphasize the effect of using military force today to enhance future payoffs.

[10]We view this equilibrium concept especially relevant in our setting since the two parties in conflict presumably communicate with one another in the process of their negotiations.

stability of peace. Otherwise, trade can matter. In particular, we find that trade reduces equilibrium arming under settlement and that greater gains from trade raise the payoff of settlement relative to war. If large enough, these gains alone render armed peace stable for all possible initial resource distributions. But, if these gains are not sufficiently large, the initial distribution plays an important role, along with the destructiveness of war and the strength of the shadow of the future. Interestingly, in this case, while armed peace dominates open conflict for sufficiently even and for sufficiently uneven distributions of resources, unilateral deviations are profitable for at least one and possibly both countries when the distribution is sufficiently even. Accordingly, peace is stable only for sufficiently uneven resource distributions.

The potential influence of the distribution of initial resources operates through two distinct channels: the gains from trade that can be realized under settlement and the possible savings in future arming that can be realized under open conflict. Specifically, an even distribution of initial resource endowments maximizes the total gains from trade. Departures from that benchmark reduce those total gains, with the smaller country enjoying relatively more over its autarky payoff. This tendency alone suggests a positive link between the evenness of international resource ownership and the relative appeal of settlement. But, due to strategic complementarity in guns choices, total arming under settlement tends to be higher precisely when the countries are similar in size. As a result, the possible savings in future arming afforded by declaring war (instead of settling) tend to be larger. Even when settlement dominates war, making it immune to coalitional deviations for more even distributions, this possible savings could render unilateral deviations profitable, thereby undermining the stability of peace. The smaller is the destructiveness of conflict and/or the gains from trade or the larger is the salience of the future, the larger are the gains from such deviations. For more uneven distributions, the smaller country has more to gain from settlement and trade and generally little to gain by deviating from settlement. While the larger country has less to gain from settlement and trade, total arming is relatively small under this mode of conflict resolution in such cases, implying that the future savings in arming that could be realized with a unilateral deviation by the larger country are also relatively small. Thus, settlement with trade remains a possible stable outcome when the initial distribution of resources is sufficiently uneven.

Aside from the literature cited above, our analysis is related to a small literature in economics that studies the emergence of peace, resulting in the status quo, possibly supported by transfers in the presence of asymmetries in the initial resources. Most notably, Beviá and Corchón (2010) show, when the initial ownership of resources is the only source of asymmetry, peace with or without transfers is more likely to emerge under more even distributions.[11] Our analysis is also related to Vesperoni and Yildizparlak (2019), who explore the importance of inequality in the emergence of (multi-prize) conflict, finding that greater inequality is more conducive to peace. Although this finding is similar to ours, their underlying logic is quite different as it is based on their result that conflict tends to be more intense and thus more costly (relative to peace that preserves the status quo) when inequality is greater. But, more generally, the differences in the results in the present study and these analyses can largely be attributed to our focus on trade and peaceful negotiations supported by arming in a dynamic setting.

In the next section, we present a basic model of trade between two countries who dispute ownership claims to a productive resource, and describe the essential features of the two types of conflict resolution they can pursue—namely, open conflict and peaceful settlement. Then, in Section 3, we study the associated outcomes and payoffs and compare them to determine which mode of conflict resolution is immune to coalitional deviations. Highlighting the importance of trade in Section 4, we examine the conditions under which settlement is immune to unilateral deviations. Section 5 discusses briefly how some of our simplifying assumptions could be relaxed to make our analysis richer without altering the thrust of our findings. Finally, we offer concluding remarks in Section 6. All technical details are relegated to the Appendix.

2 A Basic Model of Resource Conflict and Trade

Consider a global economy consisting of two countries ($i = 1, 2$) that interact over two periods. At the beginning of the first period, each country i holds a claim over an asset (e.g., land, water, or oil) that generates a stream of services per period of time denoted by R^i, where $R^1 + R^2 = \bar{R}$. However, these claims are not entirely secure. Instead, whatever resource R^i is held initially by country i is available only for the production of "military capacity" or "guns" for short. Each country i devotes G^i ($\leq R^i$) units of its resource to guns, an irreversible and non-contractable investment, to contest the remaining units $X^i = R^i - G^i$ for $i = 1, 2$ that go into a common pool: $\bar{X} \equiv \sum_i X^i = \bar{R} - \bar{G}$.[12]

[11] Jackson and Morelli (2007) also study the use of transfers to avoid the emergence of war; however, their focus is primarily on the importance of biases of the countries' leaders, where the gains from war confiscated by the leaders are disproportionately larger than the costs they bear. Absent such a bias in either country, peace supported by transfers is always feasible. This result is similar to our finding in a single-period setting that, for any given choice of guns by the two countries, they always have a short-run preference for settlement.

[12] The analysis could be modified to consider the possibility that a fraction $\kappa^i \in [0, 1]$ of X^i is *secure* and the remaining $\left(1 - \kappa^i\right) X^i$ units are subject to appropriation. For $\kappa^i = \kappa$, this modification would allow us to study the implications of various degrees of insecurity including the extreme case of perfect security ("Nirvana"), which arises when $\kappa = 1$ and is the norm in standard neoclassical theory, and other intermediate cases, where $\kappa \in (0, 1)$. We could also modify the analysis to consider $\kappa^i = 1$ while $\kappa^j = 0$ which implies that the contest is over just one country's residual resource. We abstract from these generalizations here. In the former case, they do not affect the key insights regarding the comparison between conflict and

Once ownership claims over the contested pool are resolved—either through warfare or a peaceful division—each country i produces, on a one-to-one basis with their secure holdings, a distinct and potentially tradable commodity Z^i, used as an intermediate input in the production of a consumption good.[13] Importantly, the technology for producing Z^i in each country i is unique and inalienable.[14]

In what follows, we present the details of our framework in two steps. First, we describe the mechanisms of conflict resolution that are available to the contending states. Then, we describe production and possible trade decisions, given the resources *securely* held by each country after their ownership claims have been resolved.

2.1 Arming and the Resolution of Resource Disputes

From our brief description above, it should be clear that arming is socially costly. That is to say, each country's allocation to build its military capacity G^i reduces the aggregate size of the contested pool, $\bar{X}$. Nevertheless, each country has an incentive to arm to contest those resources. The precise benefit for each country depends on the manner in which they jointly resolve their ownership claims: open conflict or peaceful settlement.

Under open conflict/war, arming is the means by which a country improves its chances of victory. To be more precise, open conflict takes the form of a "winner-take-all" contest, where the probability of winning for country i, denoted by ϕ^i, is

$$\phi^i = \phi^i(G^i, G^j) = \begin{cases} G^i/\bar{G} & \text{if} \quad \bar{G} > 0 \\ 1/2 & \text{if} \quad \bar{G} = 0, \end{cases} \tag{1}$$

for $i \neq j = 1, 2$. According to (1), country i's winning probability is increasing in its own guns ($\phi^i_{G^i} > 0$) and decreasing in the guns of its opponent ($\phi^i_{G^j} < 0$). Furthermore, the conflict technology is symmetric, such that $G^i = G^j = G \geq 0$ implies $\phi^i = \phi^j = \frac{1}{2}$.[15]

Open conflict in the first period has three other important features. First, it can be destructive, leaving only a fraction $\beta \in (0, 1]$ of the common resource pool intact in the first

settlement. The latter case, though interesting in its own right, introduces a second source of asymmetry that complicates the analysis.

[13]One could also interpret Z^i as a final tradable good of value to consumers.

[14]Put differently, the contest prize does not include access to the "blueprints" to produce the foreign intermediate good. Our strategy of focusing on nationally differentiated goods that could be traded in world markets conforms to Armington (1969) and is adopted primarily for technical reasons—essentially to bypass some complications related to discontinuities in best-response functions—without altering our main results significantly (relative to, say, a more general Ricardian type model).

[15]This functional form, first introduced by Tullock (1980), belongs to a more general class of contest success functions (CSFs), $\phi^i(G^i, G^j) = h(G^i)/\sum_j h(G^j)$ where $h(\cdot)$ is a non-negative and increasing function, axiomatized by Skaperdas (1996). See Hirshleifer (1989), who explores the properties of two important functional forms of this class, including the "ratio success function" where $h(G) = G^b$ with $b \in (0, 1]$. Though the results to follow remain qualitatively unchanged under this more general specification, we use the specification in (1), assuming $b = 1$ for simplicity (and, for our analysis of unilateral deviations from settlement, for tractability).

and second periods. That this destructive effect persists beyond the period of conflict could be viewed as permanent damage to each country's technological apparatus/infrastructure, which reduces their effectiveness to transform the resource secured in the conflict into the intermediate input.[16] In any case, for given arming choices, such destruction tends to detract from the relative appeal of conflict. Second, open conflict in the first period destroys opportunities for trade in that period and the next. This assumption, which is clearly extreme, also tends to detract from the relative appeal of conflict. Our rationale for imposing it here is to capture a salient feature of the liberal peace argument, that greater potential gains from trade imply a larger opportunity cost of conflict, thereby making it more likely that countries will opt for peaceful settlement. Finally, open conflict in the first period confers a strategic advantage on the victor in future disputes. In particular, the winner of war in the first period takes control not only of the contested pool after destruction $\beta\bar{X}$, but also of the resource that survives destruction $\beta\bar{R}$ in the next period and without having to arm at that time.[17] This last feature of open conflict is clearly extreme as well. However, it provides a useful benchmark that highlights the potential benefits of conflict under complete information.[18]

The benefits of arming under peaceful settlement ("armed peace") derive from arming's effect to enhance a country's fallback (or "threat point") payoff, thereby enabling it to secure a larger share of the contested pool $\bar{X}$ in the negotiations. A rationale for "cooperation" arises here because, for any given quantities of guns countries produce within a time period, negotiation and settlement (i) support bilateral trade and the payoff gains it yields, and (ii) avoid the destructive consequences of open conflict. Nonetheless, peaceful settlement is costly. Specifically, as each side tries to leverage its bargaining position by arming, it reduces what resources left for the production and trade of commodities and, thus, reduces the size of the bargaining set, as in the one-period settings of Skaperdas and Syropoulos (2002) and Garfinkel and Syropoulos (2018).[19] Additionally, under the reasonable assumption that countries cannot enter into binding commitments regarding the future division of the resource when they settle in the first period, the dispute between the two countries reemerges in the next period, and more arming could be called for at that time.

[16] The analysis could be extended to entertain other types of destruction that would influence arming decisions (e.g., conflict could destroy resources at different rates depending on the time period considered) without altering the key insights of our analysis. However, allowing for the possibility that war's destructive effects are greater for the defeated side could have more substantive implications for each country's decision between war and peace. (See Garfinkel and Syropoulos (2019) who allow for such differential destruction, finding that war could be the equilibrium outcome even in a one-period setting.)

[17] What we have in mind is that defeat in conflict undermines the losing side's capacity, organization, and possibly even its will to enter conflict in the future. Put differently, one could view conflict as crippling the losing side's ability to challenge the victor in future conflict. In Section 5, we discuss possible modifications to this assumption that leave our central results intact.

[18] See Fearon (1995), Garfinkel and Skaperdas (2000), Powell (2006) and Skaperdas and McBride (2014).

[19] The endogeneity of the bargaining set also arises in settings without trade (see Anbarci et al., 2002).

Indeed, since conflict involves arming only in the first period while settlement need not eliminate the incentive to arm in either period, conflict could dominate settlement in terms of payoffs. Even if settlement delivers higher *ex ante* payoffs, one or both sides could find it optimal to deviate from this outcome unilaterally. We aim to identify the conditions under which settlement is stable (i.e., immune to coalitional and unilateral deviations that produce conflict). Our focus on sequential "coalition proof" equilibria seems appropriate in this context, because negotiations involve communication between the two countries. However, before turning to that analysis, we must specify the economic environment, including production and possible trade that play a prominent role in shaping the countries' preferences over open conflict and settlement.

2.2 Production and Possible Trade

With the resources secured in the resolution of the dispute, country i produces on a one-to-one basis Z^i units of its distinct intermediate input. For ease of exposition, we refer to this quantity as country i's "effective endowment," which can in turn be used to produce a consumption good. In the case that the dispute is settled peacefully, Z^i is tradable, allowing each country i to employ the inputs produced by both countries. All markets are perfectly competitive, and the final good in each country i is produced according to the following constant elasticity of substitution (CES) technology:

$$F\left(D_1^i, D_2^i\right) = \left[\sum\nolimits_{j=1,2} (D_j^i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where D_j^i denotes the quantity of intermediate good $j \in \{1,2\}$ demanded and employed in country $i \in \{1,2\}$ and $\sigma > 1$ is the elasticity of substitution between inputs.

In this model, as in standard trade models, the gains from trade derive from the imperfect substitutability between intermediate inputs (i.e., $\sigma < \infty$). Our assumption that $\sigma > 1$, which is needed to ensure that autarky payoffs depend on arming, plays a role similar to the one in models of monopolistic competition, reflecting the value of diversity in productive inputs. Absent trade between the two countries (perhaps because war emerges between them), $D_j^i = 0$ holds for $i \neq j = 1,2$, implying that each country i produces $F(\cdot) = Z^i$ units of the final good that yield the following payoff:

$$w_A^i = Z^i, \;\; i = 1,2, \tag{3}$$

where the subscript "A" identifies the autarkic regime.

Turning to the possibility of trade, let Y^i and p_j^i respectively denote country i's income and domestic price for good j in any given time period. Then, as one can verify, country i's expenditure or cost share on good j is given by $\gamma_j^i \equiv (p_j^i/P^i)^{1-\sigma}$, where $P^i \equiv [\sum_j (p_j^i)^{1-\sigma}]^{\frac{1}{1-\sigma}}$

is the price index in country i. In turn, the technology in (2) implies that the demand for good $j=1,2$ in country i is given by $D_j^i = \gamma_j^i Y^i / p_j^i$. Thus, country i's indirect payoff is

$$w^i = Y^i / P^i, \ \ i = 1, 2, \tag{4}$$

where $Y^i = p_i^i Z^i$.

Trade of intermediate goods takes place in the presence of "iceberg" type trade costs, reflecting geographic trade barriers. In particular, for each unit of the intermediate input j that country i imports, $\tau - 1$ units "melt" or "shrink" in transit, so that $\tau \geq 1$ must be shipped by its trading partner j $(\neq i = 1, 2)$.[20] Let π_j be the "world" price of good $j = 1, 2$. Then $\pi^i \equiv \pi_j / \pi_i$ and $p^i \equiv p_j^i / p_i^i$ are the world and domestic relative prices of country i's importable, respectively. Competitive pricing and arbitrage imply these prices satisfy $p^i = \tau \pi^i$ and, naturally, are endogenously determined through a world market-clearing condition. This condition requires the value of country i's imports to be equal to the value of country j's exports (appropriately adjusted to take into account the "shrinkage" in transit); that is, $\tau \pi_j D_j^i = \tau \pi_i D_i^j$ $(i \neq j = 1, 2)$. Applying the forms of the demand functions derived earlier and the fact that $Y^i = p_i^i Z^i$, we rewrite this condition as

$$\pi^i = \gamma_j^i Z^i / \gamma_i^j Z^j, \tag{5}$$

where γ_j^i can be rewritten as $\gamma_j^i = (\tau\pi^i)^{1-\sigma} / \left[1 + (\tau\pi^i)^{1-\sigma}\right]$ for $i \neq j = 1, 2$. The (implicit) solution to (5), denoted by π_T^i where "T" indicates the outcome under trade, is the relative price of country i's importable that clears the world market.[21]

Next, define $\mu^i(\cdot) \equiv \left[1 + (\tau\pi_T^i)^{1-\sigma}\right]^{-\frac{1}{1-\sigma}}$ $(= p_i^i / P^i)$. Substituting this definition (together with $Y^i = p_i^i Z^i$) back in (4) enables us to obtain the following expression for country i's payoff under trade:

$$w_T^i = \mu^i(\cdot) Z^i, \ \ i = 1, 2. \tag{6}$$

As is the case under autarky, a country's payoff under trade depends on its capacity to produce intermediate good Z^i. However, because $\mu^i(\cdot)$ depends on the world market clearing price π_T^i and this price depends on (Z^i, Z^j), the payoff w_T^i now is a function of both countries' output levels (Z^i, Z^j).[22] A comparison of w_T^i in (6) with w_A^i in (3) reveals that country i's gains from trade (given both countries' arming choices and thus Z^i) in relative

[20] The analysis could be extended to consider the possible use of import tariffs. The analysis could also be extended to allow for asymmetric trade costs. We abstract from both possibilities here for simplicity.

[21] Inspection of (5) reveals that π_T^i depends on $\left(Z^i, Z^j\right)$, σ, and τ. It is easy to confirm that this solution exists and is unique. Furthermore, one can verify that, in the special case of unimpeded trade (where $\tau = 1$), the world market clearing price is given by $\pi_T^i = (Z^i / Z^j)^{1/\sigma}$.

[22] We describe how π_T^i, w_T^i and μ^i depend on all variables of interest below, relegating formal details to the Appendix.

terms are captured by $\mu^i(\cdot) > 1$, reflecting the diversity of distinct inputs in the production of the final good (i.e., $\sigma < \infty$). As discussed in Section 3.2, these gains are endogenously determined, decreasing in both the degree of similarity between traded goods ($\sigma \uparrow$) and trade costs ($\tau \uparrow$), with $\lim_{\sigma\to\infty} w_T^i = \lim_{\tau\to\infty} w_T^i = w_A^i$.[23]

2.3 Sequence of actions

Let λ^i be the fraction of the common pool $\bar{X}$ that state i would obtain under settlement in a given period. The sequence of actions in period $t = 1$ is as follows:

Stage 1. Each state i chooses G^i ($\leq R^i$), treating its rival's decision G^j ($j \neq i$) as given.

Stage 2. The two countries enter into negotiations about how to divide the contested pool, $\bar{X} = \sum_i (R^i - G^i) = \bar{R} - \bar{G} \geq 0$, in the current period.

- 2a. If both states agree on a division, $\bar{X}$ is distributed accordingly. Country i's effective endowment becomes $Z^i = \lambda^i \bar{X}$.
- 2b. If negotiations fail, the two sides enter into open conflict over $\bar{X}$. The effective endowments are $Z_W = \beta\bar{X}$ for the winner and $Z_L = 0$ for the loser.

Stage 3. If the two sides agree to settle their claims and no deviation from the agreement is recorded, the contenders engage in competitive trade of their intermediate goods, Z^i. Conflict and deviations from settlement foreclose on current and future trade.

What happens in period $t = 2$ depends on the outcome of the two countries' interactions in period $t = 1$. If war breaks out in period $t = 1$, the defeated side is no longer in contention. Thus, there is no arming in period $t = 2$, and the winner enjoys the stream of benefits associated with controlling $\beta\bar{R}$ units of the services of the primary resource at that time. If peaceful settlement arises in period $t = 1$, the three stages specified above are repeated in period $t = 2$. For reasons that will become apparent below, conflict can be part of the equilibrium only in period $t = 1$; that is, settlement in period $t = 1$ always leads to settlement in period $t = 2$.

3 Equilibria under Conflict and Settlement

Having specified the model's essential elements, we now go on to describe the countries' expected payoff functions when interactions occur over the two-period time horizon. We then describe countries' arming incentives and decisions both under war and peace to prepare the groundwork for our subsequent analysis of equilibria of the extended game.

[23] See Arkolakis et al. (2012) who discuss the importance of σ in this and other trade models. The analysis could be extended to allow each country to produce both tradable intermediate inputs (through differential access to the relevant technologies, as in standard Ricardian type trade models). It could also be extended to allow for the possibility of trading a fixed number of differentiated goods or an endogensouly determined number of varieties, as in Krugman (1980). Neither extension would change the key insights of our analysis.

3.1 Open Conflict

Let $u^i\left(G^i,G^j\right)$ be country i's expected one-period payoff function under open conflict in the first period. Since conflict destroys any trading opportunities, equation (3) implies that country i's payoff contingent on the outcome of the war is linear in its effective resource endowment or intermediate good output level, $Z_W \equiv \beta\bar{X} = \beta(\bar{R}-\bar{G}) \geq 0$ in the case of victory and $Z_L = 0$ in the case of defeat. Thus, country i's expected current-period payoff u^i can be written as follows:

$$u^i \equiv u^i(G^i,G^j) = \phi^i Z_W^i + \left(1-\phi^i\right) Z_L^i = \phi^i\beta\bar{X},\ i \neq j = 1,2. \tag{7}$$

The dependence of this payoff on arming by both countries operates through the probability of winning ϕ^i and through the determination of the common pool being contested $\bar{X}$.[24]

Now let $\delta \in [0,1]$ denote the countries' (common) discount factor and U^i denote country i's expected discounted lifetime payoff under overt conflict divided by $1+\delta$, which we refer to as its "average" payoff under conflict. Since country i controls $\beta\bar{R}$ with probability ϕ^i and gets nothing with probability $1-\phi^i$ in period $t=2$, its average payoff is

$$U^i \equiv U^i(G^i,G^j) = \frac{1}{1+\delta}\left[u^i(G^i,G^j) + \phi^i\beta\delta\bar{R}\right],\ i \neq j = 1,2.$$

Using (7) in the above equation and rearranging terms gives

$$U^i = \frac{\beta}{1+\delta}\phi^i\left(\bar{X}+\delta\bar{R}\right),\ \ i \neq j = 1,2, \tag{8}$$

where $\bar{X} = \bar{R}-\bar{G}$.

3.1.1 Incentives to Arm under Open Conflict

The extent to which each country i arms in period $t=1$, in anticipation of conflict, depends on the solution to $\max_{G^i} U^i$, s.t., $X^i = R^i - G^i \geq 0$ for $i = 1,2$. Differentiation of country i's expected payoff U^i in (8) with respect to G^i gives:

$$U^i_{G^i} = \frac{\beta}{1+\delta}\left[\phi^i_{G^i}\left(\bar{X}+\delta\bar{R}\right) - \phi^i\right],\ \ i = 1,2, \tag{9}$$

The first term inside the square brackets of (9) (multiplied by $\beta/(1+\delta)$) represents country i's average discounted marginal benefit to arming. This benefit captures the effect of a marginal increase in G^i to improve country i's probability of winning the war and controlling the output stream of $\beta\bar{X}$ and $\beta\bar{R}$. The second term (multiplied by $\beta/(1+\delta)$) represents country i's opportunity cost of arming due to the reduction in the size of the pool $\bar{X}$.

[24]Note that, if in period $t=1$ the countries settled their resource dispute peacefully, it is this payoff that each country would consider when choosing between war and peace in period $t=2$.

Inspection of the expression inside the square brackets of (9) reveals that (i) an increase in the aggregate initial resource ($\bar{R}$) that implies a larger prize and (ii) a stronger shadow of the future (δ) that increases the future valuation of that prize each augment the net marginal benefit to arming. By contrast, an increase in the destructiveness of open conflict ($\beta \downarrow$) has no impact on this marginal condition. Finally, observe that an increase in the rival's guns G^j influences the net marginal benefit of arming to country i through the conflict technology ϕ^i and the current-period prize $\bar{X}$.

To proceed, let $B_c^i\left(G^j;\cdot\right)$ denote country i's best reply to $G^j > 0$ ($j \neq i$) under conflict. One can easily verify that the first-order condition (FOC) implied by (9) together with the resource constraint that possibly binds in country i's arming choice imply the following best-response functions:

$$B_c^i\left(G^j;\delta,R^i,\bar{R}\right) = \min\{R^i, \widetilde{B}_c^i(G^j)\}, \quad i \neq j = 1,2, \tag{10a}$$

where $\widetilde{B}_c^i(G^j)$ is country i's *unconstrained* best-response function[25] that solves $U_{G^i}^i = 0$:

$$\widetilde{B}_c^i(G^j) \equiv -G^j + \sqrt{(1+\delta)\,\bar{R}G^j}. \tag{10b}$$

The expressions in (10) reveal the importance of the opponent's arming G^j, the aggregate quantity of the initial resource $\bar{R}$, its distribution (R^i, R^j), and the strength of the shadow of the future δ in jointly determining the shape of country i's best-response function $B_c^i(G^j)$. Inspection of (10b), in particular, reveals that the country i's incentive to arm in anticipation of open conflict depends positively on its rival's choice G^j when $\widetilde{B}_c^i(G^j) > G^j$ and negatively so when $\widetilde{B}_c^i(G^j) < G^j$.[26] Consistent with our discussion regarding (9), increases in $\bar{R}$ and in the strength of the shadow of the future δ augment country i's arming incentives. Of course, changes in these parameters need not translate into changes in equilibrium arming. Also relevant here are the countries' resource constraints and thus the distribution of initial ownership claims to $\bar{R}$.

Denote the quantity of guns country i produces in this equilibrium by G_c^i, and define the following

$$R_L^c \equiv \left[1 - \tfrac{1}{2}(1-\delta)\right]\tfrac{1}{2}\bar{R} \;\text{ and }\; R_H^c \equiv \left[1 + \tfrac{1}{2}(1-\delta)\right]\tfrac{1}{2}\bar{R}, \tag{11}$$

where "L" ("H") identifies the "low" ("high") endowment threshold in anticipation of conflict (c) that together determine the parameter space for which neither country is resource constrained in its arming choice. Clearly, $R_H^c + R_L^c = \bar{R}$ and $R_H^c - R_L^c = (1-\delta)\frac{1}{2}\bar{R} \geq 0$

[25] Here and below, to limit notational cluttering, we suppress the dependence the best-response function on resources and the other parameters of the model.

[26] One can verify that the slope of $\widetilde{B}_c^i(G^j)$ where $\widetilde{B}_c^i(G^j) = G^j$ equals 0 since $d\widetilde{B}_c^i/dG^j = -U_{G^iG^j}^i/U_{G^iG^i}^i$ and, by (1) with (9), $U_{G^iG^j}^i|_{G^i=G^j} = 0$.

for $\delta \leq 1$. Using the properties of $B_c^i\left(G^j;\cdot\right)$ implied by (10), together with the aggregate resource constraint $R^i + R^j = \bar{R}$ and (11), leads to the following characterization of equilibrium security policies when open conflict is anticipated in the second stage:

Proposition 1 (Arming under open conflict.) *Under open conflict, there exists a unique equilibrium, with strictly positive arming by both contenders: $G_c^i > 0$ for $i = 1, 2$. Given any $\bar{R}$ such that $R^i + R^j = \bar{R}$, equilibrium arming decisions are independent of conflict's rate of destruction $(1-\beta)$, but depend on the distribution of $\bar{R}$ across the two countries and the rate of discount (δ) as follows:*

(a) *If $R^i \in [R_L^c, R_H^c]$ for $i = 1$ (and thus for $i = 2$), then $G_c^i = R_L^c(\delta)$ for $i = 1, 2$.*

(b) *If $R^i \in (0, R_L^c]$ for $i = 1$ or 2, then $G_c^i = R^i < G_c^j = \widetilde{B}_c^j\left(R^i, \delta\right)$ for $j \neq i$.*

(c) *$dG_c^i/d\delta > 0$ for $R^i \in (R_L^c, \bar{R})$ and $d(R_H^c - R_L^c)/d\delta < 0$ with $\lim_{\delta\to 1} R_L^c = \lim_{\delta\to 1} R_H^c = \bar{R}/2$.*

This proposition establishes that the equilibrium outcome under open conflict involves strictly positive arming by both countries. Furthermore, an uneven ownership of initial claims to $\bar{R}$ across the two countries matters only insofar as that distribution implies one country is constrained in its production of guns.[27] Specifically, part (a) shows that, when the configuration of initial asset ownership is sufficiently even (i.e., $R^i \in [R_L^c, R_H^c]$ for $i = 1, 2$), the two countries produce an identical amount of guns (i.e., $G_c^i = G_c = R_L^c$ for $i = 1, 2$), and that quantity is invariant to changes in the initial distribution of $\bar{R}$.[28]

However, as shown in part (b), when the configuration of initial ownership claims is sufficiently uneven (i.e., $R^i \in (0, R_L^c]$), the smaller country i's guns choice is constrained by its resource endowment: $G_c^i = R^i$; at the same time, the larger country (j) continues to operate on its unconstrained best-response function shown in (10b), and generally arms by more than its smaller adversary. In such cases, a redistribution of initial resources from the larger country j to the smaller country i relaxes country i's resource constraint, causing it to increase its arming one-for-one with the increase in R^i. The decrease in the larger country's initial resource (R^j) has no direct effect on its arming choice; however, by the strategic complementarity of its best-response function (i.e., $\partial B_c^j(G^i)/\partial G^i > 0$ when $B_c^j(G^i) > G^i$), country j increases its arming in response to country i's more aggressive security policy. As a result, a redistribution of initial resource endowments towards the smaller country results in a new equilibrium outcome where both countries arm by more. An implication here is that aggregate arming under conflict $\bar{G}_c = G_c^i + G_c^j$ is maximized when neither country

[27] Observe from the definition of R_L^c in (11), at most one country can be resource constrained.

[28] That the equilibrium in guns is symmetric ($G_c^i = G_c^j$), even when the contenders have (mildly) uneven resources initially, might seem surprising. However, the result follows from the assumption that they contest the same prize ($\bar{X} + \delta\bar{R}$) and the symmetric specification of ϕ^i in (1), implying that $U_{G^i}^i$ shown in (9) can be equal to zero for both countries only if $G^i = G^j$.

is resource constrained: $\bar{G}_c = 2R_L^c$. By contrast, a redistribution of $\bar{R}$ from the smaller (constrained) country i to the larger country j implies less equilibrium arming by both and thus less aggregate arming: $\bar{G}_c < 2R_L^c$.

Part (c) shows that, while the rate of conflict's destruction $(1-\beta)$ has no effect on equilibrium arming, an increase in the strength of the shadow of the future (δ) induces greater arming by countries that are not resource constrained, as it increases the value of the contest prize. However, since the aggregate quantity of the initial resource $\bar{R}$ remains unchanged, an increase in δ shrinks the range of initial resource allocations for which both countries are unconstrained, collapsing to a single point at $R_L^c = R_H^c = \bar{R}/2$ as $\delta \to 1$; at this limit, the dispute over ownership claims results in the full dissipation of total productive resources (i.e., $\bar{X} = 0$). These results are illustrated in Fig. 1(a), which shows country i's equilibrium arming as a function of the distribution of initial resource ownership for alternative values of δ.

3.1.2 Equilibrium Payoffs under Open Conflict

Building on our characterization of equilibrium arming choices when countries anticipate open conflict, we now examine how their corresponding equilibrium payoffs, U_c^i $(i = 1, 2)$, depend on the distribution of initial resource ownership (R^i, R^j). When evaluating the effects of exogenous changes in this initial distribution on country i's payoff, it is important to account not only for the direct effects, but also the possible indirect effects that operate through the conflict technology ϕ^i as they can induce changes in the equilibrium choices, G_c^i and G_c^j. Of course, by the envelope theorem, when country i's resource constraint is not binding, the effect of a change in its own arming G^i on its own payoff U_c^i vanishes; otherwise, exogenous changes in the parameters that enable country i to move closer to its unconstrained optimum improve its payoff. In contrast, a change in country i's rival arming G_c^j always adversely affects its payoff U_c^i, first by reducing its probability of winning the war ϕ^i and second by reducing the overall size of the common resource pool $\bar{X}$.

The next proposition shows how the just described indirect effects of arming decisions combine with the direct effects of changes in conflict's rate of destruction, the shadow of the future and the distribution of initial resource ownership to influence average payoffs U_c^i.

Proposition 2 (Payoffs under open conflict.) *For all $R^i \in (0, \bar{R})$, equilibrium payoffs under open conflict are decreasing in conflict's rate of destruction $(1-\beta)$. The payoff effects of changes in the shadow of the future (δ) and in the distribution of initial resource ownership (R^i, R^j) depend on whether one of the country is resource constrained in its arming decision:*

(a) *If $R^i \in [R_L^c, R_H^c]$ for $i = 1$ or 2, then for $i = 1, 2$, $U_c^i = \beta\frac{\bar{R}}{4}$ and*

  (i) $dU_c^i/dR^i = 0$

  (ii) $dU_c^i/d\delta = 0$.

(b) *If* $R^i \in (0, R_L^c]$ *for* $i = 1$ *or* 2, *then*

(i) $dU_c^i/dR^i > 0$, $d^2U_c^i/(dR^i)^2 < 0$ *and* $\lim_{R^i \to 0} U_c^i = 0$, *whereas* $dU_c^j/dR^j > 0$, $d^2U_c^j/(dR^j)^2 > 0$ *and* $\lim_{R^j \to \bar{R}} U_c^j = \beta \bar{R}$

(ii) $dU_c^i/d\delta < 0$, *while* $dU_c^j/d\delta > 0$.

Since arming is independent of β, changes in that parameter influence payoffs of both countries only directly, and positively so. Fig. 1(b) illustrates the remaining parts of the proposition.

The intuition for part (a.i), which characterizes the countries' payoffs in anticipation of conflict in the case where the initial distribution is sufficiently even, follows from Proposition 1(a) and equation (8). Specifically, in this benchmark case, since countries arm identically, their payoffs are identical; similarly, since their arming choices are invariant to any reallocation of the initial resource (R^i, R^j) within $[R_L^c, R_H^c]$, so too are their payoffs.

The intuition behind part (b.i), which taken as a whole implies that the unconstrained country's payoff is greater than that of the constrained country (i.e., $U_c^j > U_c^i$), can be fleshed out by studying the effects of resource redistributions outside the range of $[R_L^c, R_H^c]$, using Proposition 1(b). When country i's resource constraint binds in the production of guns such that $G^i = R^i$, an exogenous shift in the total resource towards that country relaxes its resource constraint, thereby inducing it to arm more and adding to its payoff (i.e., since $U_{G^i}^i > 0$). As previously described, the larger (and unconstrained) opponent j ($\neq i$) responds by adopting a more aggressive stance in its security policy and that has a negative effect on the smaller country's payoff. In the Appendix, we show that the positive payoff effect due to increases in the smaller country's own arming $G^i = R^i$ dominates the adverse effect due to increases in the arming of its larger rival G^j, with the net marginal effect falling as R^i rises. By contrast, an exogenous shift in the total resource $\bar{R}$ towards the larger and unconstrained country j has no direct effect on that country's payoff. Furthermore, by the envelope theorem, the indirect effect on its payoff due to changes in its own arming G^j vanishes. However, the smaller opponent i ($\neq j$) behaves less aggressively as its resource endowment falls, and that indirect effect improves country j's payoff. Since, in this case, G^i falls faster than G^j, country j's payoff rises at an increasing rate.

Parts (a.ii) and (b.ii) of the proposition show that the full impact of an increase in the discount factor δ on a country's average payoff also depends on whether the country's resource constraint on its arming decision is binding or not. Of course, as can be seen from (8), for given arms and thus given $\bar{X}$, an increase in δ has a direct effect to increase each country's payoff. But, as noted in part (c) of Proposition 1, a stronger shadow of the future ($\delta \uparrow$) fuels the arming incentives of an unconstrained country which imparts a negative indirect effect on the opponent. Part (a.ii) shows that, if both countries are unconstrained, the direct and indirect (strategic) effects perfectly offset each other, such that an increase in δ leaves both countries' payoffs unchanged. Turning to part (b.ii), an increase in δ that induces (unconstrained) country j to take a more aggressive stance in its arming policy generates an adverse indirect effect on country i's payoff. In the Appendix, we show that the indirect effect of an increase in δ on the constrained country's payoff U_c^i dominates the direct effect, thus implying $dU_c^i/d\delta < 0$. By contrast, since country i's arming remains unchanged, an increase in δ has no indirect effects on the unconstrained country's (j's) payoff; only the positive direct effect matters, thus implying $dU_c^j/d\delta > 0$.

3.2 Peaceful Settlement

The outcome when countries settle their resource dispute peacefully is considerably more complex, since it also involves trade. We start by studying how the outcome and thus payoffs under trade depend on guns (G^i, G^j) and an arbitrary division of a given $\bar{X}$. Such a preliminary analysis allows us to characterize the condition that must be satisfied for that division and guns under a particular bargaining protocol—namely, one that splits the current-period surplus.[29] With that characterization, we then can study the countries' arming incentives and equilibrium payoffs in anticipation of peaceful settlement.

3.2.1 Splitting the Surplus

Let λ^i be an arbitrary division of the total residual resource $\bar{X}$, so that $Z^i = \lambda^i \bar{X}$, and let $\omega^i \equiv \mu^i \lambda^i$, where as previously defined $\mu^i \equiv \left[1 + (\tau \pi_T^i)^{1-\sigma}\right]^{-\frac{1}{1-\sigma}}$ measures the relative gains from trade for country i and $\pi_T^i = \pi_T^i(\lambda^i)$ represents the world relative price of country i's importable that clears global markets (5).[30] Using this notation, we rewrite country i's one-period payoff under settlement $w_T^i = \mu^i Z^i$, shown in (6), as

$$v^i \equiv v^i(\lambda^i, \bar{G}; \tau, \sigma) = \omega^i \bar{X}, \quad i \neq j = 1, 2. \tag{12}$$

This payoff depends on the division λ^i through ω^i and on guns through $\bar{X}$.[31]

However, the countries' arming choices in the first stage also influence payoffs under settlement indirectly through the negotiation process that determines λ^i. To see how this works under the "split-the-surplus" rule of division, suppose that the countries resolved

[29] Our focus on an even split of the current-period surplus instead of the current and future surplus helps to maintain tractability. This focus possibly induces a slight bias in the overall preference for fighting, but without changing our key insights. Also, see Garfinkel and Syropoulos (2018) who compare the efficiency properties of several prominent division rules, including rules based on splitting-the-surplus, Nash bargaining and equal sacrifice, in a setting with trade similar to the present paper, but just one period.

[30] The dependence of π_T^i on λ^i, fully described in Lemma A.1 in the Appendix, follows from (5) where $\pi_T^i = \gamma_j^i Z^i / \gamma_i^j Z^j = \gamma_j^i \lambda^i / \gamma_i^j \lambda^j$ and $\gamma_j^i = \gamma_j^i(p^i)$ where $p^i = \tau \pi_T^i$. (Note that π_T^i does not depend directly on guns.) Lemma A.2 in the Appendix then goes on to describe the dependence of γ_j^i on λ^i. Lemma A.3 shows the importance of σ and τ in shaping π_T^i.

[31] The properties of ω^i, established in Lemma A.4, are discussed in some detail below. Lemma A.5 compares $w_T^i = \omega^i \bar{X} = \mu^i \lambda^i \bar{X}$ and $w_A^i = \lambda^i \bar{X}$ for given guns (and thus $\bar{X}$) and an identical division of that resource λ^i to highlight how the gains from trade alone to country i depend on λ^i.

their resource dispute in period $t = 1$ peacefully, and consider period $t = 2$ after they have made their arming choices (G^i, G^j).[32] The surplus in that period, denoted by S, is defined as the payoff gains the two countries could realize jointly from peaceful settlement over open conflict: $S \equiv v^i + v^j - u^i - u^j$. Given the first-stage arming choices (G^i, G^j), with $\bar{G} < \bar{R}$ so that $\bar{X} > 0$, a positive surplus exists due to the avoidance of open conflict's destructive effects and the gains from trade.[33]

The split-the-surplus division defines λ^i in that period implicitly as the solution to $(v^i - u^i) - (v^j - u^j) = 0$, given (G^i, G^j). Using (7) and (12) and dividing both sides by $\bar{X}$ (> 0), that condition can be rewritten as

$$\Psi^i = \Psi(\lambda^i, G^i, G^j; \sigma, \tau, \beta) \equiv \omega^i - \omega^j - \beta(\phi^i - \phi^j) = 0, \;\; i \neq j = 1, 2. \tag{13}$$

As (13) reveals, the guns each side i brings to the negotiation table (G^i) given the rival's guns (G^j) influences its relative bargaining position through $\phi^i - \phi^j$, thereby influencing its share under settlement. Noting that λ^i is an argument in ω^i and ω^j, assume for now $\lambda^i = \lambda^i(G^i, G^j)$ is a unique solution to (13).[34]

The problem facing country i in period $t = 2$ can now be described as follows:

$$\max_{G^i} v^i(\lambda^i, \bar{G}; \cdot), \;\; \text{s.t. } G^i \in [0, R^i], \; i \neq j = 1, 2. \tag{14}$$

Let G^i_s and v^i_s ("s" for "settlement") denote country i's $(= 1, 2)$ equilibrium arming and its associated payoff, respectively.

Turning to period $t = 1$ decisions and supposing they are made in anticipation of settlement that period, country i chooses G^i to maximize

$$V^i = \frac{1}{1+\delta} \left[v^i \left(G^i, G^j, \lambda^i(G^i, G^j; \cdot); \cdot \right) + \delta \max \left(v^i_s, u^i_c \right) \right]$$

subject to $G^i \in [0, R^i]$ and $u^i_c = U^i_c|_{\delta=0}$. But, the presence of gains from trade under peace and/or the presence of destruction under war only imply $v^i_s = \max \left(v^i_s, u^i_c \right)$.[35] Thus, due to stationarity, the solution to the 2-period problem under settlement is identical to the one described in (14) in period $t = 2$. Since this implies $V^i_s = v^i_s$ for $i = 1, 2$, it suffices to examine the equilibrium of the stage game in (14).

[32]Recall that, under conflict in period $t = 1$, the defeated country effectively vanishes. Hence, settlement is feasible in period $t = 2$ only if it is preceded by settlement in period $t = 1$.

[33]The world gains from peace, per unit of $\bar{X}$ (> 0), are given by $\Omega \equiv S/\bar{X} = \omega^i + \omega^j - \beta$. Lemma A.6 shows that Ω is strictly concave in λ^i, with $\lim_{\lambda^i \to 0} \Omega = \lim_{\lambda^i \to 1} \Omega = 1 - \beta$, reaching a maximum at $\lambda^i = \frac{1}{2}$. The lemma shows further that Ω rises with increases in the degree of input heterogeneity $(\sigma \downarrow)$ and globalization $(\tau \downarrow)$ that jointly amplify the gains from trade through ω^i and ω^j. Ω also increases with increases in conflict's destructive effects $(\beta \downarrow)$, which of course can be avoided through peaceful settlement.

[34]Lemma 1 below addresses this issue.

[35]See also footnote 50.

Going back to the condition $\Psi^i = 0$ shown in (13), it is clear that λ^i depends on the payoffs ω^i and ω^j and the conflict technology ϕ^i in (1). But, as mentioned earlier, ω^i and ω^j themselves depend on λ^i. Keeping in mind that changes in λ^i are necessarily accompanied by changes in λ^j and conversely so that $d\lambda^j = -d\lambda^i$, differentiation of $\omega^i = \mu^i \lambda^i$ using (5) gives

$$\omega^i_{\lambda^i} = \mu^i \left(1 - \frac{\gamma^i_j/\lambda^j}{\Delta}\right),\ i \neq j = 1, 2, \tag{15a}$$

where

$$\Delta = 1 + (\sigma - 1)\,(\gamma^i_i + \gamma^j_j). \tag{15b}$$

Since $\sigma > 1$ by assumption, $\Delta > 1$ holds.[36] The first term inside the parentheses in (15a) (multiplied by μ^i) is the direct effect of shifting a fraction of the common pool to country i. This effect is positive because, at constant prices, an increase in λ^i expands country i's income. The second term (also multiplied by μ^i) is negative, because the expansion (contraction) of country i's (j's) resource base reduces the relative supply of i's (j's) importable (exportable), forcing the world relative price of country i's importable good π^i_T to rise and thereby adversely affecting country i's terms of trade.[37] As one can verify by differentiating (15a) with respect to λ^i, ω^i is strictly concave in λ^i, reaching a maximum at $\lambda^i_T \in (\frac{1}{2}, 1)$.[38]

Building on the properties of ω^i, Lemma A.7 presented in the Appendix characterizes the dependence of Ψ^i on λ^i, showing that Ψ^i is strictly quasiconcave (resp., quasiconvex) in λ^i for $\lambda^i \in [\frac{1}{2}, 1]$ (resp., $\lambda^i \in [0, \frac{1}{2}]$). In addition, this lemma shows that Ψ^i attains a unique maximum (resp., minimum) at $\lambda^i_{\max}$ (resp., $\lambda^i_{\min}$), such that $\lambda^i_{\max} = 1 - \lambda^i_{\min} \in (\frac{1}{2}, 1)$ for $\sigma - \tau < 1$, whereas $\lambda^i_{\max} = 1$ for $\sigma - \tau \geq 1$. In turn, we have the following:

Lemma 1 *Under the split-the-surplus sharing rule, there exist unique upper and lower bounds on the division of $\bar{X}$, labeled $\overline{\lambda}^i = \{\lambda^i \mid \Psi^i\big|_{\phi^i=1} = 0\}$ and $\underline{\lambda}^i = \{\lambda^i \mid \Psi^i\big|_{\phi^i=0} = 0\}$, such that $\overline{\lambda}^i + \underline{\lambda}^i = 1$ where $\overline{\lambda}^i \in (\frac{1}{2}, \lambda^i_{\max}]$. Notably, $\overline{\lambda}^i = \lambda^i_{\max} = 1$ only if $\sigma - \tau \geq 1$ and $\beta = 1$. Moreover, for any feasible (G^i, G^j), there is a unique division $\lambda^i = \{\lambda^i \mid \Psi^i(\lambda^i) = 0$ for $\phi^i \in (0, 1)\}$ in $[\underline{\lambda}^i, \overline{\lambda}^i]$ with the following properties:*

(a) *$\lambda^i = \lambda(G^i, G^j)$ is symmetric in G^i and G^j, so that $G^i \gtreqless G^j \Longrightarrow \lambda^i \gtreqless \lambda^j$.*

(b) $\lambda^i_{G^i} = -\frac{\Psi^i_{G^i}}{\Psi^i_{\lambda^i}} = \frac{2\beta\phi^i_{G^i}}{\omega^i_{\lambda^i} + \omega^j_{\lambda^j}} > 0$ *and* $\lambda^i_{G^j} = -\frac{\Psi^i_{G^j}}{\Psi^i_{\lambda^i}} = \frac{2\beta\phi^i_{G^j}}{\omega^i_{\lambda^i} + \omega^j_{\lambda^j}} < 0$.

[36] $\Delta > 0$ is the Marshall-Lerner condition for stability of equilibrium, given in this setting by $\Delta \equiv \varepsilon^i + \varepsilon^j - 1$, where $\varepsilon^i = -(\partial D^i_j/\partial p^i_j)/(D^i_j/p^i_j) = 1 + (\sigma - 1)\gamma^i_i$ for $i \neq j = 1, 2$ denotes country i's price-elasticity of imports.

[37] See Lemma A.1.

[38] See Lemma A.4, which also establishes that ω^i and ω^j are symmetric functions (of λ^i and λ^j) and describes the dependence of λ^i_T on the elasticity of substitution σ and trade costs τ.

(c) $\lambda^i_\beta = -\frac{\Psi^i_\beta}{\Psi^i_{\lambda^i}} = \frac{\phi^i - \phi^j}{\omega^i_{\lambda^i} + \omega^j_{\lambda^j}}$ *and* $\lambda^i_\xi = -\frac{\Psi^i_\xi}{\Psi^i_{\lambda^i}} = -\frac{\omega^i_\xi - \omega^j_\xi}{\omega^i_{\lambda^i} + \omega^j_{\lambda^j}}$ *for* $\xi \in \{\tau, \sigma\}$.

Lemma 1 shows that, under the split-the-surplus sharing rule, there is a unique division λ^i of $\bar{X}$ taking on a value in $[\underline{\lambda}^i, \overline{\lambda}^i]$, where the lower bound equals the share country i would receive if it chose $G^i = 0$ (with $G^j > 0$) and the upper bound equals the share it would receive if its rival chose $G^j = 0$ (with $G^i > 0$). Importantly, these bounds need not coincide with the corresponding winning probabilities under conflict given the countries' gun choices (i.e., respectively, $\phi^i = 0$ and $\phi^i = 1$), that would be relevant if negotiations were to break down. In particular, $[\underline{\lambda}^i, \overline{\lambda}^i] \subset [0, 1]$ if either there is destruction under conflict ($\beta < 1$) or, for given trade costs, the countries' traded goods are sufficiently dissimilar ($\sigma - \tau < 1$). This aspect of the split-the-surplus protocol is noteworthy because it implies that under settlement—in contrast to conflict—each country could secure a strictly positive fraction of the common pool, even when it does not arm. The symmetric structure of $\lambda^i = \lambda(\cdot)$ is due to the symmetric nature of $\Psi(\cdot)$ in (13). Parts (b) and (c), which also follow from the properties of $\Psi(\cdot)$, describe the dependence of λ^i on the countries' arming choices and the exogenous parameters $\xi \in \{\beta, \tau, \sigma\}$.

3.2.2 Incentives to Arm under Peaceful Settlement

With the results in Lemma 1, we now turn to study the countries' incentives to arm under peaceful settlement. Recognizing the dependence of λ^i on guns as shown in Lemma 1(b), differentiation of (12) with respect to G^i gives

$$v^i_{G^i} = \omega^i_{\lambda^i} \lambda^i_{G^i} \bar{X} - \omega^i = \frac{\omega^i_{\lambda^i}}{\omega^i_{\lambda^i} + \omega^j_{\lambda^j}} 2\beta \phi^i_{G^i} \bar{X} - \omega^i. \tag{16}$$

The second term in both equations on the RHS of (16) is country i's marginal cost of arming (MC^i). This cost is the payoff reduction country i suffers due to the reduction in the size of common pool $\bar{X}$ and coincides with its per unit of $\bar{X}$ payoff under peace. Since $MC^i \equiv \omega^i \equiv \mu^i \lambda^i$, the opportunity cost to arming varies in proportion to the country's relative gains from trade (μ^i) and its share (λ^i) of the common pool. The first term in both equations of the RHS of (16) represents country i's marginal benefit to arming (MB^i). This term reflects the effect of an increase in guns by country i to increase its production of the intermediate input (captured by $\lambda^i_{G^i} \bar{X}$) that, in turn, expands its income but also worsens its terms of trade (jointly captured by $\omega^i_{\lambda^i}$, shown in (15a)). Clearly, country i would never produce guns to the point where the deterioration in its terms of trade swamps the payoff increase due to the increase in its income, as that implies $\omega^i_{\lambda^i} < 0$ and thus $MB^i < 0$. Observe further that MB^i is increasing in the size of the common pool $\bar{X}$ and thus decreasing in $\bar{G}$. In addition, by (1), $\lim_{G^i \to 0} \phi^i_{G^i}$ is arbitrarily large for $G^j = \epsilon$

arbitrarily close to but greater than 0; at the same time, then, MB^i becomes arbitrarily large. Thus, as is true under open conflict, when country i's rival produces an arbitrarily small but positive quantity of guns, country i will produce a positive quantity of guns as well, to influence the division of the prize in its favor. This last point raises the following question: What is a country's optimal arming choice when its rival produces no guns?

We consider both possibilities, starting with the case that a country's rival produces a positive quantity of guns. Given $G^j > 0$, the extent to which country i arms under settlement, like under conflict, depends on the sensitivity of MB^i and MC^i to changes in guns G^i and on the distribution of initial resource ownership. In view of the possibility that a country's resource constraint can bind in the production of guns under settlement, we distinguish this case from the case where the country is not resource constrained again using a tilde "$\sim$" over variables and functions (e.g., $\widetilde{v}^i$ denotes country i's unconstrained per period payoff and $\widetilde{B}_s^i\left(G^j\right)$ its unconstrained best-response function).[39] In Lemma A.8, we establish some useful properties of $\widetilde{v}^i$ that ensure existence and uniqueness of equilibrium.

To proceed, suppose that neither country's arming is constrained by its initial resource endowment, which implies $\widetilde{v}^i_{G^i} = 0$ for $i = 1, 2$. Then, from the definitions of MB^i and MC^i and equations (16), (15a), and (1), the following must hold:

$$\frac{MB^i}{MB^j} = \frac{MC^i}{MC^j} \;\Rightarrow\; \frac{\omega^i_{\lambda^i}\phi^i_{G^i}}{\omega^j_{\lambda^j}\phi^j_{G^j}} = \frac{\omega^i}{\omega^j} \;\Rightarrow\; \left(\frac{G^j}{G^i}\right)\left[\frac{1-\frac{\gamma^i_j}{\lambda^j\Delta}}{1-\frac{\gamma^j_i}{\lambda^i\Delta}}\right] = \frac{\lambda^i}{\lambda^j},$$

for $i \neq j$. Now suppose, $G^j/G^i < 1$ for $i \neq j$. By Lemma 1(a), this supposition implies $\lambda^i/\lambda^j > 1$. Thus, for the last equality in the expression above to hold, the value of the expression inside the square brackets must exceed 1. Because π^i_T is increasing in λ^i and countries have symmetric technologies, $\pi^i_T > 1$ holds.[40] This inequality, in turn, implies (from the world market clearing condition (5)) $\frac{\lambda^i\gamma^i_j}{\lambda^j\gamma^j_i} > 1$, which can be rewritten as

$$\frac{\gamma^i_j}{\lambda^j} > \frac{\gamma^j_i}{\lambda^i} \;\Rightarrow\; \frac{\gamma^i_j}{\lambda^j\Delta} > \frac{\gamma^j_i}{\lambda^i\Delta} \;\Rightarrow\; \frac{1-\frac{\gamma^i_j}{\lambda^j\Delta}}{1-\frac{\gamma^j_i}{\lambda^i\Delta}} < 1,$$

thereby contradicting our supposition that $G^j/G^i < 1$. Since this argument also applies to the case of $G^j/G^i > 1$, the equilibrium in security policies must be symmetric (i.e., $G^i = G^j$) when both countries' arming decisions are unconstrained by their respective initial resources.

Maintaining our assumption that the resource constraint on guns binds for neither coun-

[39] Since a country's arming decision could be constrained by its initial resource endowment, its actual (possibly constrained) best reply function is $B^i_s\left(G^j; \sigma, \tau, \beta, R^i, \bar{R}\right) = \min\{R^i, \widetilde{B}^i_s(G^j)\}$ for $i = 1, 2$.

[40] See Lemma A.1(a).

try $i = 1, 2$, Lemma 1(a) implies that $\lambda^i = \lambda^j = \frac{1}{2}$. In turn, Lemma A.1(c) presented in the Appendix and based on the world market-clearing condition implies $\pi_T^i = 1$, while Lemma A.2(b) implies $\gamma_j^i = \gamma_i^j \leq \frac{1}{2}$ (with equality if $\tau = 1$). Then, we evaluate country i's relative gains from trade as follows: $\mu^i = \mu \equiv \left[1 + \tau^{1-\sigma}\right]^{-\frac{1}{1-\sigma}}$, $\omega^i = \mu/2$ and $\omega^i_{\lambda^i} = \omega^j_{\lambda^j}$ for $i \neq j = 1, 2$. Furthermore, since $G^i = G^j$, the conflict technology (1) implies $\phi^i_{G^i} = 1/2\bar{G}$ for $i = 1, 2$, where as previously defined $\bar{G} = G^i + G^j$. Applying these results to (16), simplifying the resulting expression, and setting it equal to 0 yield

$$\widetilde{v}^i_{G^i} = \tfrac{1}{2}\beta\left(\bar{X}/\bar{G} - m\right) = 0, \;\text{ where }\; m \equiv \mu/\beta \geq 1 \;\text{ for }\; i = 1, 2.$$

Since $\bar{X} = \bar{R} - \bar{G}$, we can solve for the symmetric solution as $G^i_s = G_s = \frac{\bar{R}/2}{1+m}$ for $i = 1, 2$. Clearly, G_s is independent of the shadow of the future δ; however, it is decreasing in the gains from trade and in the rate of destruction, jointly captured by $m = \mu/\beta$.

Moreover, resource constraints and thus the initial distribution of resources matter here. Define the following threshold values of resources under peaceful settlement:

$$R^s_L \equiv \tfrac{1}{2}\left[1 - \tfrac{m}{1+m}\right]\bar{R} \;\text{ and }\; R^s_H \equiv \tfrac{1}{2}\left[1 + \tfrac{m}{1+m}\right]\bar{R}. \tag{17}$$

Building on the ideas above along with these expressions, the next proposition characterizes the equilibrium in arming under settlement for the entire parameter space, including the possibility that neither country arms:

Proposition 3 (Arming under peaceful settlement.) *Under peaceful settlement, there exist combinations of parameter values for β, σ, and τ such that both countries choose not to arm at all: $G^i_s = 0$, for $i = 1, 2$. However, for all combinations, there also exists a unique equilibrium in which both countries arm: $0 < G^i_s \leq R^i$, with equality for at most one country. Equilibrium guns and shares in this case have the following properties:*

(a) *If $R^i \in [R^s_L, R^s_H]$, then $G^i_s = G_s = \frac{\bar{R}/2}{1+m} = R^s_L(m)$ for $i = 1, 2$, where $m \equiv \mu/\beta$, and $\lambda^i_s = \frac{1}{2}$.*

(b) *If $R^i \in (0, R^s_L)$ for $i = 1$ or 2, then $G^i_s = R^i$, $G^j_s = \widetilde{B}^j_s(R^i, \cdot)$, $\lambda^i_s \in (\underline{\lambda}^i, \frac{1}{2})$ and $\lambda^j_s = 1 - \lambda^i_s \in (\frac{1}{2}, \overline{\lambda}^i)$.*

(c) *$d\left(R^s_H - R^s_L\right)/dm > 0$, and if $R^i \in (0, R^s_L)$ for $i = 1$ or 2, then $dG^j_s/d\xi > 0$ for $\xi \in \{\sigma, \tau\}$ and $dG^j_s/d\beta > 0$ (resp., < 0) for R^i close to R^s_L (resp., 0).*

The anticipation of peaceful settlement gives rise to a unique equilibrium in arming with positive quantities of guns produced by both countries. As in the case where both countries anticipate open conflict, uneven factor ownership matters only when it causes one country's resource constraint to limit its production of guns. Specifically, part (a) shows that, when the distribution of initial resource ownership is sufficiently even, the resource constraint on arming in the first stage of period $t = 1$ binds for neither country and that implies they produce equal quantities of guns: $G_s^i = G_s = R_L^s$.[41] Part (b) shows, in contrast, if initial resource ownership is sufficiently uneven, the less endowed country specializes completely in the production of arms in period $t = 1$ whereas its more affluent adversary diversifies its production and, at the same, arms by relatively more. By the same logic in the case of open conflict, total equilibrium arming $\bar{G} = G_s^i + G_s^j$ rises with transfers of the resource to the smaller country, and is maximized at $\bar{G} = 2R_L^s$ when the distribution is sufficiently even.

How do the parameters related to the gains from trade (μ^i, which depends negatively on the elasticity of substitution σ and on barriers to trade τ) and the destructiveness of conflict $(1 - \beta)$ matter here? As one can easily see from the solution when neither country is resource constrained ($G_s^i = R_L^s = \frac{\bar{R}/2}{1+m}$), equilibrium arming is decreasing in $m \equiv \mu/\beta$ where $\mu = \mu^i$ is evaluated at $\lambda^i = \frac{1}{2}$. Thus, with a decrease in the gains from trade ($\mu \downarrow$ due to $\sigma \uparrow$ or $\tau \uparrow$) or a decrease in conflict's destructive effects ($\beta \uparrow$), equilibrium arming under settlement rises. Because such parameter changes generally amplify the incentive to arm and have no effect on $\bar{R}$, they naturally shrink the range of initial resource endowments for which neither country is resource constrained, as pointed out in the first component of Proposition 3(c). In the case that country i is resource constrained, decreases in the gains from trade or in the destructiveness of conflict do not influence its arming. However, as stated in the second component of part (c), the unconstrained country (j) responds to decreased gains from trade with higher arming. In addition, a decrease in the degree of conflict's destruction could decrease or increase the unconstrained country j's arming depending on whether the rival's resource R^i is close to R_L^s or close to zero. Fig. 2(a) illustrates the dependence of a country's arming under settlement on the distribution of initial resource ownership, and also shows how the size of the gains from trade (measured inversely by σ assuming $\beta = \tau = 1$) matters.[42]

Finally, let us turn to the outcome under settlement with no arming or "unarmed peace." For such an outcome to be a possible equilibrium (in addition to the one with arming), neither country i should have an incentive to produce a positive quantity of guns given the opponent chooses $G^j = 0$. If country i were to arm, it would do so by just an infinitesimal amount, since that is all that is needed to secure the maximum share of the contested pool, $\overline{\lambda}^i$. Thus, given $G^j = 0$, country i would be choosing between $\lambda^i = \frac{1}{2}$ (when $G^i = 0$) and $\lambda^i = \overline{\lambda}^i$ (when $G^i = \epsilon > 0$). And, for $G^i = G^j = 0$ to be a possible equilibrium given $\xi \in \{\sigma, \tau\}$, v^i evaluated at $\lambda^i = \frac{1}{2}$ and $\bar{G} = 0$ must be greater than v^i evaluated at $\lambda^i = \overline{\lambda}^i$

[41] Similar to our characterization of equilibrium arming under conflict, symmetry arises here due to the symmetric nature of the marginal benefits and costs of arming in this case. Interestingly, as suggested earlier (see footnote 33) and shown in Lemma A.6, this outcome is precisely the one a benevolent social planner would choose for any given $\bar{G} < \bar{R}$. The difference here, of course, is that arming is endogenously determined.

[42] Ignore the pink curve for now. As will become apparent shortly, assuming $\beta = \tau = 1$, $\delta = 0$ and $\sigma = \infty$ implies that $G_s^i = G_c^i$.

and $\bar{G} = \epsilon$. In the Appendix, we demonstrate that this ranking in payoffs is possible, in the case where war generates no destruction ($\beta = 1$) and there are no obstacles to trade under settlement ($\tau = 1$), but only when $\sigma \in [\frac{3}{2}, 2]$. The upper-limit restriction on σ requires that the gains from trade are sufficiently large.[43] However, the lower-limit restriction suggests that this outcome is fragile. A numerical analysis shows further that marginal increases in either the degree of conflict's destructiveness or trade costs dramatically shrinks the parameter space (in terms of σ) for which unarmed peace can be an equilibrium outcome.[44]

3.2.3 Equilibrium Payoffs under Peaceful Settlement

Let us now take a closer look at equilibrium payoffs under settlement. We focus on the payoffs that would arise if peace and settlement arose in both periods, in which case (as discussed earlier) a country's average discounted payoff V_s^i coincides with its per period (stationary) payoff v_s^i defined in (12). The next proposition summarizes the salient findings in this context.

Proposition 4 (Payoffs under peaceful settlement.) *In equilibria under peaceful settlement, with or without arming, a country's average discounted payoff V_s^i is independent of the shadow of the future, δ. In equilibria with no arming, $V_s^i = V_s^j = \mu\bar{R}/2$, which does not depend on the rate of destruction $1-\beta$, but does rise with decreases in the elasticity of substitution σ and trade costs τ that expand the gains from trade μ. In equilibria with positive arming ($G_s^i > 0$, $i = 1, 2$), V_s^i depends on initial resource ownership, the rate of conflict's destruction $1-\beta$, the elasticity of substitution σ and trade costs τ as follows:*

- (a) *If $R^i \in [R_L^s, R_H^s]$ for $i = 1, 2$, then $V_s^i = V_s^j = \frac{\beta m^2}{1+m}\bar{R}/2$, where $m \equiv \mu/\beta$ and $dV_s^i/d\xi < 0$ for $\xi \in \{\beta, \tau, \sigma\}$.*
- (b) *If $R^i \in (0, R_L^s)$ for $i = 1$ or 2, then $\lim_{R^i \nearrow R_L^s} dV_s^i/dR^i \leq 0$ and $\lim_{R^i \to 0} dV_s^i/dR^i > 0$, whereas $dV_s^j/dR^j > 0$ and $d^2V_s^j/(dR^j)^2 > 0$. In addition,*
  - (i) *$dV_s^i/d\beta < 0$ while $dV_s^i/d\xi < 0$ for $\xi \in \{\tau, \sigma\}$.*
  - (ii) *$dV_s^j/d\beta > 0$ while $dV_s^j/d\xi < 0$ for $\xi \in \{\tau, \sigma\}$.*
- (c) *If $\sigma < \infty$ and $\tau < \infty$, then $V_s^i > 0$ for R^i close to zero and $V_s^j > \bar{R}$ for R^j close to $\bar{R}$.*

The payoffs under unarmed peace and the implications stated in the proposition are precisely what one would expect based on our (static) model of trade if there were no dispute at all

[43] Our characterization of ω^i in the Appendix (see Lemma A.4(a)) shows that for sufficiently large gains from trade (or sufficiently small σ given τ), each country i would prefer $\lambda^i = \frac{1}{2}$ to $\lambda^i = 1$. Given $\beta = \tau = 1$, this critical value is $\sigma = 2$. At the same time, Lemma 1 implies that for $\sigma < 2$, $\overline{\lambda}^i < 1$, such that with $G^j = 0$ and $G^i = \epsilon > 0$, country j receives a positive share of $\bar{X}$.

[44] With a focus on the case of a symmetric distribution of initial resources ($R^i = R^j$), Garfinkel and Syropoulos (2018) show that, provided the gains from trade are sufficiently large (i.e., $\sigma \in (1, 2]$ assuming $\beta = \tau = 1$), unarmed peace is the unique equilibrium under settlement when based on the equal-sacrifice bargaining protocol (instead of the split-the-surplus protocol considered here).

over resource claims and each country held an equal share of total resources, $R^i = \bar{R}/2$. However, the actual payoffs shown under armed peace obtain even with an asymmetric distribution of resources.

Turning to the case of armed peace, part (a) establishes the welfare implications when the distribution of initial claims of ownership is sufficiently even. Specifically, since the two countries arm identically, each receives an equal share $\lambda^i = \frac{1}{2}$, implying identical payoffs. These payoffs are decreasing in the same parameters that fuel arming incentives in this case—namely, decreases in open conflict's destruction ($\beta \uparrow$) and in the gains from trade ($\sigma \uparrow$ and $\tau \uparrow$ that imply $\mu \downarrow$). Furthermore, any reallocation of $\bar{R}$ across the two countries that keeps their endowments in $[R^s_L, R^s_H]$ leave arming and thus equilibrium payoffs unchanged.[45]

Part (b) shows what happens in the case where country i is resource-constrained (and thus country j is not). In particular, the constrained country's payoff is non-monotonic in its own initial resource R^i, increasing in R^i for lower initial endowments, decreasing in R^i for larger values and thus reaching a maximum in $R^i \in (0, R^s_L)$. Meanwhile, the unconstrained country's payoff is increasing and convex in its own endowment. This set of results suggests that a redistribution of $\bar{R}$ away from the constrained country (i) towards the unconstrained country (j) could be welfare improving in a Pareto sense.

The first components of (b.i) and (b.ii) show the contrasting payoff effects of a decrease in the destructiveness of open conflict ($\beta \uparrow$) across the two countries. The unconstrained country's payoff V^j_s increases in β because (i) for given guns with $G^j_s > G^i_s$ the direct effect of an increase in this parameter is to enhance that country's leverage in negotiations (Lemma 1(c)) and (ii) the inability of the constrained country's to adjust its guns choice means there is no strategic effect. The negative effect of an increase in β on the constrained country's payoff V^i_s derives largely from the direct effect to lower that country's leverage (for $G^i_s < G^j_s$) in negotiations. When R^i is moderately high, the strategic effect is negative and reinforces the direct effect; even when R^i is small such that the strategic effect is positive, it is swamped by the negative direct effect (Proposition 3(c)). The second components of parts (b.i) and (b.ii) show that each country, whether constrained or not, benefits as the gains from trade rise (i.e., $\sigma \downarrow$ and $\tau \downarrow$). These results together with those from part (a) indicate that for all possible distributions of $\bar{R}$ where each country initially holds a strictly positive amount of the resource, both countries necessarily benefit from enhanced gains from trade. See Fig. 2(b) which illustrates (in green) a country's payoffs for various distributions of initial resource ownership and gains from trade reflected solely in different values of σ.[46]

[45] Observe that, when the gains from trade are sufficiently large such that unarmed peace is a possible outcome along with armed peace, unarmed peace dominates armed peace in a Pareto sense for such distributions of R^i. One can show such a ranking remains intact for resource distributions outside the range considered in part (a) of the proposition, provided that the distribution of initial resources is not too uneven.

[46] Ignore the pink curve for now.

Finally, part (c) establishes that the payoffs for both countries under settlement are strictly greater then their respective payoffs when one country initially has a claim to nearly all of $\bar{R}$. This result, which holds for all $\beta \leq 1$, builds on the gains from trade, suggesting that under settlement, the unconstrained country (j) finds it appealing to arm in a way that effectively permits the relatively smaller country (i) to produce more of its tradable good and thereby take greater advantage of the opportunities for trade. Since more trade is mutually advantageous, country j's rival (country i) also finds this arrangement appealing.

3.3 Armed Peace versus War

In this section we examine how arming and payoffs differ across peaceful settlement (with arming) and war. It should be clear that the possible avoidance of war's destructive effects ($\beta < 1$) and the possible realization of the gains from trade ($\mu^i > 1$ for $i = 1, 2$) add to the relative appeal of peaceful settlement. The relative appeal of war, by contrast, derives from the possibility of emerging as the winner and capturing the residual and future resources without having to arm in the future. The influence of this consideration is more important when the shadow of the future ($\delta > 0$) is larger. Also at play here is how arming incentives compare across the two modes of conflict resolution.

To start, let us consider the following benchmark case: suppose there is no destruction ($\beta = 1$), no gains from trade (i.e., either $\sigma \to \infty$ or $\tau \to \infty$), and countries do not value the future ($\delta = 0$). Since there are no gains from trade, $\mu^i = 1$ and thus $\omega^i \equiv \mu^i \lambda^i = \lambda^i$ for both $i = 1, 2$. As such, the condition in (13) that must hold under the split-the-surplus rule for any feasible pair of guns (G^i, G^j) implies $\lambda^i = \phi^i$. Furthermore, note from (15a) these assumptions imply that $\omega^i_{\lambda^i} = 1$. One can see, then, that the unconstrained country's incentives to arm under war and settlement, respectively shown in equations (9) and (16), are identical, implying that $[R^c_L, R^c_H] = [R^s_L, R^s_H]$ and $G^i_c = G^i_s$ for $i = 1, 2$ and any initial distribution of resoures $\bar{R}$. The next proposition establishes how departures from this benchmark case matter for equilibrium arming under the two regimes.

Proposition 5 (A comparison of arming.) *If $\delta > 0$, $\mu^i > 1$ for $i = 1, 2$, and/or $\beta < 1$, then we have the following:*

(a) *the resource constraint on arms binds for a larger set of factor allocations under conflict than under settlement (i.e., $[R^c_L, R^c_H] \subset [R^s_L, R^s_H]$);*

(b) *$G^i_c \geq G^i_s$, with strict inequality for at least one country.*

Parts (a) and (b) of the proposition can be visualized with the help of Fig. 3, which illustrates how arming under conflict and settlement differ when $\delta = 1$ and either $\mu^i > 1$ for $i = 1, 2$ or $\beta < 1$. The color pink is reserved for variables related to conflict and green for variables related to settlement.[47] Importantly, settlement reduces each country i's incentive to arm given the other country's (j) choice and, thus, induces lower arming by both than under conflict in period $t = 1$. As such, settlement reduces social waste in period $t = 1$. However, because arming is a recurrent use of resources under settlement but not under war, a comparison of payoffs under these regimes is a bit more involved.

How payoffs under conflict U_c^i and settlement V_s^i compare is an interesting question in its own right, but such a comparison also sheds light on which outcome is more likely to be observed when both are possible. Suppose country i expects country j to declare war and to prepare accordingly in the first period. Then, country i's best reply would be to arm for war and make the same declaration. Country j's best strategy would be the same. Hence, open conflict is always a possible outcome. However, because countries can communicate freely in the process of their negotiations, one would expect them to pursue a mode of conflict resolution that best advances both their mutual and self interests. Thus, if both countries' average discounted payoffs under settlement V_s^i exceeded those under open conflict U_c^i, open conflict would not be a coalition-proof equilibrium, and we would expect the two countries to coordinate on the settlement outcome.

Let us return momentarily to the benchmark case where countries do not value the future ($\delta = 0$), there is no destruction under war ($\beta = 1$) and no gains from trade to be realized under settlement (i.e., either $\sigma = \infty$ or $\tau = \infty$ implying $\mu^i = 1$ for $i = 1, 2$). Clearly, in this benchmark case, war and settlement are equivalent, so that $U_c^i = V_s^i$ for all $R^i \in (0, \bar{R})$, as indicated by the solid green curve in Fig. 2(b) associated with $\sigma = \infty$.

Now suppose the shadow of the future δ increases above zero. Proposition 4 establishes that an increase in δ leaves V_s^i unchanged. By contrast, the increase in δ causes U_c^i to decrease for $R^i \in (0, R_L^c)$, remain unchanged at $U_c^i = V_s^i = \beta \frac{\bar{R}}{4}$ for $R^i \in [R_L^c, R_H^c]$ (while shrinking the size of that range), and to increase for $R^i \in (R_H^c, \bar{R})$, as shown in Proposition 2 and illustrated in the context of Fig. 1(b). Thus, an increase in δ leaves U_c^i unchanged and equal to V_s^i for sufficiently even distributions of $\bar{R}$; for sufficiently uneven distributions, we have $U_c^i < V_s^i$ when $R^i \in (0, R_L^c)$ and $U_c^i > V_s^i$ when $R^i \in (R_H^c, \bar{R})$. Fig. 2(b) illustrates this ranking of payoffs with the pink curve representing U_c^i when $\delta = 1$, in which case the range (R_L^c, R_H^c) shrinks to a single point, $R_L^c = R_H^c = \bar{R}/2$. In less extreme cases, given $\mu^i = 1$ and $\beta = 1$, an increase in δ above 0 alone makes war more appealing relative to settlement for one country i when its rival (j) is resource constrained under conflict, $R^i \in (R_H^c, \bar{R})$; meanwhile, its constrained rival (j) has a preference for settlement.

Next, consider a series of increases in the gains from trade μ from 1, due to a reduction in τ and/or in σ, keeping $\delta = \beta = 1$ fixed in the background.[48] Although U_c^i (again, depicted

[47] Ignore the blue curves for now.

[48] Keep in mind that, while μ is defined as μ^i evaluated at a symmetric distribution $\lambda^i = \frac{1}{2}$, μ^i differs

by the pink curve in Fig. 2(b)) is independent of such gains, Proposition 4 establishes that V_s^i increases at each $R^i \in (0, \bar{R})$, as shown by the upward shift of the green curves in Fig. 2(b). Importantly, initial increases in these gains (starting from $\mu = 1$) imply $V_s^i > U_c^i$ for both i only when the initial distribution of $\bar{R}$ across the two countries is sufficiently even (up to point A in the figure) and when it is sufficiently uneven (beyond point B in the figure). But, once μ rises above a threshold level (associated with point C), settlement dominates conflict for all possible $R^i \in (0, \bar{R})$. Note that, while Fig. 2(b) shows the extreme cases of $\delta = 0$ and $\delta = 1$, the thrust of the above argument holds true for any $\delta \in [0, 1]$.

Finally, let us consider the destructiveness of war. Returning to our benchmark assumptions that $\delta = 0$, $\mu = 1$, and $\beta = 1$, an increase in war's destructive effects ($\beta \downarrow$) expands the set of asset allocations for which neither country is constrained in its arming under peaceful settlement; and, more importantly for our purposes here, a decrease in β implies $V_s^i > U_c^i$ for all allocations of $R^i \in (0, \bar{R})$.[49] Thus, when $\delta = 0$ and $\beta < 1$, settlement dominates war under all resource allocations $R^i \in (0, \bar{R})$ even when there are no gains from trade ($\mu = 1$). For larger values of $\delta > 0$, there exists a threshold rate of destruction $1 - \beta_0$, such that when $\beta < \beta_0$ peaceful settlement dominates war for all $R^i \in (0, \bar{R})$, again even when $\mu = 1$.

The next proposition builds on and extends these ideas:

Proposition 6 (Immunity to coalitional deviations.) *For any given $\delta \in (0, 1]$, there exists a threshold level of destruction $1 - \beta_0 \equiv 1 - \beta_0(\delta) \in (0, 1)$ with $\partial\beta_0/\partial\delta < 0$ and a gains-from-trade threshold level $\mu_0 \equiv \mu_0(\delta, \beta) > 1$ with $\partial\mu_0/\partial\delta > 0$ and $\partial\mu_0/\partial\beta > 0$, such that settlement dominates conflict (i.e., $V_s^i > U_c^i$ for $i = 1, 2$) under the following circumstances:*

(a) *if $\beta \in (0, \beta_0]$, then for any $R^i \in (0, \bar{R})$;*

(b) *if $\beta \in (\beta_0, 1]$ and*

  (i) *$\mu \geq \mu_0$, then for any $R^i \in (0, \bar{R})$*

  (ii) *$\mu < \mu_0$, then only for sufficiently even and sufficiently uneven international allocations of asset ownership.*

Thus, peaceful settlement is immune to coalitional deviations under a variety of conditions. In particular, when war is sufficiently destructive $\beta \leq \beta_0$, settlement dominates war for all $R^i \in (0, \bar{R})$ regardless of the size of the gains from trade. The threshold rate of destruction $1 - \beta_0$ is increasing in the salience of the future δ. Even when war is not sufficiently destructive, sufficiently large gains from trade render settlement dominant over trade. The threshold level μ_0 is increasing in β and increasing in δ. But, when conflict is not very destructive and the gains from trade are moderate such that $\mu < \mu_0$, settlement dominates

across countries for $R^i \notin [R_L^s, R_H^s]$.

[49] For $R^i \in [0, R_H^s]$, V_s^i rises while U_c^i falls as β falls. For $R^i \in (R_H^s, \bar{R})$, V_s^i falls, but is tempered by the favorable strategic effect as $R^i \to 0$; at the same time, U_c^i falls in proportion with the fall in β.

conflict for both countries under two distinct types of situations, illustrated in Fig. 2(b): (i) when the international distribution of resource ownership is sufficiently even; and (ii) when this distribution is sufficiently uneven.[50]

4 Unilateral Deviations and the Stability of Armed Peace

For settlement to arise as a *perfectly* coalition-proof equilibrium, it must not only dominate open conflict, but also be immune to unilateral deviations from it. Suppose in the first stage of period $t = 1$, each country i anticipates that it will choose settlement in the second stage and that its rival j will do the same. Given that expectation, each country $i = 1, 2$ will produce G_s^i. Now, let us suppose that one country i contemplates a possible deviation from this configuration in period $t = 1$.[51] There are two possible deviations. First, given both countries' first-stage gun choices made in anticipation of settlement (G_s^i, G_s^j), country i could choose to declare "war" in the second stage. Second, a country i could deviate by choosing another quantity of guns in the first stage of period $t = 1$ (G_d^i, where "d" stands for "deviation," given $G^j = G_s^j$) in anticipation that it will choose war in the second stage, and then proceed to declare war. Clearly, the first possibility is relevant when country i's arming decision under settlement is limited by its resource endowment R^i ($\leq R_L^s$). The second possibility arises when the country i's resource constraint on its arming is not binding. Because conflict inevitably breaks out once country i violates the negotiated settlement either way, its optimal arming under a unilateral deviation is given by its best reply to G_s^j ($j \neq i = 1, 2$) shown in (10), $G_d^i = B_c^i(G_s^j; \delta, R^i, \bar{R})$.[52]

We now describe the salient features of this best reply with the help of Fig. 3 in the special case of $\delta = 1$ that implies $R_L^c = R_H^c = \bar{R}/2$.[53] Following our earlier convention, green (pink) curves in this figure represent the best-response functions under settlement (open conflict) for all possible allocations of asset ownership. The blue curve shows country 1's payoff-maximizing arming $G_d^1 = B_c^1(G_s^2; \cdot)$ given G_s^2 under a unilateral deviation for

[50] To gain insight on how payoffs under open conflict and settlement would compare in period $t = 2$ (provided that war did not break out in period $t = 1$), one could compare these payoffs by setting $\delta = 0$. Such a comparison would show that, if either $\mu > 1$ while $\beta = 1$ or if $\mu = 1$ while $\beta < 1$, then each country would prefer peaceful settlement over war in the second stage in $t = 2$ for any given pair of feasible guns. This preference is magnified by the effect of the anticipation of settlement to induce less arming in the first stage (Proposition 5(b)). As such, settlement strictly dominates conflict in the second period whenever $\beta < 1$ and/or $\mu > 1$.

[51] Settlement is immune to unilateral deviations in period $t = 2$ because when $\delta = 0$ each country's average discounted payoff under an optimal deviation coincides which its payoff under conflict. Thus, for settlement to be the unique equilibrium of the second period (given settlement in the first period), it is sufficient that settlement strictly dominates conflict, which is the case when either $\beta < 1$, $\mu > 1$ or both (see footnote 50).

[52] Note that a country would not deviate by choosing another quantity of guns $G_d^i \neq G_s^i = B_s^i(G_s^j; \cdot)$, without also declaring war in the second stage. That is to say, if country i anticipates choosing (along with country j) settlement in the second stage, then it's optimizing choice of guns is given by $G_s^i = B(G_s^j; \cdot)$.

[53] Consideration of other values of δ is straightforward and thus omitted. Note that this figure holds for any $\beta \leq 1$ and $\mu \geq 1$ as long as one is satisfied as a strict inequality.

alternative configurations of initial resource ownership. The arrows indicate the direction of change in country 1's guns as R^1 increases and R^2 decreases along the $\bar{R}\bar{R}$ line.

To characterize G_d^i, we distinguish between four intervals of asset allocations for R^i: (i) $(0, R_L^s)$; (ii) $\left(R_L^s, R_L^d\right)$; (iii) $\left(R_L^d, R_H^s\right)$; and (iv) $\left(R_H^s, \bar{R}\right)$, where R_L^d denotes the threshold level of country i's resource, below which it is resource-constrained in its arming under a unilateral deviation.[54] For the first two intervals, country i's optimal, unilateral deviation in arming is constrained by its resource endowment. More precisely, in case (i) illustrated in the figure by asset allocations on the segment $\bar{R}D_2'$ of the $\bar{R}\bar{R}$ line, country i is constrained under settlement, implying that its unilateral deviation involves no adjustment in arming, only a declaration of war: $G_d^i = B_c^i(G_s^j; \cdot) = G_s^i = R^i$. In case (ii) depicted by allocations on the $D_2'D_1''$ segment, country i's optimal deviation entails both a declaration of war and producing a larger quantity of guns as compared with settlement, but only as much as its endowment: $G_d^i = B_c^i(G_s^j; \cdot) = R^i > G_s^i = G_s^j = R_L^s$. For the last two intervals, country i is no longer resource constrained. Specifically, in case (iii) shown by allocations on the $D_1''D_2''$ segment, country i's optimal deviation is given by its unconstrained best-response function under conflict, while its rival's arming remains at the unconstrained equilibrium under settlement, $G_s^j = G_s = R_L^s$: $G_d^i = \widetilde{B}_c^i(R_L^s) = R_L^d$. In case (iv) illustrated by allocations on the $D_1'\bar{R}$ segment, country j's arming G_s^j is constrained by its endowment, while country i operates along its best-response function under conflict: $G_d^i = \widetilde{B}_c^i(R^j)$, which equals precisely the amount of its arming under conflict. For additional clarity, we illustrate the above ideas in Fig. 4(a) where G_d^1 is shown with the blue curves in the special case of $\delta = 1$, $\beta = 1$ and $\mu = \mu_0$, the threshold value of the gains from trade that ensure settlement dominates conflict. The figure also shows, to allow for easier comparisons, the quantities of guns produced under settlement (green curves) and conflict (pink curves).

Next, we ask: when are the unilateral deviations described above profitable? Let $W_d^i \equiv U^i(G_d^i, G_s^j; \cdot)$ denote the payoff to country i under such deviations, including the case where $G_d^i = G_s^i$ (i.e., i declares war without adjusting its guns relative to settlement.) Again considering the special case where $\delta = 1$, $\beta = 1$ and $\mu = \mu_0$, Fig. 4(b) shows both countries' payoffs under settlement and conflict for various distributions of initial resource ownership. It also illustrates country 1's payoff under an optimal deviation (the blue curve). Key here is our starting point: $\mu = \mu_0$. In particular, this assumption implies by the definition of μ_0 that, for allocations of the resource $R^i \in (R_L^s, R_H^s)$ (or intervals (ii) and (iii)), $V_s^i(R^i) = U_c^i(R_H^s)$ holds; in addition, since country i's optimal deviation is given by $G_d^i = \widetilde{B}_c^i(G_s) = R_L^d$ for $R^i \in (R_L^d, R_H^s)$ (interval (iii)) where $G_s^j = R_L^s$, we have $W_d^i(R^i) = U_c^i(R_H^s)$. Thus, for interval (iii) when $\mu = \mu_0$, $W_d^i(R^i) = V_s^i(R^i)$, meaning that a unilateral deviation by

[54] That $R_L^d > R_L^s$ follows from our finding in Proposition 5, that country i's incentive to arm for any given G^j is higher under open conflict than under settlement.

country i provides no payoff gains relative to settlement. In such cases, the gains from trade that can be realized under settlement match precisely the expected gains from a unilateral deviation that involve the possibility of emerging as the victor in conflict and not having to arm in the second period.[55] In interval (iv) where $R^i \in \left(R_H^s, \bar{R}\right)$, $G_d^i = G_c^i$, such that the payoff to a country that deviates unilaterally coincides with its payoff under conflict: $W_d^i(R^i) = U_c^i(R^i)$. But, by the definition of $\mu = \mu_0$, settlement dominates open conflict for this range of resource allocations, implying that the larger country i has no incentive deviate from settlement.

What about the other intervals where i is the smaller country? We already know from above that the definition of μ_0 implies, when $R^i = R_L^d$, $W_d^i(R_L^d) = V_s^i(R_L^d) = V_s^i(R_H^s) = U_c^i(R_H^s)$, such that country i has no incentive to deviate from settlement at that point. We also know that, as R^i falls into interval (ii) where $R^i \in (R_L^s, R_L^d)$, country i's payoff under settlement remains unchanged. At the same time, its resource constraint on arming under a unilateral deviation kicks in, becoming increasingly severe and thereby pushing its deviation payoff $W_d^i(R^i)$ further and further below $V_s^i(R^i)$ as $R^i \to R_L^s$. As R^i falls further, moving into interval (i) where $R^i \in (0, R_L^s)$, so does its deviation payoff and eventually it approaches 0 as $R^i \to 0$.[56] While country i's settlement payoff eventually starts to decline as well as R^i falls within interval (i), that payoff remains above its deviation payoff, approaching a positive amount by virtue of the gains from trade $\mu > 1$ as $R^i \to 0$.[57] As such, when $\mu = \mu_0$, neither country has an incentive to deviate from settlement for any resource allocation $R^i \in (0, \bar{R})$; furthermore, any increase in μ above μ_0, implying greater gains from trade, tilts the balance even more towards settlement.

Given the benchmark established above, we now describe the vulnerability of settlement to unilateral deviations as follows:

Proposition 7 (Immunity to unilateral deviations.) *For any given $\delta \in (0, 1]$, settlement is immune to unilateral deviations (i.e., $V_s^i > W_d^i$ for $i = 1, 2$) under the following circumstances:*

(a) *if $\beta \in (0, \beta_0]$, then for any $R^i \in (0, \bar{R})$;*

(b) *if $\beta \in (\beta_0, 1]$ and*

 (i) *$\mu \geq \mu_0$, then for any $R^i \in (0, \bar{R})$*

 (ii) *$\mu < \mu_0$, then only for sufficiently uneven international distributions of resource ownership.*

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[55] More generally, if $\beta < 1$ which would imply a smaller value of μ_0 relative to what is drawn in Fig. 2(b), the equality $W_d^i(R^i) = V_s^i(R^i)$ would also reflect the benefit of settlement to avoid war's destruction.

[56] Observe $U_c^1(R^1) < W_d^1(R^1)$ for $R^1 \in (0, R_L^s)$ as shown in Fig. 4(b), simply because the optimizing deviation operates on $B_c^1(G^2; \cdot)$ but with $G^2 = G_s^2 < G_c^2$.

[57] Even when $\mu = \mu_0 = 1$, any positive destruction $\beta < 1$ under war implies the same ranking for $R^i > 0$.

Consistent with the liberal peace hypothesis, when the gains from trade are sufficiently large (i.e., $\mu \geq \mu_0$), peace is stable under all possible configurations of initial resource distributions. Even for $\mu < \mu_0$, settlement remains immune to unilateral deviations, but only for asset allocations within the intervals $[0, R_L^s]$ and $[R_H^s, \bar{R}]$. For such allocations, the gains from trade to the smaller country alone are sufficiently large to make unilateral deviations unprofitable. The gains from trade to the larger country are not very large; but, because arming tends to be smaller for such allocations, the potential future savings in arming from wiping out the adversary are not large enough to make a unilateral deviation profitable. For more symmetric asset allocations that lie within the interval $[R_L^s, R_H^s]$, the overall gains from peaceful settlement fall short of the expected benefits of unilaterally deviating from that outcome—namely, to gain an edge in conflict by arming more and to enjoy the resource savings associated with not having to arm in the subsequent period. Although armed peace Pareto dominates war for such allocations, at least one country and possibly both are eager to deviate from settlement, thereby undermining its stability.[58] A larger degree of dissimilarity between traded commodities ($\sigma \downarrow$) and/or lower trade costs ($\tau \downarrow$) imply greater gains from trade, thereby making it more likely that peace prevails as the stable equilibrium for any given distribution of initial resource ownership. Yet, the stronger is shadow of the future ($\delta \uparrow$) and/or the lower is the destructiveness of war ($\beta \uparrow$), the less likely it is that peace arises as the stable equilibrium.

5 Generalizations and Qualifications

While our analysis is based on a very stylized model, relaxing some of the simplifying assumptions would not change the analysis substantively. For example, extending the model to include more than two periods, or even an infinite horizon, and supposing that the victor in open conflict holds a strategic advantage forever, or just for a finite number of periods (more than two), would amplify the relative appeal of war and thus shrink the parameter space for which peace prevails. Relaxing the assumption that war's destructive effects are exogenous and supposing, in particular, they depend positively on arms deployed could work in the opposite direction, adding to the stability of peace. In addition, consideration of rules of division under peace that are less sensitive to the threat-point payoff (e.g., rules based on equal-sacrifice or Nash bargaining protocols) would similarly make the stability of peace more likely, whereas rules based more directly on the conflict technology would tend to make the stability of peace less likely.[59] However, with any of these modifications, the implications for how the key parameters matter in determining the stability of peace would

[58] But, one can show that, when unarmed peace is another possible outcome (i.e., assuming $\beta = \tau = 1$ and $\sigma \in [\frac{3}{2}, 1]$), it is immune to such deviations. Since, as noted above, unarmed peace Pareto dominates armed peace in such cases, it emerges as the stable outcome.

[59] Indeed, a rule of division based only on the CSF in (1) would preclude the possibility of unarmed peace.

not change qualitatively.

Let us consider other modifications that would allow us to address possible objections related to the assumption that the defeated country is put out of contention in the future. One possibility is to relax the assumption that all of the countries' residual resources are contestable. To the extent that a fraction of the countries' resources is secure, the defeated nation would be able to threaten the victor of the first-period war via "rebellion" in future interactions. Accordingly, the victor would have to devote some of its second-period resources to suppress such activity and more generally maintain "law and order," whereby it could protect its own resource and extract its winnings from the losing side. But, this future expense for the victor would reduce the expected payoff of war relative to peaceful settlement as well as the expected profitability of unilateral deviations, such that the parameter space for which war is the stable equilibrium shrinks.

In a similar but distinct approach, we could suppose instead that the winner of war (e.g., an imperial power) can, in both periods, extract the distinct intermediate good that the defeated country would have produced with its resources (net of destruction) and traded under peace; but, to do so successfully, the victor may have to incur a sunk cost in each period. Financed with some of the victor's output of the intermediate good it produces, this sunk cost can be interpreted as an investment in facilitating local production, monitoring order, and punishing insurgents. Since the victor appropriates the produced and potentially tradable intermediate good (as opposed to the defeated side's effective resource), one could view this alternative scenario as "colonial" or "forced" trade.[60] Giving the victor access to the other country's distinct intermediate good increases the value of the prize under open conflict relative to what we studied above. However, the investments required of the victor to obtain those goods reduces the value of the prize. Indeed, although such expenditures bring benefits to the imperial power, they could over time cause a significant strain on that country's economy—what Kennedy (1987) has coined "imperial overstretch." Insofar as this effect is likely to dominate, one would expect the amount of equilibrium arming under the anticipation of war to fall. While this indirect effect alone would raises the expected payoff under conflict, it could be swamped by the direct effect of the required investment by the victor in each period. Hence, under these modifications to our model, open conflict could be less appealing relative to armed peace. These modifications could also reduce the payoff under a unilateral deviation, and thus expand the parameter space under which peace arises as the stable equilibrium outcome.

Finally, let us suppose that victory in a war in the first period brings no direct future reward in terms of the defeated country's resources in the second period, only the victor's

[60] Under this alternative scenario, the winner's resource net of destruction in the second period would be fully secure, while the loser's resource would be fully insecure.

own resource net of destruction. Under this modification, with a reduced prize from victory in open conflict, the parameter space for which peace is the stable equilibrium naturally expands. However, underscoring the importance of the costs of peace and the possible value of war to avoid these costs, this modification would not eliminate the possibility that war emerges as the stable equilibrium.

6 Concluding Remarks

The liberal peace hypothesis has much intuitive appeal. Greater interdependence between national economies implies larger potential gains from trade; and, insofar as interstate conflict disrupts the realization of those gains, one would expect countries in potential conflict to resolve their differences peacefully. In a setting where countries dispute initial resource ownership claims and with a focus on equilibria that are immune to both coalitional and unlateral deviations, our analysis shows how the endogenous choice of conflict resolution depends on the potential gains from trade as determined jointly by trade costs and the elasticity of substitution between traded commodities. If those gains are sufficiently large, then peaceful settlement will emerge as the stable equilibrium outcome for all distributions of initial ownership claims. However, the actual threshold level of gains itself depends on other factors, including the destruction of open conflict and the salience of the future. The less destructive is open conflict and the more salient is the future, the larger is that threshold. Moreover, there also exists a threshold degree of destruction, above which peaceful settlement emerges for any initial distribution of resources, regardless of the magnitude of the gains from trade. Even when conflict is not sufficiently destructive and the gains from trade are small, peace could emerge as the stable equilibrium, but only when the international distribution of initial resource ownership is sharply uneven.

An interesting extension of the analysis would be to allow one country to make a pure resource transfer to the other country in advance of their arming decisions. The aim of this line of research would be to sort out the set of conditions under which transfers help to promote peace as the stable equilibrium outcome. For example, a transfer from the larger country to the smaller country would, for given guns, augment the possible gains from trade, and that effect alone would increase the chances for peace. However, if the smaller country is resource constrained in its arming prior to the transfer, the countries' arming choices and their threat-point payoffs would also be affected, possibly undermining the stability of peace. Alternatively, a transfer from the smaller country to the larger country that would result in a greater disparity in resources could make peaceful settlement a more likely outcome.

Another potentially fruitful extension left for future research involves the consideration of trade policies. In particular, allowing countries to use trade policies would influence the size and the disposition of the surplus in the shadow of war. Such an extension would make it possible to explore possible interactions between security and trade policies in dynamic environments.

The analysis could also be extended to consider more than the two countries (say, three) each possessing a unique technology for producing an intermediate good distinct from that produced by the others. Assuming that the third country is not in dispute with the other two, one could ask how the possibility of trade between all three influences the prospects for peace. Furthermore, one could study the opportunities and incentives of the third, friendly country to intervene in the conflict between the two adversaries as well as the importance of alliances.

References

Amano, A., (1966). "International Factor Movements and the Terms of Trade," *Canadian Journal of Economics* 32(4), 510-519.

Anbarci, N., Skaperdas, S., and Syropoulos, C., (2002). "Comparing Bargaining Solutions in the Shadow of Conflict: How Norms against Threats Can Have Real Effects," *Journal of Economic Theory* 106(1), 1–16.

Arkolakis, C., Costinot, A., and Rodríguez-Clare, A., (2012). "New Trade Models, Same Old Gains?" *American Economic Review* 102(1), 94–130.

Armington, P.S., (1969). "A Theory of Demand for Products Distinguished by Place of Production," *International Monetary Fund Staff Papers* 16(1), 159-176.

Barbieri, K., (2002). *The Liberal Illusion: Does Trade Promote Peace?* Ann Arbor: The University of Michigan Press.

Barbieri, K. and Levy, J.S., (1999). "Sleeping with the Enemy: The Impact of War on Trade," *Journal of Peace Research* 36(4), 463–479.

Bernheim, B. Douglas, Peleg, B., and Whinston, M.D., (1987). "Coalition-Proof Nash Equilibria: I. Concepts," *Journal of Economic Theory* 42(1), 1–12.

Beviá, C. and Corchón, L., (2010). "Peace Agreements without Commitment," *Games and Economic Behavior* 68(2), 469–487.

Bhagwati, J.N, Brecher, R.A. and Hatta, T., (1983). "The Generalized Theory of Transfers and Welfare: Bilateral Transfers in a Multilateral World," *American Economic Review*, 73(4), 606–618.

Brecher, R.A. and Bhagwati, J.N., (1982). "Immiserizing Transfers from Abroad," *Journal of International Economics* 13(3-4), 353–364.

Copeland, D.C., (2015) *Economic Interdependence and War.* Princeton: Princeton University Press.

Fearon, J., (1995). "Rationalist Explanations for War," *International Organization* 49(3), 379–414.

Garfinkel, M.R. and Skaperdas S., (2000). "Conflict without Misperceptions or Incomplete Information: How the Future Matters," *Journal of Conflict Resolution* 44(6), 793–807.

Garfinkel, M.R., Skaperdas S., and Syropoulos, C., (2015). "Trade and Insecure Resources," *Journal of International Economics* 95(1), 98–114.

Garfinkel, M.R. and Syropoulos, C., (2018). "Rules for Dividing a Disputed Resource in the Context of the Classical Liberal Argument for Peace," *Peace Economics, Peace Science and Public Policy* 24(1), 2–17.

Garfinkel, M.R. and Syropoulos, C., (2019). "Problems of Commitment in Arming and War: How Insecurity and Destruction Matter," *Public Choice* 178(3-4), 349–369.

Garfinkel, M.R., Syropoulos, C., and Yotov, Y., (2018). "Arming in the Global Economy: The Importance of Trade with Enemies and Friends," unpublished paper.

Glick, R and Taylor, A.M., (2010) "Collateral Damage: Trade Disruption and the Economic Impact of War," *Review of Economics and Statistics* 92(1), 102–127.

Grossman, G.M., (1984). "The Gains from International Factor Factor Movements," *Journal of International Economics* 17(1-2), 73–83.

Hirshleifer, J., (1989)."Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success," *Public Choice* 63(2), 101–112.

Jackson, M.O. and Morelli, M., (2007). "Political Bias and War," *American Economic Review* 97(4), 1353–1373.

Jackson, M.O. and Nei, S., (2015). "Networks of Military Alliances, Wars and International Trade," *PNAS* 112(50), 15277–15284.

Kennedy, P. , (1987). *The Rise and Fall of the Great Powers: Economic Change and Military Conflict from 1500 to 2000*, New York: Random House.

Krugman, P., (1980). "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review* 70(5), 950–959.

Martin, P., Mayer, T., and Thoenig, M., (2008). "Make Trade Not War," *Review of Economic Studies* 75(3), 865–900.

McBride, M. and Skaperdas, S., (2014). "Conflict, Settlement and the Shadow of the Future," *Journal of Economic Behavior and Organization* 105, 75–89.

Oneal, J.R. and Russett, B.M., (1997). "The Classical Liberals Were Right: Democracy, Interdependence and Conflict, 1950–1985," *International Studies Quarterly* 41(2), 267–293.

Oneal, J.R. and Russett, B.M., (1999). "The Kantian Peace: The Pacific Benefits of Democracy, Interdependence and International Organizations, 1855–1992" *World Politics* 52(1), 1–37.

Polachek, S.W., (1980). "Conflict and Trade," *Journal of Conflict Resolution* 24(1), 55–78.

Powell, R., (2006). "War as a Commitment Problem," *International Organization* 60(1), 169–203.

Skaperdas, S., (1996). "Contest Success Functions," *Economic Theory* 7(2), 283-290.

Skaperdas, S. and Syropoulos, C., (1996). "Can the Shadow of the Future Harm Cooperation?" *Journal of Economic Behavior and Organization* 29(3), 355–372.

Skaperdas, S. and Syropoulos, C., (2001). "Guns, Butter, and Openness: On the Relationship between Security and Trade," *American Economic Review: Papers and Proceedings* 91(2), 353–357.

Skaperdas, S. and Syropoulos, C., (2002). "Insecure Property and the Efficiency of Exchange," *Economic Journal* 112(476), 133–146.

Tian, N., Fleurant, A., Kuimova, A., Wezeman, P.D. and Wezeman, S.T., (2018). "Trends in World Military Expenditure, 2017," *SIPRI Fact Sheet, May 2018.*

Tullock, G., (1980). "Efficient Rent Seeking," in Buchanan, J.M., Tollison, R.D. and Tullock, G. (eds), *Toward a Theory of the Rent Seeking Society,* College Station: Texas A&M University Press, 3–15.

Vesperoni, A. and Yildizparlak, A., (2019). "Inequality and Conflict Outbreak," *Social Choice and Welfare* (forthcoming).

von Mises, L., (1948). *Human Action*, New Haven: Yale University Press.

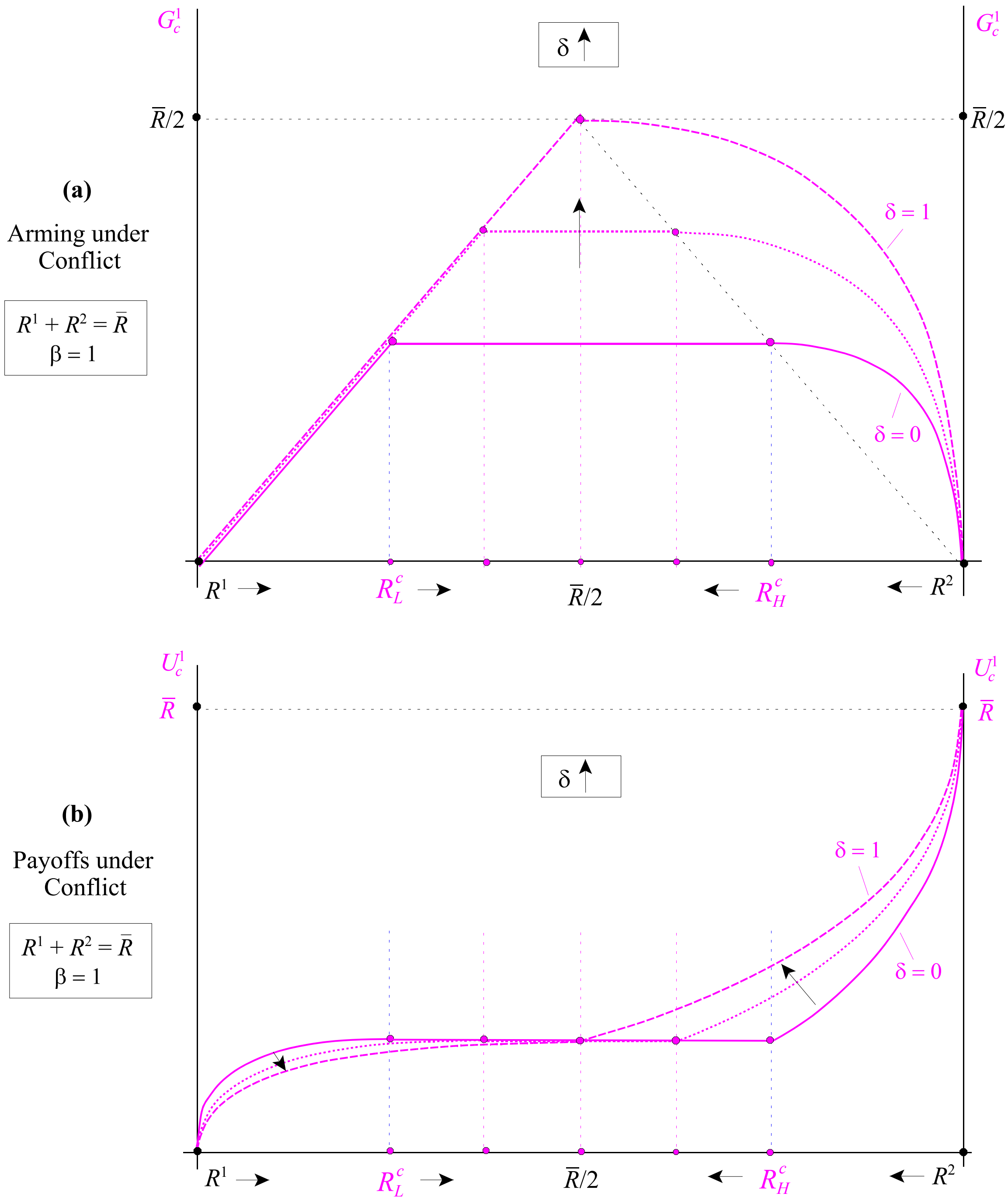

Figure 1: Arming and Payoffs under Conflict for Alternative Distributions of Initial Resource Ownership

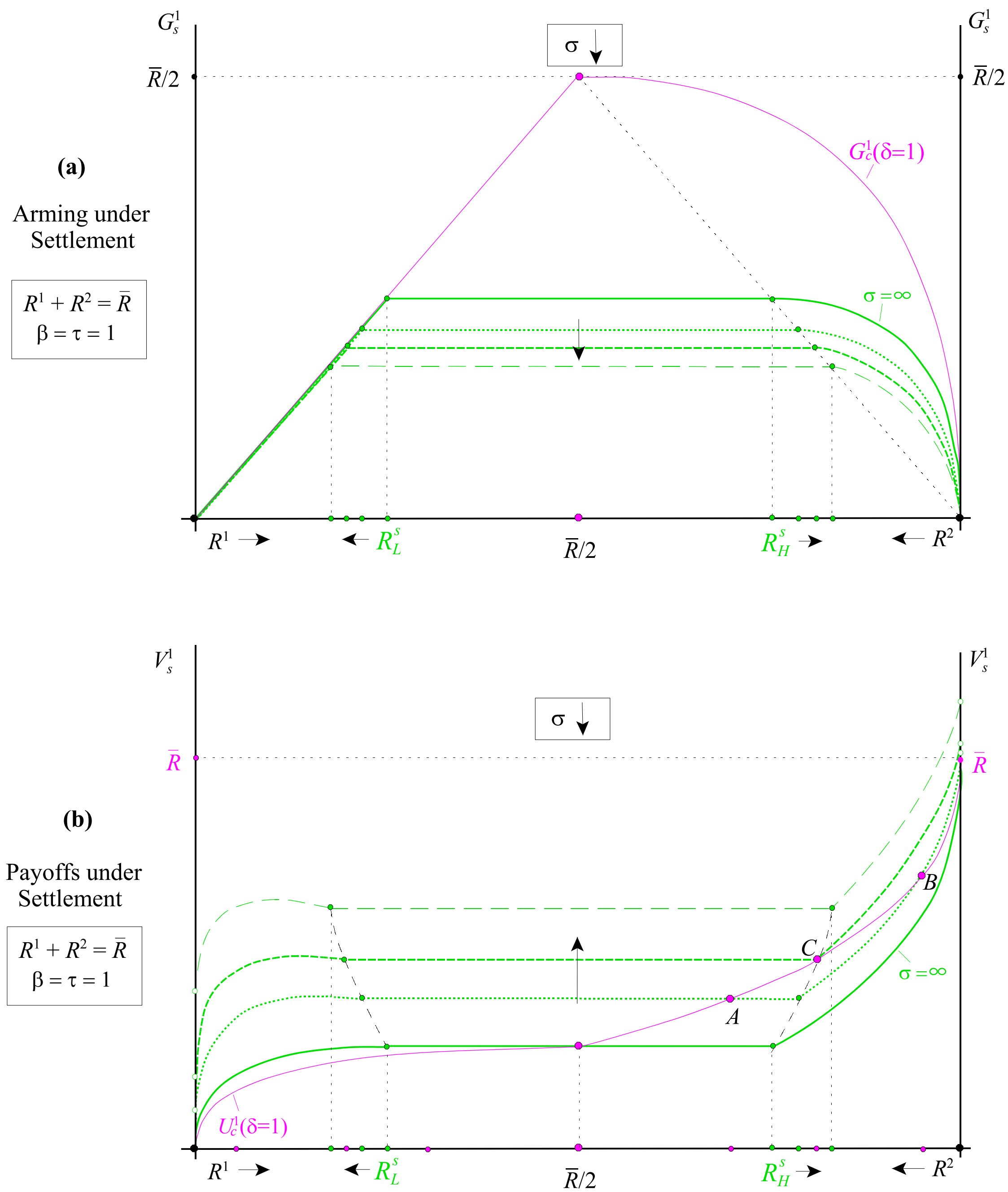

Figure 2: Arming and Payoffs under Settlement for Alternative Distributions of Initial Resource Ownership

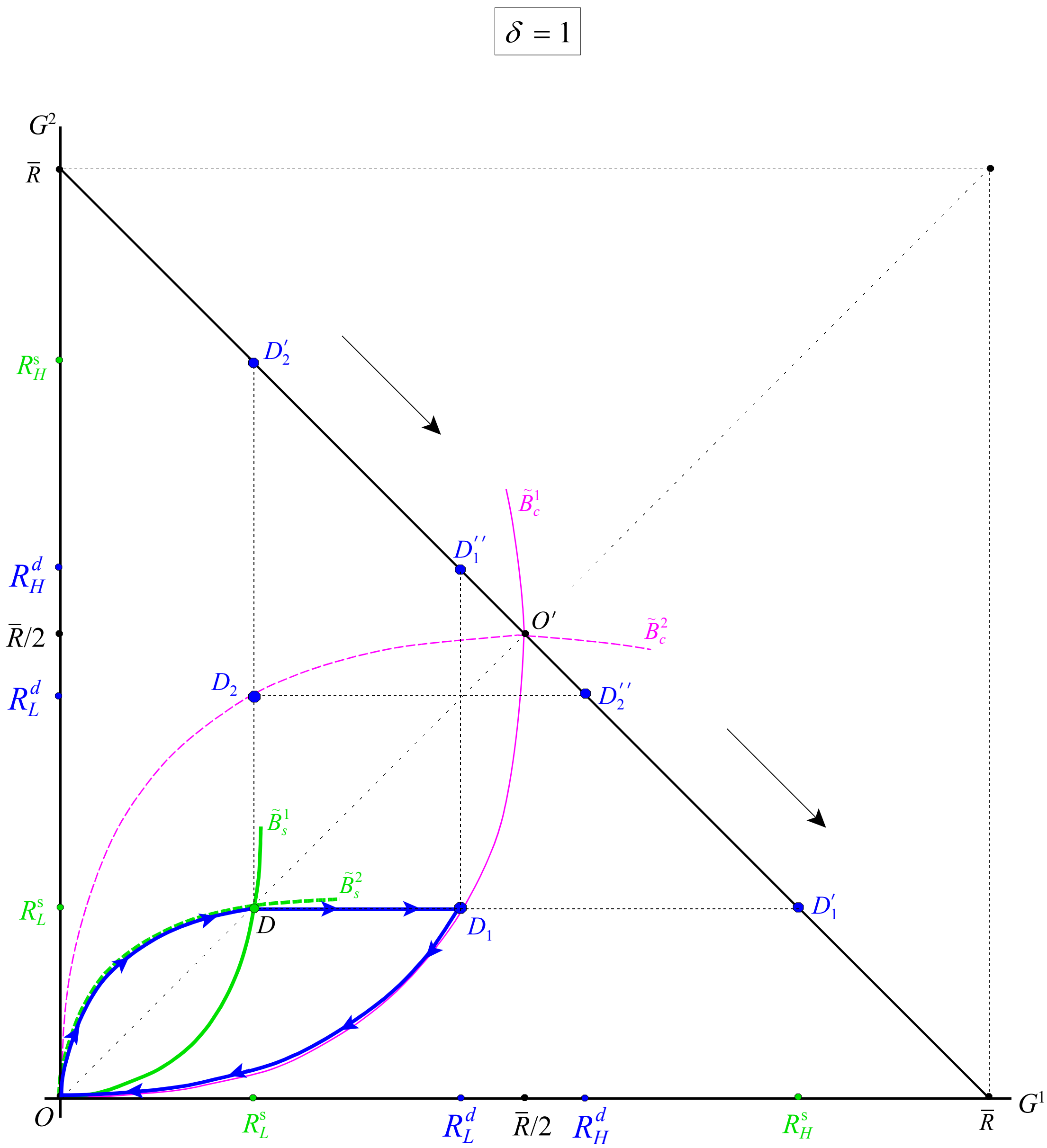

Figure 3: Best-Response Functions under War and Settlement and Unilateral Deviations from Settlement

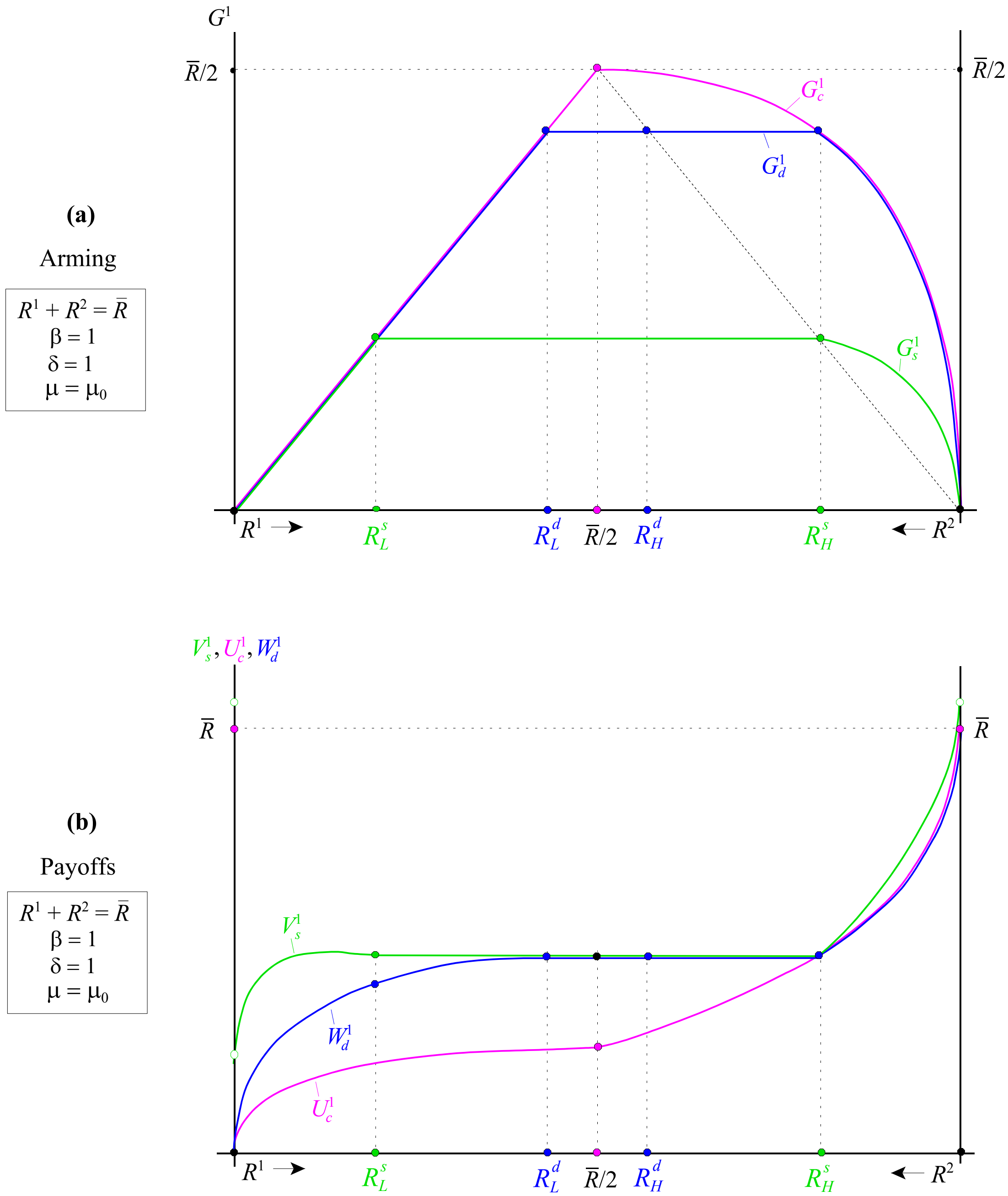

Figure 4: Arming and Payoffs under Conflict, Settlement, and Unilateral Deviations for Alternative Distributions of Initial Resource Ownership

A Appendix

Proof of Proposition 1. The proof follows from the discussion in the text, along with the best-response functions shown in (10) and the critical values for R shown in (11).

Proof of Proposition 2. That U_c^i for $i = 1, 2$ depends positively on β for all $R^i \in (0, \bar{R})$ follows from the finding in Proposition 1, that equilibrium arming is independent of β, and the fact that an increase β implies more of the residual resource remains after conflict for employment by the victor in the production of its intermediate good.

Part (a). From Proposition 1(a), when neither country is resource constrained in its arming decision, we have the symmetric solution $G_c = R_L \equiv \frac{1}{4}(1+\delta)\bar{R}$ implying $\phi^i = \phi^j = \frac{1}{2}$ and $\bar{X} = \bar{R} - 2G_c = \frac{1}{2}(1-\delta)\bar{R}$. Then, the specification for average payoffs in (8) gives $U_c^i = \beta\frac{\bar{R}}{4}$, which is independent of the distribution of $\bar{R}$ and time preferences δ.

Part (b). Turning to the case of sufficiently uneven distributions that imply country i is resource constrained, while country j is not, Proposition 1(b) shows that $G_c^i = R^i$ and, with (10b), implies

$$G_c^j = \widetilde{B}_c^j(R^i) = -R^i + \sqrt{(1+\delta)\bar{R}R^i}.$$

Appropriately differentiating the expression above, imposing $d\bar{R} = 0$, gives:

$$\frac{dG_c^j}{dR^i} = \frac{G_c^j - R^i}{2R^i} > 0 \tag{A.1a}$$

$$\frac{dG_c^j}{d\delta} = \frac{R^i}{\bar{G}_c}\frac{\bar{R}}{2} > 0, \tag{A.1b}$$

where $\bar{G}_c = R^i + G_c^j$.

A change in a country's own initial resource affects its payoff shown in (8) directly only through its effect on $\bar{R}$. But, since our focus here is on changes in the distribution of $\bar{R}$ such that $dR^i = -dR^j$, we need to consider only the indirect effects as follows:

$$\frac{dU_c^i}{dR^i} = U_{G^i}^i + U_{G^j}^i\frac{dG_c^j}{dR^i} = \frac{\beta}{1+\delta}\left(\left[\phi_{G^i}^i\left(\bar{X}+\delta\bar{R}\right) - \phi^i\right] + \left[\phi_{G^j}^i\left(\bar{X}+\delta\bar{R}\right) - \phi^i\right]\frac{dG_c^j}{dR^i}\right)$$

$$= \frac{\beta}{1+\delta}\left[\frac{G_c^j - R^i}{2R^i}\right] > 0 \tag{A.2a}$$

$$\frac{dU_c^j}{dR^i} = U_{G^i}^j\frac{dG_c^i}{dR^i} + U_{G^j}^j\frac{dG_c^j}{dR^i} = \frac{\beta}{1+\delta}\left\{\left[\phi_{G^i}^j\left(\bar{X}+\delta\bar{R}\right) - \phi^j\right] + [0]\frac{dG_c^j}{dR^i}\right\}$$

$$= -\frac{\beta}{1+\delta}\left[\frac{G_c^j}{R^i}\right] < 0. \tag{A.2b}$$

The above expressions can be obtained using country j's FOC based on (9), $U^j_{G^j} = 0$, to eliminate $(\bar{X} + \delta\bar{R})$, (A.1a) and the properties of the conflict technology in (1). To establish the limit results, we use (8) keeping in mind that, while $\lim_{R^i \to 0} G^i = 0$ and $\lim_{R^i \to 0} G^j = 0$, $\lim_{R^i \to 0} \phi^j = \lim_{R^i \to 0}\{1 - \sqrt{R^i/(1+\delta)\bar{R}}\} = 1$.

Differentiation of (A.2a) and (A.2b) with respect to R^i while using (A.1a) gives

$$\frac{d^2 U^i_c}{(dR^i)^2} = -\frac{\beta}{1+\delta}\left[\frac{\bar{G}_c}{2\left(R^i\right)^2}\right] < 0 \tag{A.3a}$$

$$\frac{d^2 U^j_c}{(dR^i)^2} = \quad \frac{\beta}{1+\delta}\left[\frac{\bar{G}_c}{2\left(R^i\right)^2}\right] > 0. \tag{A.3b}$$

Equation (A.3a) reveals that the constrained country's average payoff U^i_c is concave in R^i. Furthermore, since $dR^j = -dR^i$ implies, by (A.3b), $d^2U^j_c/(dR^j)^2 = d^2U^j_c/(dR^i)^2 > 0$, it follows that the unconstrained country's equilibrium average payoff is convex in its own endowment R^j. This completes the proof of (b.i) that deals with endowment redistributions.

Turning to part (b.ii), we examine the influence of δ on payoffs in (8). Accounting for both the direct and indirect welfare effects, we have for the constrained country i:

$$\begin{aligned}\frac{dU^i_c}{d\delta} &= U^i_\delta + U^i_{G^j}\frac{dG^j_c}{d\delta} = \beta\frac{\phi^i\bar{R}\,(1+\delta) - \phi^i\left(\bar{X}+\delta\bar{R}\right)}{(1+\delta)^2} + \beta\left[\frac{\phi^i_{G^j}\left(\bar{X}+\delta\bar{R}\right) - \phi^i}{1+\delta}\right]\frac{dG^j_c}{d\delta}\\ &= \frac{\beta\phi^i}{(1+\delta)^2}\left[\bar{G}_c - \tfrac{1}{2}\,(1+\delta)\,\bar{R}\right].\end{aligned} \tag{A.4}$$

The last expression above makes use of the definitions $\bar{X} = \bar{R} - \bar{G}_c$, $\phi^i = G^i_c/\bar{G}_c = R^i/\bar{G}_c$, and $\phi^j + \phi^i = 1$, along with the implication of (1) that $\phi^i_{G^j} = -\phi^j_{G^j}$ in country j's FOC (which requires $\phi^j_{G^j}\left(\bar{X}+\delta\bar{R}\right) = \phi^j$) and (A.1b). To evaluate the sign of (A.4), observe that when neither country's arming decision is constrained by its initial resource endowment, the aggregate quantity of guns under conflict $\bar{G}_c$ equals $2R^c_L = \frac{1}{2}(1+\delta)\bar{R}$. But, assuming that country i is resource constrained $G^i_c = R^i$, we have $R^i < R^c_L$, which implies $G^j_c > R^i$. Owing to strategic complementarity exhibited by the unconstrained country's best-response function, it follows that $\bar{G}_c < 2R^c_L$, such that the expression shown in (A.4) is negative. Thus, the direct (and positive) effect of an increase in δ on U^i_c is dominated by the indirect (and negative) effect of δ on the unconstrained rival's arming (which rises), implying that the constrained country's average discounted payoff necessarily falls. For the unconstrained country j, the direct effect of an increase in δ on U^j_c is strictly positive, while there is no indirect effect since $G^i_c = R^i$. As such, an increase in δ necessarily augments the unconstrained country's average discounted payoff U^j_c. ||

Some Comparative Statics under Trade. Let a "ˆ" over variables denote percent change (e.g., $\widehat{x} \equiv dx/x$). The definitions of expenditure shares $\gamma_j^i = (p^i)^{1-\sigma}/\left[1 + (p^i)^{1-\sigma}\right]$ and domestic prices $p^i = \tau\pi_T^i$ imply

$$\widehat{\gamma}_j^i = -(\sigma-1)\gamma_i^i(\widehat{\pi}_T^i + \widehat{\tau}) - \gamma_i^i \ln\left(p^i\right) d\sigma.$$

Logarithmically differentiating (5) and simplifying the resulting expression gives

$$\widehat{\pi}_T^i = \frac{1}{\Delta}\left\{\widehat{Z}^i - \widehat{Z}^j + (\sigma-1)(\gamma_j^i - \gamma_i^j)\widehat{\tau} - \left[\gamma_i^i \ln(p^i) - \gamma_j^j \ln(p^j)\right] d\sigma\right\}, \tag{A.5}$$

where, as previously shown in (15b), $\Delta = 1 + (\sigma-1)(\gamma_i^i + \gamma_j^j) > 1$. Hence, increases in country i's effective endowment Z^i affects its terms of trade adversely. Exactly the opposite is true for an increase in Z^j. Equation (A.5) also reveals that the effect of an increase in trade costs τ on country i's terms of trade π_T^i depends qualitatively on the ranking of the two countries' expenditure shares of their respective importables. Similarly, the impact of the elasticity of substitution σ on π_T^i depends on the manner in which internal prices compare internationally. As we will see shortly, both rankings depend on the distribution of $\bar{X}$ or the effective endowments, Z^i and Z^j.

To gain some understanding of how this distribution of $\bar{X}$ matters not only for equilibrium prices, but also for the countries' payoffs and their gains from trade, we suppose for now that $Z^i = \lambda^i \bar{X}$ for $i = 1, 2$, where $\lambda^i \geq 0$ (implying $\lambda^j = 1 - \lambda^i \leq 1$) is an arbitrary division of the common pool $\bar{X}$ $(= \bar{R} - \bar{G} > 0)$. Additionally, keep $\bar{R}$, $\bar{G}$ (and thus $\bar{X}$) fixed in the background. The next two lemmas describe how λ^i affects π_T^i and γ_j^i respectively.

Lemma A.1 *Country i's terms of trade π_T^i depends on the division λ^i of a given $\bar{X}$ for $i \neq j = 1, 2$, as follows:*

(a) $\partial\pi_T^i/\partial\lambda^i > 0$ *and, for* $\lambda^i > \lambda^j$, $\partial^2\pi_T^i/(\partial\lambda^i)^2 \geq 0$.

(b) $\lim_{\lambda^i \to 0} \pi_T^i = 0$, $\lim_{\lambda^i \to 1/2} \pi_T^i = 1$ *and* $\lim_{\lambda^i \to 1} \pi_T^i = \infty$.

(c) *If* $\lambda^i \gtreqqless \frac{1}{2}$ *then* $\pi_T^i \gtreqqless 1$ *and* $p^i \gtreqqless p^j$.

Proof: *Part (a).* Because the supply of country i's intermediate input is $Z^i = \lambda^i \bar{X}$ we have $\widehat{Z}^i - \widehat{Z}^j = \widehat{\lambda}^i - \widehat{\lambda}^j = \left(\frac{1}{\lambda^i} + \frac{1}{\lambda^j}\right) d\lambda^i$ in (A.5) for any given $\bar{X}$. Since $\Delta > 0$, we thus have

$$\frac{\partial\pi_T^i}{\partial\lambda^i} = \frac{1}{\lambda^i\lambda^j\Delta} > 0,$$

which proves the first portion of part (a).

To prove the convexity of π_T^i in λ^i for $\lambda^i \geq \lambda^j$, we differentiate the expression above with respect to λ^i, keeping in mind that $\lambda^j = 1 - \lambda^i$ and using the definition of Δ in (15b) and the facts that $p^i = \tau\pi_T^i$, $\pi_T^j = 1/\pi_T^i$ and $\pi_T^i\left(\partial\gamma_j^i/\partial\pi_T^i\right) = -(\sigma-1)\,\gamma_i^i\gamma_j^i$ for $i \neq j = 1, 2$:

$$\begin{aligned}\frac{\partial^2\pi_T^i}{(\partial\lambda^i)^2} &= \frac{\lambda^i-\lambda^j}{(\lambda^i\lambda^j)^2\Delta} + \frac{(\sigma-1)}{\lambda^i\lambda^j\Delta^2}\left[\pi_T^i\left(\frac{\partial\gamma_j^i}{\partial\pi_T^i}\right) + \pi_T^j\left(\frac{\partial\gamma_i^j}{\partial\pi_T^j}\right)\right]\frac{\partial\pi_T^i/\partial\lambda^i}{\pi_T^i} \\ &= \frac{\lambda^i-\lambda^j}{(\lambda^i\lambda^j)^2\Delta} + \frac{(\sigma-1)^2}{\pi_T^i\,(\lambda^i\lambda^j)^2\Delta^3}\left(\gamma_j^j\gamma_i^j - \gamma_i^i\gamma_j^i\right).\end{aligned}$$

The first term in the last line of the expression above is non-negative due to our assumption that $\lambda^i \geq \lambda^j$. Hence, it is sufficient to show that the second term is non-negative as well. Using the definitions of the expenditure shares in terms of internal prices gives

$$\begin{aligned}\gamma_j^j\gamma_i^j - \gamma_i^i\gamma_j^i &= \frac{(p^j)^{1-\sigma}}{[1+(p^j)^{1-\sigma}]^2} - \frac{(p^i)^{1-\sigma}}{[1+(p^i)^{1-\sigma}]^2} = \frac{\left[(p^i)^{\sigma-1}-(p^j)^{\sigma-1}\right]\left[\left(p^ip^j\right)^{\sigma-1}-1\right]}{[1+(p^i)^{\sigma-1}]^2\,[1+(p^j)^{\sigma-1}]^2} \\ &= \frac{(p^j)^{\sigma-1}\left[\left(\pi_T^i\right)^{2(\sigma-1)}-1\right]\left[\tau^{2(\sigma-1)}-1\right]}{[1+(p^i)^{\sigma-1}]\,[1+(p^j)^{\sigma-1}]}.\end{aligned}$$

Since $\tau \geq 1$, the desired result follows from part (c) of the lemma (shown below), that $\lambda^i \gtreqless \frac{1}{2}$ implies $\pi_T^i \gtreqless 1$, thereby establishing the convexity of π_T^i in λ_i for $\lambda^i > \lambda^j$.

Part (b). The expenditure shares can be written, using $p^i = \tau\pi_T^i$ and noting $\pi_T^j = 1/\pi_T^i$, as

$$\gamma_j^i = \frac{1}{1+\tau^{\sigma-1}\left(\pi_T^i\right)^{\sigma-1}} \quad \text{and} \quad \gamma_i^j = \frac{\left(\pi_T^i\right)^{\sigma-1}}{\left(\pi_T^i\right)^{\sigma-1}+\tau^{\sigma-1}}, \tag{A.6}$$

where $\widehat{\gamma}_j^i = -(\sigma-1)\left(1-\gamma_j^i\right)\widehat{\pi}_T^i$. Substituting these expressions along with $Z^i = \lambda^i\bar{X}$ and $Z^j = (1-\lambda^i)\bar{X}$ into the world market clearing condition (5), after some rearranging, gives

$$\left[\frac{1+\tau^{\sigma-1}\left(\pi_T^i\right)^{\sigma-1}}{\tau^{\sigma-1}+\left(\pi_T^i\right)^{\sigma-1}}\right]\left(\pi_T^i\right)^{\sigma} = \frac{\lambda^i}{1-\lambda^i}. \tag{A.7}$$

To proceed, we study the behavior of π_T^i on the LHS of the condition above as λ^i varies on the RHS. Now observe that the RHS behaves as follows: (i) $\lim_{\lambda^i\to 0}\text{RHS} = 0$, (ii) $\lim_{\lambda^i\to 1/2}\text{RHS} = 1$, and (iii) $\lim_{\lambda^i\to 1}\text{RHS} = \infty$. Clearly, the limits of the LHS must match the respective limits of the RHS in all three cases. In what follows, keep in mind that, for any finite $\tau \geq 1$ and $\sigma > 1$, $\tau^{\sigma-1}$ in the LHS is finitely positive. In case (i), the expression inside the square brackets of the LHS is finitely positive for all $\pi_T^i \geq 0$. Therefore, $\lim_{\lambda^i\to 0}\text{LHS} = 0$ only if $\lim_{\lambda^i\to 0}\pi_T^i = 0$. Similarly, in case (ii), $\lim_{\lambda^i\to 1/2}\pi_T^i = 1$ because no other value of π_T^i ensures $\lim_{\lambda^i\to 1/2}\text{LHS} = 1$. Lastly, in case (iii), $\lim_{\lambda^i\to 1}\pi_T^i = \infty$ because the expression inside the square brackets of the LHS is finitely positive for all $\tau \geq 1$ and $\pi_T^i \geq 0$ (including the case of $\pi_T^i \to \infty$).

Part (c). This part follows readily from the first component of part (a) and part (b). ||

Lemma A.2 *For $i \neq j = 1, 2$, the division λ^i of $\bar{X}$ has the following implications for the expenditure shares:*

(a) $\lim_{\lambda^i \to 1/2} \gamma^i_j \leq \frac{1}{2}$, $\lim_{\lambda^i \to 1} \gamma^i_j = 0$ *and* $\lim_{\lambda^i \to 1} \gamma^i_j / \lambda^j = \infty$.

(b) *If* $\lambda^i \gtreqless \frac{1}{2}$ *then* $\gamma^i_i \gtreqless \gamma^j_j$ *and* $\gamma^i_j \lesseqgtr \gamma^j_i$.

Proof: *Part (a).* The first component of part (a) follows from the second component of Lemma A.1(b), the definition of γ^i_j in (A.6), and the assumptions that $\tau \geq 1$ and that $\sigma > 1$. The second component follows from the third component of Lemma A.1(b), which implies $\lim_{\lambda^i \to 1} \left(\pi^i_T\right)^{\sigma - 1} = \infty$ and (A.6). The last component of part (a) follows by rewriting (5) as $\gamma^i_j / \lambda^j = \pi^i_T \gamma^j_i / \lambda^i$ and by noting that the limit of the RHS is

$$\lim_{\lambda^i \to 1} \left(\frac{\gamma^j_i}{\lambda^i} \right) \times \lim_{\lambda^i \to 1} \left(\pi^i_T \right) = \left[\frac{\lim_{\lambda^i \to 1} (\gamma^j_i)}{\lim_{\lambda^i \to 1} (\lambda^i)} \right] \times \lim_{\lambda^i \to 1} \left(\pi^i_T \right) = \left[\frac{1}{1} \right] \times \infty,$$

which implies $\lim_{\lambda^i \to 1} \text{LHS} = \lim_{\lambda^i \to 1} (\gamma^i_j / \lambda^j) = \infty$. Thus, the convergence of γ^i_j to 0 is slower that the convergence of λ^j to 0 as $\lambda^i \to 0$.

Part (b). The two components of this part follow from straightforward calculations using Lemma A.1(b) and the expressions for the expenditure shares in (A.6). ||

Lemma A.3 *If* $\lambda^i \gtreqless \frac{1}{2}$ *then* $d\pi^i_T / d\tau \lesseqgtr 0$ *and* $d\pi^i_T / d\sigma \lesseqgtr 0$.

Proof: This lemma follows from (A.5), which shows how π^i_T depends on τ and σ, with Lemmas A.1(c) and A.2(b), conditional on the division of $\bar{X}$. It suggests that larger trade costs and a greater distinction between traded commodities impart a home bias in favor of the country with the largest effective endowment. ||

To identify the effect of changes in countries' effects endowments Z^i and Z^j on country i's payoff w^i_T under trade, we use (6) along with (A.5) and the fact that $p^i \mu^i_{p^i} / \mu^i = -\gamma^i_j$:

$$\widehat{w}^i_T = \widehat{Z}^i - \gamma^i_j \widehat{\pi}^i_T = \left(1 - \frac{\gamma^i_j}{\Delta} \right) \widehat{Z}^i + \frac{\gamma^i_j}{\Delta} \widehat{Z}^j.$$

Since $0 < \gamma^i_j < 1$ whereas $\Delta > 1$, we have $0 < \gamma^i_j / \Delta < 1$, which implies w^i_T unambiguously rises with increases in country i's effective endowment Z^i. As such, immiserizing growth (due to an adverse terms-of-trade effect) does not arise in this context. Similarly, an increase in country j's effective endowment Z^j increases w^i_T because of a favorable (to country i) terms-of-trade effect.[61]

[61] Equi-proportionate increases in Z^i and Z^j would cause both countries' welfare to rise proportionately because they expand each country's income, while leaving world prices unchanged.

Letting $Z^i = \lambda^i \bar{X}$ so that $w_T^i = \omega^i \bar{X}$ where as previously defined $\omega^i \equiv \mu^i \lambda^i$, we now explore how an arbitrary division λ^i of the common pool $\bar{X}$ and the quantity of guns $\bar{G}$ affect w_T^i. Naturally, $dw_T^i/d\bar{G} = -\omega^i$ and $dw_T^i/d\lambda^i = \bar{X}\omega_{\lambda^i}^i$. As noted in (15a), one can show (from the definition of ω^i and (A.5)) that

$$\omega_{\lambda^i}^i = \mu^i \left(1 - \frac{\gamma_j^i/\lambda^j}{\Delta}\right). \tag{A.8a}$$

In addition, $dw_T^i/d\xi = \bar{X}\omega_\xi^i$ for $\xi \in \{\tau, \sigma\}$, so that the dependence of ω^i, not just on λ^i and $\bar{G}$, but also on trade costs and the elasticity of substitution are important. After some algebra, using facts that $p^i = \tau\pi_T^i$ and $p^i\mu_{p^i}^i/\mu^i = -\gamma_j^i$, along with (A.5) and the expression for $\Delta > 0$ in (15b), we find:

$$\omega_\tau^i = \lambda^i \mu_{p^i}^i p_\tau^i = \omega^i \left(p^i \mu_{p^i}^i/\mu^i\right)\left(\tau p_\tau^i/p^i\right)\frac{1}{\tau} \tag{A.8b}$$

$$= -\frac{\omega^i\gamma_j^i}{\tau\Delta}\left[1 + 2\left(\sigma - 1\right)\left(1 - \gamma_i^j\right)\right] < 0. \tag{A.8c}$$

In short, the (direct) effect of an increase in trade costs on a country's payoff under trade, keeping λ^i and $\bar{G}$ fixed, is negative.

The effect of σ on ω^i is a bit more involved as in this case we have

$$\begin{aligned}
\omega_\sigma^i &= \lambda^i \left\{\mu_\sigma^i + \mu_{p^i}^i p_\sigma^i\right\} = \omega^i \left\{\mu_\sigma^i/\mu^i + \left(p^i\mu_{p^i}^i/\mu^i\right)\left(p_\sigma^i/p^i\right)\right\} \\
&= \omega^i \left\{-\frac{1}{\left(\sigma-1\right)^2}\left[\left(\sigma - 1\right)\gamma_j^i \ln p^i + \ln\left[1 + \left(p^i\right)^{1-\sigma}\right]\right]\right. \\
&\quad \left. + \frac{\gamma_j^i}{\Delta}\left[\gamma_i^i \ln p^i - \gamma_j^j \ln p^j\right]\right\}.
\end{aligned}$$

Using (15b) and the properties of logarithms, the above equation can be rewritten (after some additional algebra) as

$$\begin{aligned}
\omega_\sigma^i &= -\frac{\omega^i}{\Delta\left(\sigma-1\right)^2}\left\{\Delta \ln[1 + (p^i)^{1-\sigma}] - \gamma_j^i \ln\left(p^i\right)^{1-\sigma}\right. \\
&\quad \left. - \left(\sigma - 1\right)\gamma_j^i\gamma_j^j \ln(p^i)^{1-\sigma} - (\sigma-1)\gamma_j^i\gamma_j^j \ln(p^j)^{1-\sigma}\right\} \\
&= -\frac{\omega^i}{\Delta\left(\sigma-1\right)^2}\left\{\Delta \ln[1 + (p^i)^{1-\sigma}] - \gamma_j^i \ln(p^i)^{1-\sigma} + 2(\sigma-1)^2\gamma_j^i\gamma_j^j \ln\tau\right\}.
\end{aligned} \tag{A.8d}$$

Inspection of the RHS of the last expression reveals that $\omega_\sigma^i < 0$ for the following reasons: (i) $\Delta > \gamma_j^i$ implies $\Delta \ln[1 + (p^i)^{1-\sigma}] - \gamma_j^i \ln(p^i)^{1-\sigma} > 0$; and (ii) $\tau \geq 1$ implies the last term in the curly brackets is positive.

Let us now study in finer detail the dependence of ω^i on the division λ^i of $\bar{X}$. In particular, starting with $\lambda^i = 0$, let us ask how arbitrary reallocations of $\bar{X}$ from country j to country i $(i \neq j)$ affect ω^i and ω^j. Going back to (A.8a), one can see that the direct effect of such a resource transfer is to improve (worsen) the recipient's (donor's) purchasing power and thus its payoff. However, the transfer also causes the recipient (donor) country's terms-of-trade to deteriorate (improve), so this indirect effect works against the direct effect.[62] The presence of this trade-off raises the following questions. Is there an optimal division, λ_T^i, of the common pool that would maximize country i's payoff ω^i? If there is, what are its properties? Furthermore, is it possible for resource transfers to immiserize both the recipient and the donor countries?[63] These questions are of interest in their own right. But, as we will see later, they are of special interest in the context of the resource disputes we are studying due to their consequences for arming and the division of the common pool which in turn have strong implications for countries' preferences over war and peace.

The next lemma summarizes several noteworthy properties of ω^i.

Lemma A.4 *For any given guns, the payoff ω^i $(\equiv \mu^i \lambda^i)$ depends on the division λ^i of $\bar{X}$, the elasticity of substitution $\sigma \in (1, \infty)$ and trade costs $\tau \in [1, \infty)$ as follows:*

(a) (i) $\lim_{\lambda^i \to 0} \omega^i = 0$ *and* $\lim_{\lambda^i \to 1} \omega^i = 1$.

(ii) $\lim_{\lambda^i \to 1/2} \omega^i \gtreqless \lim_{\lambda^i \to 1} \omega^i = 1$ *as* $\sigma \lesseqgtr \breve{\sigma}(\tau)$, *where* $\breve{\sigma}'(\tau) < 0$, $\breve{\sigma}(1) = 2$ *and* $\lim_{\tau \to \infty} \breve{\sigma}(\tau) = 1$.

(iii) $\lambda^i \gtreqless \frac{1}{2} \;\Rightarrow\; \omega^i \gtreqless \omega^j$.

(b) ω^i *is strictly concave in* $\lambda^i \in (0,1)$ *and attains a maximum* $\lambda_T^i = \arg\max_{\lambda^i} \omega^i$.

(c) $\lambda_T^i \in (\frac{1}{2}, 1)$ *is increasing in* σ *and* τ, $\lim_{\sigma \to 1} \lambda_T^i = \frac{1}{2}$ *and* $\lim_{\sigma \to \infty} \lambda_T^i = \lim_{\tau \to \infty} \lambda_T^i = 1$.

(d) $\omega_{\lambda^i}^i < 0$ *and* $\omega_{\lambda^i}^j < 0$ *for all* $\lambda^i \in \left(\lambda_T^i, 1\right)$ $(i \neq j = 1, 2)$.

(e) $\partial \omega^i / \partial \xi < 0$ *for* $\xi \in \{\tau, \sigma\}$.

Proof: *Part (a).* For any $\lambda^i \in (0,1)$, given $1 < \sigma < \infty$ and $\tau < \infty$, we have

$$\mu^i \equiv \left[1 + \tau^{1-\sigma} \left(\pi_T^i\right)^{1-\sigma}\right]^{1/(\sigma-1)} > 1. \tag{A.9}$$

From Lemma A.1(b), we know that (i) $\lim_{\lambda^i \to 1} \pi_T^i = \infty$, which implies $\lim_{\lambda^i \to 1} \mu^i = 1$; and

[62] Once again, for now guns $\bar{G}$ and thus $\bar{X}$ are kept fixed in the background. One can also think of such reallocations as a resource gift from country j (the donor) to country i (the recipient). Amano (1966) addresses this terms-of-trade issue in a variety of contexts. However, he does not study the welfare implications of resource transfers for both donor and recipient countries. Garfinkel et al. (2018) examine a variant of this issue in the context of a modified Ricardian model of trade and conflict.

[63] In the standard trade literature that considers pure *income* transfers between two trading partners, this possibility does not arise. In fact, stability of the world trading equilibrium necessarily implies that the recipient enjoys a welfare improvement while the donor suffers a welfare loss. Prior work in this area (e.g., Brecher and Bhagwati, 1982; Bhagwati et al. 1983) also emphasized the idea that, indeed, transfers could worsen the recipient's welfare in the presence of distrortions. Grossman (1984) argued that, when goods are already traded freely, trade in factors can be welfare-reducing. However, his analysis was in the context of factor movements that require earnings in the host country to be transferred back to the country of origin. Moreover, he did not study the possible existence of immisering factor movements in the Pareto sense.

(ii) $\lim_{\lambda^i\to 0}\pi_T^i = 0$, which implies $\lim_{\lambda^i\to 0}\mu^i = \infty$. Then, the definition of ω^i ($\equiv \mu^i\lambda^i$) readily implies $\lim_{\lambda^i\to 1}\omega^i = 1$. To prove $\lim_{\lambda^i\to 0}\omega^i = 0$, rewrite ω^i as

$$\omega^i = \left(\lambda^i/\pi_T^i\right)\left[\left(\pi_T^i\right)^{\sigma-1} + \tau^{1-\sigma}\right]^{1/(\sigma-1)}$$

and note that the expression inside the square brackets is finitely positive as $\lambda^i \to 0$ because $\lim_{\lambda^i\to 0}\pi_T^i = 0$ and $\tau \in [1,\infty)$. Let us rearrange the world market clearing condition (5) using $Z^i = \lambda^i\bar{X}$ for $i = 1, 2$ as $\lambda^i/\pi_T^i = \lambda^j\gamma_i^j/\gamma_j^i$. From Lemma A.2(a), we have $\lim_{\lambda^i\to 0}\gamma_j^i = 1$ and $\lim_{\lambda^i\to 0}\gamma_i^j = 0$ while $\lambda^j \to 1$; thus, $\lim_{\lambda^i\to 0}\left(\lambda^i/\pi_T^i\right) = 0$, thereby completing the proof of (a.i).

To prove part (a.ii), recall that, by Lemma A.1(b), $\lim_{\lambda^i\to 1/2}\pi_T^i = 1$, which implies

$$\lim_{\lambda^i\to 1/2}\omega^i = \tfrac{1}{2}\left[1+\tau^{1-\sigma}\right]^{1/(\sigma-1)}.$$

Equating the expression above to $\lim_{\lambda^i\to 1}\omega^i = 1$ and rearranging terms yields $g(\sigma,\tau) = 1+\tau^{1-\sigma}-2^{\sigma-1} = 0$, which implicitly defines the critical value of σ as a function of τ, $\breve{\sigma}(\tau)$, introduced in the lemma. One can now verify the following: $\breve{\sigma}(1) = 2$, $\lim_{\tau\to\infty}\breve{\sigma}(\tau) = 1$ and, by the implicit function theorem, $\breve{\sigma}'(\tau) = -g_\tau/g_\sigma < 0$.[64]

Part (a.iii) follows from parts (a.i) and (a.ii) and part (b) that follows.

Part (b). We know from Lemma A.2(a) that $\lim_{\lambda^i\to 1}\gamma_j^i = 0$ while $\lim_{\lambda^i\to 1}\gamma_i^j = 1$. Using these observations in the expression for Δ in (15b) readily implies $\lim_{\lambda^i\to 1}\Delta = \sigma > 1$. But from Lemma A.2(a), we also have $\lim_{\lambda^i\to 1}\gamma_j^i/\lambda^j = \infty$. Thus, the expression for $\omega_{\lambda^i}^i$ inside the parentheses in (A.8a) becomes negative as $\lambda^i \to 1$ (i.e., $\lim_{\lambda^i\to 1}\omega_{\lambda^i}^i < 0$), so $\omega^i \to 1$ from above as $\lambda^i \to 1$. Yet, ω^i is continuous in λ^i and from part (a), we know that $\lim_{\lambda^i\to 0}\omega^i = 0$; therefore, ω^i attains a maximum at some λ^i, denoted by λ_T^i, that solves $\omega_{\lambda^i}^i = 0$ in (A.8a).

We now prove that $\omega_{\lambda^i\lambda^i}^i < 0$, which implies that ω^i is concave in λ^i and thereby establishes the uniqueness of λ_T^i. Differentiation of $\omega_{\lambda^i}^i$ in (A.8a) gives

$$\omega_{\lambda^i\lambda^i}^i = \mu^i\left\{\frac{p^i\mu_{p^i}^i}{\mu^i}\left(\frac{p_{\lambda^i}^i}{p^i}-\frac{p_{\lambda^j}^i}{p^i}\right)\left(1-\frac{\gamma_j^i}{\lambda^j\Delta}\right)-\frac{\gamma_j^i}{\left(\lambda^j\right)^2\Delta}-\frac{p^i\left(\partial\gamma_j^i/\partial p^i\right)}{\lambda^j\Delta}\left(\frac{p_{\lambda^i}^i}{p^i}-\frac{p_{\lambda^j}^i}{p^i}\right)\right.$$
$$\left.+\frac{\gamma_j^i}{\lambda^j\Delta^2}\left[p^i\Delta_{p^i}\left(\frac{p_{\lambda^i}^i}{p^i}-\frac{p_{\lambda^j}^i}{p^i}\right)+p^j\Delta_{p^j}\left(\frac{p_{\lambda^i}^j}{p^j}-\frac{p_{\lambda^j}^j}{p^j}\right)\right]\right\}.$$

Using the facts that $p^i\mu_{p^i}^i/\mu^i = -\gamma_j^i$, $p_{\lambda^i}^i/p^i - p_{\lambda^j}^i/p^i = 1/(\lambda^i\lambda^j\Delta)$, $p^i(\partial\gamma_j^i/\partial p^i) = -(\sigma-1)\gamma_i^i\gamma_j^i$,

[64] An alternative approach here would be to find the critical value of τ as a function of σ. For any $\sigma \in (1,2]$, there exists a range of trade cost values $[1,\breve{\tau}(\sigma)]$ such that $\lim_{\lambda^i\to 1/2}\omega^i \geq \lim_{\lambda^i\to 1}\omega^i = 1$ for all $\tau \in [1,\breve{\tau}(\sigma)]$, whereas $\lim_{\lambda^i\to 1/2}\omega^i < \lim_{\lambda^i\to 1}\omega^i = 1$ for all $(\sigma,\tau) \in (1,2]\times(\breve{\tau}(\sigma),\infty)\cup(2,\infty)\times(1,\infty)$. In this case, $\breve{\tau}(\sigma)$ solves $g(\sigma,\tau) = 0$, $\breve{\tau}'(\sigma) < 0$, and $\breve{\tau}(2) = 1$.

and $p^i \Delta_{p^i} = (\sigma - 1)^2 \gamma_i^i \gamma_j^i$ for $i \neq j$ enables us to transform the above expression into:

$$\omega^i_{\lambda^i\lambda^i} = \mu^i \left[-\frac{\gamma_j^i}{\lambda^i \lambda^j \Delta} \left(1 - \frac{\gamma_j^i}{\lambda^j \Delta} \right) - \frac{\gamma_j^i}{(\lambda^j)^2 \Delta} + \frac{(\sigma-1)\gamma_i^i \gamma_j^i}{\lambda^i (\lambda^j \Delta)^2} + \frac{\gamma_j^i (\sigma-1)^2}{\lambda^i (\lambda^j)^2 \Delta^3} \left(\gamma_i^i \gamma_j^i - \gamma_j^j \gamma_i^j \right) \right]$$

$$= -\frac{\gamma_j^i \omega^i}{(\lambda^i \lambda^j \Delta)^2} \left[\gamma_i^i A + \gamma_j^j B \right], \tag{A.10a}$$

where

$$A = 1 + \frac{(\sigma-1)^2}{\Delta} \left(\gamma_i^i + \gamma_j^j - 1 \right) \quad \text{and} \quad B = \frac{\sigma(\sigma-1)}{\Delta}. \tag{A.10b}$$

We now establish that $A > 0$ buy showing that $\gamma_i^i + \gamma_j^j - 1 \geq 0$. Using the definition of the expenditure shares as a function of internal prices, we calculate the following:

$$\gamma_i^i + \gamma_j^j - 1 = \frac{1}{1 + (p^i)^{1-\sigma}} + \frac{1}{1 + (p^j)^{1-\sigma}} - 1 = \frac{(p^i p^j)^{\sigma-1} - 1}{[1 + (p^i)^{\sigma-1}]\,[1 + (p^j)^{\sigma-1}]}$$

$$= \frac{\tau^{2(\sigma-1)} - 1}{[1 + (p^i)^{\sigma-1}]\,[1 + (p^j)^{\sigma-1}]} \geq 0,$$

for $i \neq j = 1, 2$, since $\tau \geq 1$. Thus, $A > 0$. In addition, $B > 0$ holds, because $\sigma > 1$. Thus $\omega^i_{\lambda^i\lambda^i} < 0$ for $i = 1, 2$.

Part (c). Having shown in part (b) that $\lambda_T^i < 1$, we now prove that $\lambda_T^i > \frac{1}{2}$. Evaluating $\omega^i_{\lambda^i}$ at $\lambda^i = \frac{1}{2}$ gives

$$\omega^i_{\lambda^i}\big|_{\lambda^i = \frac{1}{2}} = 2\mu^i \left(\frac{1}{2} - \frac{\gamma_j^i}{\Delta} \right) > 0,$$

where the sign follows from the finding that $\gamma_j^i \leq \frac{1}{2}$ (by Lemma A.2(a)), while $\Delta > 1$. Therefore, $\lim_{\lambda^i \to \frac{1}{2}} \omega^i_{\lambda^i} > 0$.

The dependence of λ_T^i on σ and τ can be studied by using the implicit function theorem, which implies $d\lambda_T^i / d\xi = -\omega^i_{\lambda^i \xi} / \omega^i_{\lambda^i\lambda^i}$, where $\omega^i_{\lambda^i\lambda^i} < 0$. It is straightforward for one to show (by differentiating $\omega^i_{\lambda^i}$ with respect to ξ and evaluating the resulting expression at $\lambda^i = \lambda_T^i$) that $\omega^i_{\lambda^i \xi} > 0$ for $\xi \in \{\sigma, \tau\}$ which proves that $d\lambda_T^i / d\xi > 0$. The last two portions of part (c) follow readily by the taking the appropriate limits of (A.8a) and finding λ_T^i.[65]

Part (d). This part follows from part (c). It is interesting in that it indicates that giving the larger economy an even bigger share in the specified range hurts both countries.

Part (e). This part follows from equations (A.8c) and (A.8d). ||

Next we turn to compare the countries' payoffs under trade with those under autarky for

[65]For the limit as $\tau \to \infty$, the last component of Lemma A.2(a) implies $\omega^i_{\lambda^i} > 0$ for all $\lambda^i \in (0, 1]$.

given arbitrary distributions (λ^i) of fixed $\bar{X}$ across the two regimes. Since $w_T^i = \omega^i \bar{X}$ and $w_A^i = \lambda^i \bar{X}$, the behavior of these payoffs can be understood by the behavior of ω^i relative to that of λ^i.[66]

Lemma A.5 *For any given $\bar{X} > 0$, payoffs under trade (w_T^i) and under autarky (w_A^i) have the following properties:*

(a) (i) $w_T^i > w_A^i$ *for all* $\lambda^i \in (0,1)$.

(ii) $\lim_{\lambda^i \to 0} w_T^i = \lim_{\lambda^i \to 0} w_A^i = 0$.

(iii) $\lim_{\lambda^i \to 1} w_T^i = \lim_{\lambda^i \to 1} w_A^i = \bar{X}$.

(b) $\lim_{\lambda^i \to 1/2} w_T^i \gtreqless \bar{X}$ *as* $\sigma \lesseqgtr \breve{\sigma}(\tau)$, *where* $\breve{\sigma}(1) = 2$ *and* $\breve{\sigma}'(\tau) < 0$.

Proof: Each part follows readily from Lemma A.4. ||

The importance of this lemma for our analysis is twofold. First, when guns are exogenously determined (a situation that serves as a valuable benchmark), trade dominates autarky in payoffs for all possible allocations of the common pool except in the extreme cases where $\lambda^i = 0$ and $\lambda^i = 1$. Second, if the elasticity of substitution σ is sufficiently close to 1 (so that the gains from trade are sufficiently high), a country enjoys a higher payoff under an even split of the common pool relative to a situation in which it controls the entire pool. As will become evident, this finding helps explain the emergence of an equilibrium under settlement with less and, under some circumstances, no arming at all.

The next lemma characterizes the global gains from peace per unit of $\bar{X}$, defined as $\Omega\left(\lambda^i; \sigma, \tau, \beta\right) \equiv S/\bar{X} = \omega^i + \omega^j - \beta$:

Lemma A.6 *For any feasible quantity of guns $\bar{G}$ that yields a common pool of non-negligible size $\bar{X} = \bar{R} - \bar{G} > 0$, the global gains from peace function, $\Omega(\lambda^i; \sigma, \tau, \beta)$, is strictly concave in λ^i and maximized at $\lambda^i = \frac{1}{2}$. Furthermore,*

(a) $\lim_{\lambda^i \to 0} \Omega = \lim_{\lambda^i \to 1} \Omega = 1 - \beta$

(b) $\partial \Omega / \partial(-\xi) > 0$ *for* $\xi \in \{\sigma, \tau, \beta\}$.

Proof: The proof follows in a straightforward way from the properties of the individual components of Ω, studied in Lemma A.4. In particular, the strict concavity of Ω in λ^i is due to the fact that it is the sum of two strictly concave functions ω^i and ω^j. The reason a benevolent social planner would choose $\lambda^i = \frac{1}{2}$ is threefold: (i) the production function $F(\cdot, \cdot)$ of the final good (2) is symmetric across countries $i = 1, 2$; (ii) the technologies of countries' respective intermediate goods are identical; and (iii) the rate of destruction $1 - \beta$ is fixed. Part (a) is fairly obvious: if all of $\bar{X}$ is allocated to a single country, there are no

[66] One should keep in mind, though, that our upcoming comparison of conflict and settlement will be complicated by the following facts: (i) The size of the common pool $\bar{X}$ under these regimes will differ because arming incentives across these regimes will differ. (ii) In addition to guns, the division of $\bar{X}$ under these regimes will be endogenous and thus will differ.

gains from trade. Nonetheless, peace could still generate global gains through the avoidance of destruction. Part (b) follows from Lemma A.4(e) and the definition of Ω above. $\quad ||$

Lemma A.7 *The function $\Psi^i(\lambda^i, G^i, G^j; \sigma, \tau, \beta)$ which appears in the split-the-surplus condition (13), is strictly quasiconcave (resp., quasiconvex) in $\lambda^i \in [\frac{1}{2}, 1]$ (resp., $\lambda^i \in [0, \frac{1}{2}]$); hence Ψ^i admits a unique maximum $\lambda^i_{\max} = \arg\max_{\lambda^i} \Psi^i$ (resp., unique minimum $\lambda^i_{\min} = \arg\min_{\lambda^i} \Psi^i$), such that $\Psi^i_{\lambda^i} > 0$ for $\lambda^i \in (\lambda^i_{\min}, \lambda^i_{\max})$. Additionally, $\lambda^i_{\min} = 1 - \lambda^i_{\max}$ and*

(a) *$\lambda^i_{\max} \in (\frac{1}{2}, 1)$ and $\Psi^i\big|_{\lambda^i=\lambda^i_{\max}} > 0$ for $\sigma - \tau < 1$, whereas*

(b) *$\lambda^i_{\max} = 1$ and $\Psi^i\big|_{\lambda^i=\lambda^i_{\max}} \geq 0$ (with equality if $\beta = 1$ and $\phi^i = 1$) for $\sigma - \tau \geq 1$.*

Proof: To prove that Ψ^i is strictly quasiconcave in $\lambda^i \in [\frac{1}{2}, 1]$ it suffices to show that there exists a $\lambda^i_{\max} \in [\frac{1}{2}, 1]$ such that $\Psi^i_{\lambda^i} > 0$ for $\lambda^i \in [\frac{1}{2}, \lambda^i_{\max})$ and $\Psi^i_{\lambda^i} < 0$ for $\lambda^i \in (\lambda^i_{\max}, 1]$. First note, from Lemma A.4(c), that $\omega^i_{\lambda^i} > 0$ and $\omega^j_{\lambda^j} > 0$ for $\lambda^i \in [\frac{1}{2}, \lambda^i_T)$, while $\omega^i_{\lambda^i} < 0$ as $\lambda^i \to 1$; therefore, $\Psi^i_{\lambda^i} = \omega^i_{\lambda^i} + \omega^j_{\lambda^j} > 0$ for $\lambda^i \in [\frac{1}{2}, \lambda^i_T + \varepsilon)$ for some $\varepsilon > 0$. We do not know the sign of $\Psi^i_{\lambda^i}$ for $\lambda^i \in [\lambda^i_T + \varepsilon, 1]$. There are two possibilities: Either $\Psi^i_{\lambda^i} > 0$ for all $\lambda^i \in [\frac{1}{2}, 1]$, in which case $\lambda^i_{\max} = 1$ and we are done, or there exists at least one $\lambda^i_{\max} \in (\lambda^i_T, 1]$ such that $\Psi^i_{\lambda^i} = 0$. Focusing on the latter case, we prove that $\lambda^i_{\max}$ is a unique maximizer of Ψ^i because $\Psi^i_{\lambda^i\lambda^i}\big|_{\lambda^i=\lambda^i_{\max}} < 0$.

By its definition, $\Psi^i_{\lambda^i\lambda^i} = \omega^i_{\lambda^i\lambda^i} - \omega^j_{\lambda^j\lambda^j}$ $(i \neq j)$, where $\omega^i_{\lambda^i\lambda^i} < 0$ for $i = 1, 2$ from Lemma A.4(b). Thus, to prove $\Psi^i_{\lambda^i\lambda^i}\big|_{\lambda^i=\lambda^i_{\max}} < 0$, it suffices to prove that $(\omega^i_{\lambda^i\lambda^i}/\omega^j_{\lambda^j\lambda^j})|_{\Psi_{\lambda^i}=0} > 1$. From the expression for $\omega^i_{\lambda^i\lambda^i}$ in (A.10a) and the definition $\omega^i = \mu^i\lambda^i$ for $i = 1, 2$, we have

$$\frac{\omega^i_{\lambda^i\lambda^i}}{\omega^j_{\lambda^j\lambda^j}} = \frac{\omega^i\gamma^i_j[\gamma^i_i A + \gamma^j_j B]}{\omega^j\gamma^j_i[\gamma^j_j A + \gamma^i_i B]} = \frac{\mu^i\lambda^i\gamma^i_j[\gamma^i_i A + \gamma^j_j B]}{\mu^j\lambda^j\gamma^j_i[\gamma^j_j A + \gamma^i_i B]}, \tag{A.11}$$

where A and B were defined in (A.10b). Furthermore, $\omega^i_{\lambda^i} < 0$ and $\omega^j_{\lambda^j} > 0$ at $\lambda^i_{\max}$. Applying (A.8a) to $\Psi_{\lambda^i} = \omega^i_{\lambda^i} + \omega^j_{\lambda^j} = 0$ and rearranging terms gives

$$\frac{\mu^i}{\mu^j} = \left(1 - \frac{\gamma^j_i/\lambda^i}{\Delta}\right) \bigg/ \left(-1 + \frac{\gamma^i_j/\lambda^j}{\Delta}\right) > 0.$$

Substituting the above expression in (A.11) implies

$$\left.\frac{\omega^i_{\lambda^i\lambda^i}}{\omega^j_{\lambda^j\lambda^j}}\right|_{\Psi_{\lambda^i}=0} = \frac{\left(\lambda^i\Delta - \gamma^j_i\right)\gamma^i_j\left[\gamma^i_i A + \gamma^j_j B\right]}{\left(-\lambda^j\Delta + \gamma^i_j\right)\gamma^j_i\left[\gamma^j_j A + \gamma^i_i B\right]}. \tag{A.11$'$}$$

As one can confirm, both the numerator and the denominator of the above expression are positive. As such, we subtract the latter from the former to obtain

$$C = \left(\lambda^i\Delta - \gamma^j_i\right)\gamma^i_j\left[\gamma^i_i A + \gamma^j_j B\right] - \left(-\lambda^j\Delta + \gamma^i_j\right)\gamma^j_i\left[\gamma^j_j A + \gamma^i_i B\right].$$

Thus, to prove that the expression in (A.11′) is larger that 1 it suffices to prove that $C > 0$.

Applying the definitions of A and B from (A.10b) and $\Delta = 1 + (\sigma - 1)(\gamma_i^i + \gamma_j^j)$ in the above expression gives, after rearrangement and simplification,

$$\text{sign}\,(C) = \text{sign}\,(C_1 + C_2 + C_3)$$

where

$$\begin{aligned} C_1 &\equiv \gamma_i^i \gamma_j^i (\lambda^i - \gamma_i^j) + \gamma_j^j \gamma_i^j (\lambda^j - \gamma_j^i) = \frac{\left[(p^i p^j)^{\sigma-1} - 1\right][\lambda^i (p^j)^{\sigma-1} + \lambda^j (p^i)^{\sigma-1}\right]}{\left[1 + (p^i)^{\sigma-1}]^2 [1 + (p^j)^{\sigma-1}\right]^2} > 0 \\ C_2 &\equiv (\sigma - 1)(\lambda^i \gamma_j^j \gamma_j^i + \lambda^j \gamma_i^i \gamma_i^j) > 0 \\ C_3 &\equiv (\sigma - 1)(\gamma_i^i + \gamma_j^j - 1)(\lambda^i \gamma_i^i \gamma_j^i + \lambda^j \gamma_j^j \gamma_i^j) > 0. \end{aligned}$$

C_1 is positive since the facts that $p^i = \tau \pi_T^i$ for $i = 1, 2$ and $\pi_T^j = 1/\pi_T^i$ imply $(p^i p^j)^{\sigma-1} = (\tau)^{2(\sigma-1)} > 1$. The sign of C_2 is clearly positive. Lastly, one can confirm that C_3 is also positive, using our earlier finding in the proof of Lemma A.4(b) that $\gamma_i^i + \gamma_j^j - 1 > 0$. Thus, $C > 0$ and $\Psi^i_{\lambda^i \lambda^i}\big|_{\lambda^i = \lambda^i_{\max}} < 0$.

Observe that, since $\lim_{\lambda^i \to 1} \omega^j_{\lambda^j} > 0$, $\text{sign}\big[\lim_{\lambda^i \to 1} \Psi^i_{\lambda^i}\big] = \text{sign}\big[\lim_{\lambda^i \to 1}(1 + \omega^i_{\lambda^i}/\omega^j_{\lambda^j})\big]$. We now show that $\text{sign}\big[\lim_{\lambda^i \to 1}(1 + \omega^i_{\lambda^i}/\omega^j_{\lambda^j})\big] = \text{sign}(\sigma - 1 - \tau)$. From (A.8a) we have

$$1 + \frac{\omega^i_{\lambda^i}}{\omega^j_{\lambda^j}} = 1 + \frac{\mu^i \left(1 - \frac{\gamma_j^i/\lambda^j}{\Delta}\right)}{\mu^j \left(1 - \frac{\gamma_i^j/\lambda^i}{\Delta}\right)}, \quad \text{for} \quad i \neq j = 1, 2. \tag{A.12}$$

Using the definition of μ^i in (A.9) with $p^j = \tau/\pi_T^i$, we rewrite μ^j as

$$\mu^j = \left[1 + (p^j)^{1-\sigma}\right]^{\frac{1}{\sigma-1}} = \left(p^j\right)^{-1} \left[1 + (p^j)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} = \frac{\pi_T^i}{\tau} \left[1 + (p^j)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}.$$

Now, substitute $\pi_T^i = \frac{\lambda^i \gamma_j^i}{\lambda^j \gamma_i^j} = \frac{\gamma_j^i/\lambda^j}{\gamma_i^j/\lambda^i}$ from the world market clearing condition into the above expression, and then use the resulting expression to rewrite (A.12) as

$$1 + \frac{\mu^i \left(1 - \frac{\gamma_j^i/\lambda^j}{\Delta}\right) \tau \frac{\gamma_i^j/\lambda^i}{\gamma_j^i/\lambda^j}}{\left(1 - \frac{\gamma_i^j/\lambda^i}{\Delta}\right) \left[1 + (\tau/\pi_T^i)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}} = 1 + \frac{\mu^i \left(\frac{1}{\gamma_j^i/\lambda^j} - \frac{1}{\Delta}\right) \gamma_i^j/\lambda^i \tau}{\left(1 - \frac{\gamma_i^j/\lambda^i}{\Delta}\right) \left[1 + (\tau/\pi_T^i)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}}.$$

Using the facts that $p^i = \tau \pi_T^i$ and $\pi_T^j = \tau/\pi_T^i$, along with Lemmas A.1(b) and A.2(a), we have $\lim_{\lambda^i \to 1} \mu^i = 1$, $\lim_{\lambda^i \to 1} \mu^j = \infty$, $\lim_{\lambda^i \to 1}(\gamma_j^i/\lambda^j) = \infty$, $\lim_{\lambda^i \to 1}(\gamma_i^j/\lambda^i) = 1$, and

$\lim_{\lambda^i \to 1} \Delta = \sigma$. Based on these results, one can then verify the following:

$$\lim_{\lambda^i \to 1} \left(1 + \omega^i_{\lambda^i} / \omega^j_{\lambda^j}\right) = 1 + \frac{1 \times \left[\frac{1}{\infty} - \frac{1}{\sigma}\right] \times 1 \times \tau}{\left(1 - \frac{1}{\sigma}\right) \left[1 + (\frac{\tau}{\infty})^{\sigma-1}\right]^{\frac{1}{\sigma-1}}} = \frac{\sigma - 1 - \tau}{\sigma - 1}.$$

Since $\sigma > 1$, we have $\text{sign}\{\lim_{\lambda^i \to 1} \Psi^i_{\lambda^i}\} = \text{sign}\{\lim_{\lambda^i \to 1} \left(1 + \omega^i_{\lambda^i} / \omega^j_{\lambda^j}\right)\} = \text{sign}\{\sigma - 1 - \tau\}$.

Because $\Psi(\cdot)$ in (13) is a symmetric function with respect to λ^i and guns (so that the properties of Ψ^i also hold true for Ψ^j), Ψ^j is strictly quasiconcave in $\lambda^j \in [\frac{1}{2}, 1]$; thus Ψ^i is strictly quasiconvex in $\lambda^i \in [0, \frac{1}{2}]$ and attains a minimum at $\lambda^i_{\text{min}} = 1 - \lambda^i_{\text{max}}$. Furthermore, $\lambda^j_{\text{max}} = \lambda^i_{\text{max}}$ for $i \neq j = 1, 2$. We prove uniqueness below.

Part (a). Suppose $\sigma - 1 - \tau < 0$, so that the $\omega^i - \omega^j$ component of Ψ^i approaches 1 from above as $\lambda^i \to 1$. Also note that $\omega^i - \omega^j = 0$ at $\lambda^i = \frac{1}{2}$. It follows that $\omega^i - \omega^j$ (and thus Ψ^i) will attain a maximum at some $\lambda^i_{\text{max}} \in (\lambda^i_T, 1)$. This maximum has to be unique because $\Psi^i_{\lambda^i\lambda^i}\big|_{\lambda^i = \lambda^i_{\text{max}}} < 0$ at all extrema in $[\frac{1}{2}, 1]$, so $\left(\omega^i - \omega^j\right)\big|_{\lambda^i = \lambda^i_{\text{max}}} > 1$. The definition of Ψ^i now implies

$$\Psi^i\big|_{\lambda^i = \lambda^i_{\text{max}}} = \left(\omega^i - \omega^j\right)\big|_{\lambda^i = \lambda^i_{\text{max}}} - \beta\left(\phi^i - \phi^j\right) > 1 - \beta\left(\phi^i - \phi^j\right) \geq 0,$$

thus completing the proof to this part.

Part (b). Now suppose $\sigma - 1 - \tau > 0$, so that the $\omega^i - \omega^j$ component of Ψ^i approaches 1 from below. To confirm the claim in this part, suppose that, in addition to $\lambda^i_{\text{max}} = 1$, there exists another (local) interior maximum such that $\overline{\lambda}^i_{\text{max}} < 1$. Because $\lim_{\lambda^i \to 1} \Psi^i_{\lambda^i} > 0$ and $\lim_{\lambda^i \to 1/2} \Psi^i_{\lambda^i} > 0$, this type of maximum can exist only if, along with it, there also exists a local (interior) minimum at which $\Psi^i_{\lambda^i} = 0$. But this is impossible because we have shown that $\Psi^i_{\lambda^i\lambda^i}\big|_{\lambda^i = \lambda^i_{\text{max}}} < 0$ at any interior extremum in $[\frac{1}{2}, 1]$. Thus, $\lambda^i_{\text{max}} = 1$ is unique, $\left(\omega^i - \omega^j\right)\big|_{\lambda^i = \lambda^i_{\text{max}}} = 1$ and $\Psi^i\big|_{\lambda^i = \lambda^i_{\text{max}}} = 1 - \beta\left(\phi^i - \phi^j\right) \geq 0$ with equality only if $\beta = 1$ and $\phi^i = 1$. ||

Proof of Lemma 1. The unique division $\overline{\lambda}^i$ follows from the fact (established in Lemma A.7) that the $\omega^i - \omega^j$ component of Ψ^i is continuously increasing in $\lambda^i \in [\frac{1}{2}, \lambda^i_{\text{max}}]$, taking values between 0 and a number ≥ 1. Thus, if $\phi^i = 1$ (which, from our specification in (1), arises if $G^i > 0$ and $G^j = 0$), then there will exist a unique solution $\overline{\lambda}^i$ such that $\Psi^i(\lambda^i) = 0$. That $\underline{\lambda}^i = 1- \overline{\lambda}^i$ holds is due to the symmetric structure of $\Psi(\cdot)$. Clearly, $\overline{\lambda}^i \in (\frac{1}{2}, \lambda^i_{\text{max}}]$. The reason $\overline{\lambda}^i = \lambda^i_{\text{max}} = 1$ only if $\sigma - \tau \geq 1$ and $\beta = 1$ is a consequence of Lemma A.7(b). The existence of a unique division $\lambda^i(\cdot) = \{\lambda^i \mid \Psi^i\left(\lambda^i\right) = 0\}$, too, follows straightforwardly from the intermediate value theorem and Lemma A.7.

Part (a). Share $\lambda^i(\cdot)$ is symmetric in G^i and G^j because $\Psi(\cdot)$ is symmetric in these variables. The second component follows again from symmetry and part (b) below.

Part (b). The proof follows from the properties of the conflict technology in (1) (i.e., that $\phi^i_{G^i} > 0$ and $\phi^i_{G^j} < 0$ for $i \neq j = 1, 2$) and the fact that $\Psi^i_{\lambda^i} = \omega^i_{\lambda^i} + \omega^j_{\lambda^j} > 0$ for any feasible $G^i \in [0, R^i)$ and $G^j \in [0, R^j)$.

Part (c). This part follows readily by differentiating (13) totally and applying the implicit function theorem. $||$

Fig. A.1 illustrates the central ideas of Lemma A.7 as well as the determination of the shares taken up in Lemma 1. The next lemma establishes several useful properties of $\widetilde{v}^i$ that depend on the properties of λ^i and, thus, on those of Ψ^i and that, in turn, ensure existence and uniqueness of an interior equilibrium.

Lemma A.8 *Country i's unconstrained per period payoff function $\widetilde{v}^i$ has the following properties for $G^j > 0$ ($i \neq j = 1, 2$):*

(a) $\widetilde{v}^i_{G^iG^i} < 0$

(b) $\widetilde{v}^i_{G^iG^j} \geq 0$ *for* $G^i \geq G^j$

(c) $\widetilde{v}^i_{G^jG^j}|_{\widetilde{B}^i_s = G^j} < 1$.

Proof: *Part (a).* To prove the strict concavity of $\widetilde{v}^i$, we differentiate (16) with respect to G^i. Simplifying the resulting expression yields

$$\widetilde{v}^i_{G^iG^i} = \left[\frac{\omega^i_{\lambda^i}\omega^j_{\lambda^j\lambda^j} + \omega^j_{\lambda^j}\omega^i_{\lambda^i\lambda^i}}{(\omega^i_{\lambda^i} + \omega^j_{\lambda^j})^2} \left(2\beta\bar{X}\phi^i_{G^i}\right) + \omega^i_{\lambda^i}\left(-2 + \bar{X}\phi^i_{G^iG^i}/\phi^i_{G^i}\right) \right] \lambda^i_{G^i}, \tag{A.13a}$$

where $\omega^i_{\lambda^i\lambda^i} < 0$ and $\omega^j_{\lambda^j\lambda^j} < 0$ hold by Lemma A.4, while $\omega^i_{\lambda^i} > 0$ and $\omega^j_{\lambda^j} > 0$ follow from Lemma 1. Since we also have $\phi^i_{G^i} > 0$, the first expression inside the square brackets is negative. But the second expression inside these brackets is also negative since $\phi^i_{G^iG^i} < 0$ for $i = 1, 2$. Lastly, from Lemma 1(b), we have $\lambda^i_{G^i} > 0$. It is, thus, clear that $\widetilde{v}^i_{G^iG^i} < 0$ for $i = 1, 2$.

Part (b). Differentiating (16) with respect to G^j gives

$$\widetilde{v}^i_{G^iG^j} = \left[\frac{\omega^i_{\lambda^i}\omega^j_{\lambda^j\lambda^j} + \omega^j_{\lambda^j}\omega^i_{\lambda^i\lambda^i}}{(\omega^i_{\lambda^i} + \omega^j_{\lambda^j})^2} \left(2\beta\bar{X}\phi^i_{G^i}\right) + \omega^i_{\lambda^i}\left(-2 + \bar{X}\phi^i_{G^iG^j}/\phi^i_{G^j}\right) \right] \lambda^i_{G^j}. \tag{A.13b}$$

First, observe that $\lambda^i_{G^j} < 0$ for $i \neq j$ from Lemma 1(b). Second, from the proof to part (a), the first term inside the square brackets is negative. Finally, the specification of the conflict technology in (1) implies $\phi^i_{G^j} < 0$ generally and $\phi^i_{G^iG^j} \geq 0$ for $G^i \geq G^j$ ($i \neq j$), such that the second term is non-positive for $G^i \geq G^j$. Thus, $\widetilde{v}^i_{G^iG^j} > 0$ for $G^i \geq G^j$.[67] For future

[67] The result holds more generally, even for values of $G^i < G^j$ but sufficiently close to G^j.

purposes, we also calculate

$$\widetilde{v}^i_{G^jG^j} = \left[\frac{\omega^i_{\lambda^i}\omega^j_{\lambda^j\lambda^j} + \omega^j_{\lambda^j}\omega^i_{\lambda^i\lambda^i}}{\omega^i_{\lambda^i} + \omega^j_{\lambda^j}} \left(2\beta\bar{X}\phi^i_{G^j}\right) + \omega^i_{\lambda^i}\left(-2 + \bar{X}\phi^i_{G^jG^j}/\phi^i_{G^j}\right)\right]\lambda^i_{G^j}, \quad \text{(A.13c)}$$

the sign of which is ambiguous by the properties of ϕ^i in (1). Specifically, since $\phi^i_{G^j} < 0$ and $\phi^i_{G^jG^j} > 0$, the first term inside the square brackets is positive while the second term is negative.

Part (c). The slope of country i's unconstrained best-response function under settlement is $\partial\widetilde{B}^i_s/\partial G^j = -\widetilde{v}^i_{G^iG^j}/\widetilde{v}^i_{G^iG^i} > 0$ for $G^i \geq G^j$, where the positive sign follows from parts (a) and (b). Thus, G^i is a strategic complement for G^j for all $G^i \geq G^j$. To complete the proof to this part, divide (A.13b) by minus the value of (A.13a) and note from the expressions in Lemma 1(b), $\lambda^i_{G^j}/\lambda^i_{G^i} = \phi^i_{G^j}/\phi^i_{G^i} < 0$. Doing so gives

$$\frac{\partial\widetilde{B}^i_s}{\partial G^j} = -\left(\frac{\phi^i_{G^j}}{\phi^i_{G^i}}\right) \frac{\left[-\frac{\omega^i_{\lambda^i}\omega^j_{\lambda^j\lambda^j}+\omega^j_{\lambda^j}\omega^i_{\lambda^i\lambda^i}}{(\omega^i_{\lambda^i}+\omega^j_{\lambda^j})^2}\left(2\beta\bar{X}\phi^i_{G^i}\right) + \omega^i_{\lambda^i}\left(2 - \bar{X}\phi^i_{G^iG^j}/\phi^i_{G^j}\right)\right]}{\left[-\frac{\omega^i_{\lambda^i}\omega^j_{\lambda^j\lambda^j}+\omega^j_{\lambda^j}\omega^i_{\lambda^i\lambda^i}}{(\omega^i_{\lambda^i}+\omega^j_{\lambda^j})^2}\left(2\beta\bar{X}\phi^i_{G^i}\right) + \omega^i_{\lambda^i}\left(2 - \bar{X}\phi^i_{G^iG^i}/\phi^i_{G^i}\right)\right]}. \quad \text{(A.14)}$$

Once again, the properties of the conflict technology (1) imply that, at $G^i = G^j$, we have $\phi^i_{G^j}/\phi^i_{G^i} = -1$ and $\phi^i_{G^iG^j} = 0$. It should now be easy to see that, at $G^i = G^j$, the above expression satisfies $\partial\widetilde{B}^i_s/\partial G^j \in (0, 1)$. ||

Proof of Proposition 3. We break the proof of this proposition in two parts. First, we establish the results regarding peaceful settlement with strictly positive arming ("armed peace"), and then we proceed with the results that relate to the possibility of no arming under peaceful settlement ("unarmed peace").

Armed peace. The strict concavity of $\widetilde{v}^i$ in G^i shown in Lemma A.8(a) together with the continuity of $\partial\widetilde{v}^i/\partial G^i \equiv \widetilde{v}^i_{G^i}$ in G^i guarantee, by an argument similar to that made in Garfinkel et al. (2018) and Garfinkel et al. (2015), uniqueness of a country's unconstrained best-response function, $\widetilde{B}^i_s(G^j)$, for $G^j > 0$.

Existence of equilibrium in the case that neither country is resource-constrained in its arming decision under settlement then follows readily. With Lemma A.8(a), Lemma A.8(b) establishes strategic complementarity of country i's best-response function when $G^i \geq G^j$. In turn, this property and the property stated in Lemma A.8(c) that the slope of an unconstrained country's best-response function is less than 1 at $\widetilde{B}^i_s(G^j) = G^j$ ensure uniqueness of the unconstrained (interior) equilibrium.

To establish existence and uniqueness of equilibrium when one country is resource constrained, observe that the quantity of arms produced by country i when its resource constraint does not bind, $\widetilde{G}_s^i$ $(i = 1, 2)$, is no greater than $\bar{R}/2$. Now suppose that $\widetilde{G}_s^j > R^j$, such that country j is resource constrained. By the definition of a country's best-response function, we have $B_s^i(G^j; \cdot) = \min\{R^i, \widetilde{B}_s^i(G^j)\}$ for any G^j.[68] In particular, if $B_s^j = R^j$, then $B_s^i = \widetilde{B}_s^i(R^j)$. Because $(\widetilde{B}_s^i(R^j), R^j)$ lies on both countries' best-response functions, neither country has an incentive to (or can) deviate from it. Thus, this point represents an equilibrium. Uniqueness of equilibrium follows from the definition of the best response functions whose shape ensures that they intersect only once (excluding, of course, the no-arming equilibrium).

Parts (a) and (b). These parts follow readily from the discussion in the text and the definitions of the threshold values R_L^s, R_H^s in (17) and $m \equiv \mu/\beta$.

Part (c). The first component also follows from the definitions of R_L^s and R_H^s. Unfortunately, not all of the remaining components of this part can be established analytically for reasons that will become evident shortly. However, the validity of the findings in this part have been confirmed numerically for a wide range of parameter values. In particular, we computed the equilibrium using Newton's method for the non-linear systems of equations involving the world market clearing condition (5) that defines π_T^i, the split-the-surplus solution (13) that defines λ^i, and the FOC (16) that defines a country's optimal production of guns. We considered parameter values $(\beta, \tau, \sigma) \in (0, 1] \times (1, 100) \times [1, 100)$.

Since our focus is on $R^i \in (0, R_L^s)$, only country j's arming decision will be unconstrained by its initial resource endowment and thus its optimal arming will be on its best-response function, $\widetilde{B}_s^j\left(G^i\right)$, which is implicitly defined by the FOC in (16). To identify the effect of changes in $\xi \in \{\beta, \tau, \sigma\}$ on country j's guns, we need to differentiate $V_{G^j}^j = 0$ totally to obtain $dV_{G^j}^j = V_{G^jG^j}^j dG^j + V_{G^j\xi}^j d\xi = 0$. We can then use the implicit function theorem to show that $dG_s^j/d\xi = -V_{G^j\xi}^j/V_{G^jG^j}^j$. Since we have shown that $V_{G^jG^j}^j < 0$, $\text{sign}\{dG_s^j/d\xi\} = \text{sign}\{V_{G^j\xi}^j\} = \text{sign}\{\widetilde{v}_{G^j\xi}^j\}$ holds. As such, we compute $\widetilde{v}_{G^j\xi}^j$. To this end, define

$$H^j \equiv -\frac{\omega_{\lambda^j}^j \omega_{\lambda^i\lambda^i}^i + \omega_{\lambda^i}^i \omega_{\lambda^j\lambda^j}^j}{\omega_{\lambda^j}^j + \omega_{\lambda^i}^i}\left(\omega^j/\omega_{\lambda^j}^j\right) > 0. \tag{A.15}$$

Tedious calculus using (13), (16) and (A.15) gives us the following for $\xi = \beta$:

$$\begin{aligned}
\widetilde{v}_{G^j\beta}^j &= \frac{1}{\beta}\left\{\omega^j - \left[\omega_{\lambda^j}^j - \frac{\omega_{\lambda^i}^i \omega_{\lambda^j\lambda^j}^j + \omega_{\lambda^j}^j \omega_{\lambda^i\lambda^i}^i}{\omega_{\lambda^j}^j + \omega_{\lambda^i}^i}\left(\omega^j/\omega_{\lambda^j}^j\right)\right]\frac{\omega^j - \omega^i}{\omega_{\lambda^j}^j + \omega_{\lambda^i}^i}\right\} \\
&= \left(\frac{\omega^j - \omega^i}{\beta}\right)\left[\frac{\omega^j}{\omega^j - \omega^i} - \frac{\omega_{\lambda^j}^j + H^j}{\omega_{\lambda^j}^j + \omega_{\lambda^i}^i}\right].
\end{aligned} \tag{A.16a}$$

[68] $B_s^i(G^j)$ has a kink at R^j where G^j reaches a certain threshold, and this threshold quantity is less than $\widetilde{G}_s^i$.

Observe from part (a), if $R^i = R_L^s$, then $G^i = G_s^i = R_L^s$, such that $\omega^j - \omega^i = 0$; in this case, from the first line of the expression above, $\widetilde{v}^j_{G^j\beta} > 0$. Our focus, however, is on initial distributions $R^j \in (0, R_L^s)$, which imply $\omega^j - \omega^i > 0$ in (A.16a). For G^i less than but close to R_L^s, the first term inside the square brackets on the second line becomes infinitely large, while the second term is finitely positive. Thus, $\widetilde{v}^j_{G^j\beta} > 0$ in this case, confirming the first inequality related to $dG_s^j/d\beta$. For $G^i = R^i$ very close to 0, we know that $\omega^i \to 0$ and the first term inside the square bracket converges to 1. Numerical analysis reveals that, in this case, the second term becomes larger than 1, thereby implying $\widetilde{v}^j_{G^j\beta} < 0$ and confirming the second inequality related to $dG_s^j/d\beta$.

Similarly, we compute $\widetilde{v}^j_{G^j\xi}$ for $\xi \in \{\tau, \sigma\}$, finding that

$$\begin{aligned} \widetilde{v}^j_{G^j\xi} \quad &= \quad -\left(\frac{\omega^j_{\lambda^i}}{\omega^j_{\lambda^j} + \omega^i_{\lambda^i}}\omega^j_\xi + \frac{\omega^j_{\lambda^j}}{\omega^j_{\lambda^j} + \omega^i_{\lambda^i}}\omega^i_\xi\right) + H^j\left(\omega^j_\xi - \omega^i_\xi\right) \\ &\quad + \frac{\omega^i_{\lambda^i}\omega^j_{\lambda^j\xi} + \omega^j_{\lambda^j}\omega^i_{\lambda^i\xi}}{\omega^j_{\lambda^j} + \omega^i_{\lambda^i}}\left(\omega^j/\omega^j_{\lambda^j}\right). \end{aligned} \tag{A.16b}$$

Inspection of the above expression reveals that, while the first term is positive, the signs of the other two terms are analytically intractable. Nonetheless, numerical analysis reveals that $\widetilde{v}^i_{G^i\xi} > 0$ for $\xi \in \{\tau, \sigma\}$, thus confirming the remaining components of part (c).

Unarmed peace. Let us now turn to the claim made at the beginning of the proposition, that there are conditions that ensure the existence of an equilibrium with no arming ($G_s^i = 0$ for $i = 1, 2$). For what follows, we define the following:

$$\Lambda^i \;= \Lambda^i(\lambda^i, \sigma, \tau) \equiv \omega^i\left(\lambda^i, \sigma, \tau\right) - \omega^i(\tfrac{1}{2}, \sigma, \tau) \tag{A.17a}$$

$$\overline{\Psi}^i = \Psi^i(\lambda^i, \sigma, \tau, \beta)|_{\phi^i=1} \equiv \omega^i\left(\lambda^i, \sigma, \tau\right) - \omega^j\left(\lambda^j, \sigma, \tau\right) - \beta. \tag{A.17b}$$

On the one hand, Lemma A.4 points out that $\Lambda^i(\frac{1}{2}, \sigma, \tau) = 0$, $\Lambda^i_{\lambda^i}(\frac{1}{2}, \sigma, \tau) > 0$ and $\Lambda^i(1, \sigma, \tau) < 0$ for $\sigma < \breve{\sigma}(\tau)$. These findings, along with the concavity of ω^i in λ^i, imply there exists a unique $\lambda^i_e(\cdot) \in (\lambda^i_T, 1]$ that solves $\Lambda^i(\lambda^i_e, \cdot) = 0$. On the other hand, the properties of Ψ^i established in Lemma A.7 imply that $\overline{\Psi}^i(\frac{1}{2}, \sigma, \tau, \beta) = -\beta < 0$, $\overline{\Psi}^i(1, \sigma, \tau, \beta) = 1 - \beta > 0$ and, as we have already seen in Lemma 1, there exists a unique $\overline{\lambda}^i$ that solves $\overline{\Psi}^i = 0$.[69]

Key for our purposes is the comparison of $\lambda^i_e(\sigma, \tau)$ with $\overline{\lambda}^i(\sigma, \tau, \beta)$ and the corresponding payoffs. In particular, we show under that, under some circumstances, $\lambda^i_e \lesseqqgtr \overline{\lambda}^i$ implies $\omega^i(\overline{\lambda}^i, \sigma, \tau) \lesseqqgtr \omega^i\left(\lambda^i_e, \sigma, \tau\right)$, which in turn implies $\omega^i(\overline{\lambda}^i, \sigma, \tau) \lesseqqgtr \omega^i(\frac{1}{2}, \sigma, \tau)$ (since $\Lambda^i(\lambda^i_e, \cdot) = 0$). The nature of the latter inequality will enable us to determine whether country i finds

[69]Recall that $\overline{\lambda}^i$ is the share of $\bar{X}$ country i would secure if $G^j = 0$ and G^i were positive and infinitesimal. Also recall that the payoff $\omega^i(\overline{\lambda}^i, \cdot)$ fully describes the behavior of v^i because $v^i = \omega^i\bar{X}$ and $\bar{X} \approx \bar{R}$.

no arming (or $\lambda^i = \frac{1}{2}$) superior to positive arming when its rival j ($\neq i$) chooses $G^j = 0$ (or $\lambda^i = \overline{\lambda}^i$).

Suppose $\tau = 1$, so that $\breve{\sigma} = 2$ (Lemma A.4(a.ii)). Focusing on $\sigma \in (1, 2)$, we now show that, if $\tau = \beta = 1$ then $\lambda_e^i \lesseqqgtr \overline{\lambda}^i$ as $\sigma \gtreqqless \sigma_0$, where $\sigma_0 = \frac{3}{2}$. First note that under free trade we have $\pi_T^i = (\lambda^i/\lambda^j)^{1/\sigma} = 1/\pi_T^j$ for $i \neq j = 1, 2$, which allows us to write ω^i as

$$\omega^i\left(\lambda^i, \sigma, 1\right) = \left(\lambda^i\right)^{\frac{\sigma-1}{\sigma}} \left[\left(\lambda^i\right)^{\frac{\sigma-1}{\sigma}} + \left(\lambda^j\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}.$$

Therefore, (i) $\omega^j/\omega^i = (\lambda^j/\lambda^i)^{(\sigma-1)/\sigma}$. Now suppose there exists a σ_0 that ensures $\lambda_0^i \equiv \lambda_e^i(\sigma_0) = \overline{\lambda}^i(\sigma_0)$ or, equivalently, $\overline{\Psi}^i(\overline{\lambda}^i(\sigma_0), \cdot) = \Lambda^i(\lambda_e^i(\sigma_0), \cdot)) = 0$. Using the definitions of $\overline{\Psi}^i$ and Λ^i in the latter equality readily implies (ii) $\omega^j/\omega^i = (\eta - \beta)/\beta$, where $\eta \equiv 2^{-1+\frac{1}{\sigma-1}} = \omega^i(\frac{1}{2}, \sigma, 1)$. Setting the RHS of (i) equal to the RHS of (ii) and solving for λ_0^i give

$$\lambda_0^i \equiv \lambda_0^i\left(\sigma, \beta\right) = \frac{\eta^{\frac{\sigma}{\sigma-1}}}{\eta^{\frac{\sigma}{\sigma-1}} + \left(\eta - \beta\right)^{\frac{\sigma}{\sigma-1}}}. \tag{A.18}$$

Then, substitution of λ_0^i into the definition of ω^i in $\Lambda^i = 0$, after simplifying, gives

$$f(\sigma, \beta) = 2\left[1 - 2^{-\frac{1}{\sigma-1}}\beta\right]^{\frac{1}{\sigma-1}} - \left[1 - 2^{1-\frac{1}{\sigma-1}}\beta\right]^{1+\frac{1}{\sigma-1}} - 1 = 0, \tag{A.19}$$

which implicitly defines σ_0. Next observe that $\lim_{\sigma\to 1} f\left(\sigma, \beta\right) = 0$ and $f\left(2, \beta\right) = \beta\left(1 - \beta\right) \geq 0$. Now set $\sigma_0 \equiv 3/2$ and $\beta = 1$. It is easy to verify that substitution of these values in $f\left(\sigma, \beta\right)$ implies $f\left(\sigma_0, 1\right) = 0$. Thus, given $\beta = 1$, σ_0 solves (A.19). One can also verify that $f_\sigma\left(\sigma_0, 1\right) = 2\left[8\ln\left(2\right) - 9\ln\left(3\right)\right] < 0$.[70] The blue curve in Fig. A.2 depicts $f\left(\sigma, 1\right)$ for $\sigma \in (1, 2)$, with σ_0 being the unique solution. Substitution of σ_0 into (A.19), after simplifying the resulting expression, gives $\lambda_0^i\left(\sigma_0, 1\right) = \frac{8}{9}$.

Keeping in mind that $\overline{\lambda}^i = \lambda_e^i = \lambda_0^i$ (and $\omega^i(\overline{\lambda}^i) = \omega^i(\lambda_e^i)$) at $\sigma = \sigma_0$, let us now study the behavior of $\overline{\lambda}^i$ and λ_e^i in the neighborhood of σ_0. Differentiating $\overline{\Psi}^i = 0$ and $\Lambda^i = 0$ totally, applying the implicit function theorem and evaluating the resulting expressions at $\lambda_0^i\left(\sigma_0, 1\right)$ give

$$\left.\frac{d\overline{\lambda}^i}{d\sigma}\right|_{\overline{\lambda}^i = \lambda_0^i(\sigma_0, 1)} = -\frac{\omega_\sigma^i - \omega_\sigma^j}{\omega_{\lambda^i}^i + \omega_{\lambda^j}^j} = \frac{16}{81}\left[9\ln\left(3\right) - 10\ln\left(2\right)\right] \approx 0.584 \tag{A.20a}$$

$$\left.\frac{d\lambda_e^i}{d\sigma}\right|_{\lambda_e^i = \lambda_0^i(\sigma_0, 1)} = -\frac{\omega_\sigma^i - \omega_\sigma^i\left(\frac{1}{2}, \sigma_0\right)}{\omega_{\lambda^i}^i} = \frac{32}{81}\left[16\ln\left(2\right) - 9\ln\left(3\right)\right] \approx 0.475. \tag{A.20b}$$

Thus, the direction of change in both $\overline{\lambda}^i$ and λ_e^i is identical to the direction of the change in σ

[70] One can also show (i) $\lim_{\sigma\to 2} f_\sigma\left(\sigma, 1\right) = 0$, and (ii) $\lim_{\sigma\to 2} f_{\sigma\sigma}\left(\sigma, 1\right) < 0$. These relationships imply that f approaches 0 (as $\sigma \to 2$) from below.

relative to σ_0. However, the response of $\overline{\lambda}^i$ to a change in σ is larger than the corresponding response of λ_e^i, causing $\omega^i(\overline{\lambda}^i, \sigma, 1)$ to fall below $\omega^i\left(\frac{1}{2}, \sigma, 1\right)$ for an increase in σ and conversely for a decrease in σ.[71] Because σ_0 is unique, it is not possible for the implied ranking of payoffs to get reversed in the $(1, \sigma_0)$ and $[\sigma_0, 2]$ ranges.

To see why $\overline{\lambda}^i$ and λ_e^i move in the same direction, note first that $\omega_{\lambda^i}^i + \omega_{\lambda^j}^j > 0$ in (A.20a). Second, $\omega_\sigma^i < 0$ for $i = 1, 2$ (see (A.8d). Moreover, one can show that $|\omega_\sigma^i\left(\lambda_0^i, \sigma_0, 1\right)| > |\omega_\sigma^j\left(1 - \lambda_0^i, \sigma_0, 1\right)|$; that is, the direct effect of σ on the economy that obtains the larger share is relatively stronger (recall $\lambda_0^i > \frac{1}{2}$), and thereby explains the positive sign of $d\overline{\lambda}^i/d\sigma$. Turning to λ_e^i, first note that $\lambda_0^i \in (\lambda_T^i, 1)$ which implies $\omega_{\lambda^i}^i < 0$. Additionally, one can confirm that $|\omega_\sigma^i\left(\lambda_0^i, \sigma_0, 1\right)| < |\omega_\sigma^i\left(1/2, \sigma_0, 1\right)|$ which suggests that the direct effect of σ on ω^i is relatively stronger when λ^i is closer to $\frac{1}{2}$ and thus explains the positive sign of $d\lambda_e^i/d\sigma$.

How do the rates of destruction and trade costs affect the analysis? The pink and green curves in Fig. A.2 depict how the value of $f(\sigma, \beta)$ changes when moderate rates of destruction $(1 - \beta)$ are considered. The key message here is that the range of values in σ under which $G_s^i = 0$ (represented by the interval in the figure that implies $f(\cdot) < 0$) shrinks. As β falls below a (relatively large) threshold level, the dominance of $G_s^i = 0$ vanishes. We can also study the importance of trade costs in this context. Numerical analysis reveals that its role is similar to the one associated with destruction. Overall, this analysis reveals that the no arming equilibrium is very fragile, in that even mild rates of destruction and trade cost values create incentives for countries to arm under settlement. ||

Proof of Proposition 4. The independence of average discounted payoffs from δ under settlement in general is due to the stationarity of the structure of the model under this form of conflict resolution. The payoffs under settlement without arming follow easily from the definitions of $V_s^i = v_s^i$ in (12) and $\omega^i \equiv \lambda^i \mu^i$, noting that $G^i = G^j = 0$ implies $\bar{X} = \bar{R}$, $\lambda^i = \frac{1}{2}$, $\pi_T^i = 1$, and thus $\mu^i = \mu = [1 + \tau^{1-\sigma}]^{-1/(1-\sigma)}$ for $i = 1, 2$. The effects of decreases in σ and τ, then, follow by noting that $\mu_\sigma < 0$ and $\mu_\tau < 0$. The remaining parts of the proposition deal with the case of positive equilibrium arming.

Part (a). The first component follows from Proposition 3(a), which implies $\bar{X} = \bar{R} - \bar{G}_s = \mu\bar{R}/(\mu + \beta)$ and $\omega^i = \frac{1}{2}\mu$, where $\mu_\sigma < 0$ and $\mu_\tau < 0$. The effects stated for β, σ, τ on V_s^i readily follow.

Part (b). To prove this part, we start by noting that R^i affects payoffs solely through its

[71] Eventually, as $\sigma \to 0$, both $\overline{\lambda}^i$ and λ_e^i converge to 1. But, one can easily show that the convergence of $\overline{\lambda}^i$ is faster than the convergence of λ_e^i. This difference in the rate of convergence can be explained by the the behavior of $f(\sigma, 1)$ in the neighborhood of $\sigma = 2$ described earlier.

impact on both countries' guns. Thus,

$$\begin{aligned}\frac{dV_s^i\left(R^i,\widetilde{B}_s^j(R^i)\right)}{dR^i} &= V_{G^i}^i + V_{G^j}^i\left[\frac{d\widetilde{B}_s^j(R^i)}{dR^i}\right]\\ &= \left[\bar{X}\omega_{\lambda^i}^i\lambda_{G^i}^i-\omega^i\right]+\left[\bar{X}\omega_{\lambda^i}^i\lambda_{G^j}^i-\omega^i\right]\left[\frac{d\widetilde{B}_s^j(R^i)}{dR^i}\right]\\ &= \left[\bar{X}\omega_{\lambda^i}^i\lambda_{G^i}^i+\omega^i\right]\left[\frac{\bar{X}\omega_{\lambda^i}^i\lambda_{G^i}^i-\omega^i}{\bar{X}\omega_{\lambda^i}^i\lambda_{G^i}^i+\omega^i}-\frac{d\widetilde{B}_s^j(R^i)}{dR^i}\right], \end{aligned} \tag{A.21}$$

for $i \neq j = 1, 2$. Since $\bar{X}\omega_{\lambda^i}^i\lambda_{G^i}^i+\omega^i > 0$ in (A.21) we have

$$\operatorname{sign}\left[dV_s^i/dR^i\right] = \operatorname{sign}\left[\frac{\bar{X}\omega_{\lambda^i}^i\lambda_{G^i}^i-\omega^i}{\bar{X}\omega_{\lambda^i}^i\lambda_{G^i}^i+\omega^i}-\frac{d\widetilde{B}_s^j(R^i)}{dR^i}\right]. \tag{A.22}$$

Now suppose $R^i \to R_L^s$, which implies $V_{G^i}^i \to 0$ and thus $\left(\bar{X}\omega_{\lambda^i}^i\lambda_{G^i}^i-\omega^i\right) \to 0$ in (A.22). But, by Lemma A.8(c), we know that $\lim_{R^i\to R_L^s} d\widetilde{B}_s^j(R^i)/dR^i \in (0,1)$. Thus, the expression in the brackets in the RHS of (A.22) is negative, as required. The case of $R^i \to 0$ follows by noting that the first term in the brackets in the RHS of (A.22) is less than 1 while the second term attains a value larger than 1 for R^i sufficiently close to 0.

Turning to the endowment effects for the unconstrained country j, we have $V_{G^j}^j = 0$ and

$$\frac{dV_s^j(\widetilde{B}_s^j(G^i),R^i)}{dR^j} = V_{G^i}^j\left(\frac{dR^i}{dR^j}\right) = -\widetilde{v}_{G^i}^j > 0$$

along $\widetilde{B}_s^j(G^i)$. Since $\widetilde{v}_{G^i}^j < 0$, country j's payoff rises along $\widetilde{B}_s^i(G^j)$ when it is allocated a larger initial endowment. Differentiating the above expression with respect to R^j gives

$$\begin{aligned}\frac{d^2V_s^j(\widetilde{B}_s^j(G^i),R^i)}{(dR^j)^2} &= \left[-\widetilde{v}_{G^iG^j}^i\frac{d\widetilde{B}_s^j(G^i)}{dR^i}-\widetilde{v}_{G^iG^i}^j\right]\left(\frac{dR^i}{dR^j}\right)\\ &= \widetilde{v}_{G^iG^j}^j\left[\frac{d\widetilde{B}_s^j(G^i)}{dR^i}\right]+\widetilde{v}_{G^iG^i}^j\\ &= \widetilde{v}_{G^jG^i}^j\left[-\frac{\widetilde{v}_{G^jG^i}^j}{\widetilde{v}_{G^jG^j}^j}\right]+\widetilde{v}_{G^iG^i}^j, \qquad \text{from (A.14)}\\ &= -\frac{\Theta^j}{\widetilde{v}_{G^jG^j}^j}, \text{ where } \Theta^j \equiv \left(\widetilde{v}_{G^jG^i}^j\right)^2-\widetilde{v}_{G^jG^j}^j\widetilde{v}_{G^iG^i}^j. \end{aligned} \tag{A.23}$$

Since $\widetilde{v}_{G^jG^j}^j < 0$, the sign of the above expression coincides with the sign of Θ^j. This sign appears to be ambiguous because, as noted earlier in the context of (A.13c), the sign of $\widetilde{v}_{G^iG^i}^j$ is ambiguous. We, thus, need to show that $\Theta^j > 0$. Using country j's FOC (which can be rewritten as $2\beta\bar{X}\phi_{G^j}^j/(\omega_{\lambda^j}^j+\omega_{\lambda^i}^i) = \omega^j/\omega_{\lambda^j}^j$) and recalling that $\lambda_{G^k}^i = 2\beta\phi_{G^k}^i/(\omega_{\lambda^i}^i+\omega_{\lambda^j}^j)$

$(k = 1, 2)$, the expressions in (A.13a)–(A.13c) for country j can be rewritten as

$$\widetilde{v}^j_{G^jG^i} = \left[H^j + 2 - \bar{X}\phi^j_{G^jG^i}/\phi^j_{G^i}\right]\left(-\omega^j_{\lambda^j}\lambda^j_{G^i}\right) \quad \text{(A.13a}'\text{)}$$

$$\widetilde{v}^j_{G^jG^j} = \left[H^j + 2 - \bar{X}\phi^j_{G^jG^j}/\phi^j_{G^j}\right]\left(-\omega^j_{\lambda^j}\lambda^i_{G^i}\right)\left(\lambda^j_{G^j}/\lambda^j_{G^i}\right) \quad \text{(A.13b}'\text{)}$$

$$\widetilde{v}^j_{G^iG^i} = \left[H^j + \left(\phi^j_{G^j}/\phi^j_{G^i}\right)\left(2 - \bar{X}\phi^j_{G^iG^i}/\phi^j_{G^i}\right)\right]\left(-\omega^j_{\lambda^j}\lambda^j_{G^i}\right)\left(\phi^j_{G^i}/\phi^j_{G^j}\right), \quad \text{(A.13c}'\text{)}$$

where H^j (> 0) was defined in (A.15). Now define $r \equiv -\lambda^j_{G^j}/\lambda^j_{G^i}$ $(= -\phi^j_{G^j}/\phi^j_{G^i} > 0$ from Lemma 1(b)). Substituting the above expressions in the definition of Θ^j in (A.23) and simplifying the resulting expression give

$$\Theta^j = \left(\omega^j_{\lambda^j}\lambda^j_{G^i}\right)^2\Bigg\{\left[H^j + 2 - \bar{X}\phi^j_{G^jG^i}/\phi^j_{G^j}\right]^2 - \left[H^j + 2 - \bar{X}\phi^j_{G^jG^j}/\phi^i_{G^j}\right]\left[H^j - r\left(2 - \bar{X}\phi^j_{G^iG^i}/\phi^j_{G^i}\right)\right]\Bigg\}.$$

Next, noting that $\bar{G} \equiv G^i + G^j$, we can apply the properties of the conflict technology in (1) to find

$$-\phi^j_{G^jG^i}/\phi^j_{G^j} = (1 - r)/\bar{G}$$
$$-\phi^i_{G^jG^j}/\phi^j_{G^j} = 2/\bar{G}$$
$$-\phi^j_{G^iG^i}/\phi^j_{G^i} = 2/\bar{G}.$$

Let $x \equiv \bar{X}/\bar{G}$. Applying the above relations to definition of Θ^j gives

$$\begin{aligned}\Theta^j &= \left(\omega^j_{\lambda^j}\lambda^j_{G^i}\right)^2\left\{\left[H^i + 2 + (1 - r)\,x\right]^2 - \left[H^i + 2\,(1 + x)\right]\left[H^i - 2r\,(1 + x)\right]\right\}\\ &= \left(\omega^j_{\lambda^j}\lambda^j_{G^i}\right)^2(1 + r)\left[2H^i + 4(1 + x) + (1 + r)\,x^2\right] > 0.\end{aligned}$$

The positive sign of Θ^j proves that V^j_s rises at an increasing rate along its best-response function when country j is allocated a larger initial endowment.

We now prove parts (b.i) and (b.ii). Before we impose any restrictions on the countries' initial resource endowments and, therefore, their ability to produce guns a few general remarks are in order. Generally, $V^i = \omega^i\left(\lambda^i(\beta, \tau, \sigma), \tau, \sigma\right)\bar{X}$ for $i = 1, 2$, the effect of a change in $\xi \in \{\beta, \tau, \sigma\}$ on V^i_s is given by

$$\frac{dV^i_s}{d\xi} = V^i_\xi + V^i_{G^j}\frac{dG^j_s}{d\xi} + V^i_{G^i}\frac{dG^i_s}{d\xi}. \quad \text{(A.24)}$$

The first term in the RHS of (A.24) captures the direct effect of ξ on the payoff and is present under all circumstances. The second term is a strategic effect that arises only if country i's rival (j) is unconstrained by its initial endowment. The third term represents the payoff effects from an induced change in country i's own arming, and vanishes either because $dG^i_s/d\xi = 0$ when country i is constrained by its initial resource endowment or because $V^i_{G^i} = 0$ when it is not (provided $G^j > 0$). Dropping this third term implies

$$\frac{dV^i_s}{d\xi} = V^i_\xi + V^i_{G^j}\frac{dG^j_s}{d\xi}, \tag{A.24$'$}$$

where the conflict technology (1) implies $V^i_{G^j} < 0$. The sign of the direct effect V^i_ξ, for each country whether resource-constrained or not is determined by the sign of ξ's total effect on ω^i: $d\omega^i/d\xi = \omega^i_\xi + \omega^i_{\lambda^i}\lambda^i_\xi$, where $\omega^i_\beta = 0$ and $\omega^i_\xi < 0$ for $\xi \in \{\tau, \sigma\}$ (see (A.8c) and (A.8d)). From Lemma 1(c) we also have that $\lambda^i_\beta = \left(\phi^i - \phi^j\right)/(\omega^i_{\lambda^i} + \omega^j_{\lambda^j})$ and $\lambda^i_\xi = -(\omega^i_\xi - \omega^j_\xi)/(\omega^i_{\lambda^i} + \omega^j_{\lambda^j})$ for $\xi \in \{\tau, \sigma\}$. Consequently,

$$\frac{d\omega^i}{d\beta} = \frac{\omega^i_{\lambda^i}}{\omega^i_{\lambda^i} + \omega^j_{\lambda^j}}\left(\phi^i - \phi^j\right) \tag{A.25a}$$

$$\frac{d\omega^i}{d\xi} = \frac{\omega^j_{\lambda^j}}{\omega^i_{\lambda^i} + \omega^j_{\lambda^j}}\omega^i_\xi + \frac{\omega^i_{\lambda^i}}{\omega^i_{\lambda^i} + \omega^j_{\lambda^j}}\omega^j_\xi, \text{ for } \xi \in \{\tau, \sigma\}, \tag{A.25b}$$

where $\omega^i_{\lambda^i} > 0$ for both countries that arm. Since our focus is on $R^i \in (0, R^s_L)$ (which is equivalent to $R^j \in (R^s_H, \bar{R})$), country j's arming decision is unconstrained by its endowment. Therefore, only the direct effect V^j_ξ will be operational. By contrast, we must take into account both effects for (resource-constrained) country i.[72]

Starting with the constrained country i, since $G^i_s < G^j_s$ and thus $\phi^i < \phi^j$, equation (A.25a) implies that the direct effect of an increase in β in (A.24$'$), V^i_β, is negative. To evaluate the strategic effect, recall from Proposition 3(c) that an increase in β induces (unconstrained) country j to produce more guns if country i's resource endowment is of moderate size, implying from (A.24$'$) that $dV^i_s/d\beta < 0$ unambiguously holds in this case. While Proposition 3(c) also points out that country j produces fewer guns when country i's initial resource endowment is small (thereby implying a positive strategic effect), numerical analysis of the model reveals that the direct effect always dominates this strategic effect, such that $dV^i_s/d\beta < 0$ holds true even when country i's endowment is small. The following expression shows the precise nature of the two effects:

$$\frac{dV^i_s}{d\beta} = -\left(\phi^j - \phi^i\right)\bar{X}\left[\frac{\omega^i_{\lambda^i}}{\omega^j_{\lambda^j} + \omega^i_{\lambda^i}} + \frac{\omega^i\left(\frac{\omega^j}{\omega^j - \omega^i} - \frac{\omega^j_{\lambda^j} + H^j}{\omega^j_{\lambda^j} + \omega^i_{\lambda^i}}\right)}{\phi^j\omega^j\left(H^j + 2 - \bar{X}\phi^j_{G^jG^j}/\phi^j_{G^j}\right)}\right].$$

[72]One must also consider the possibility that one side's endowment, say i's, satisfies $R^i = R^s_L$ at the time that parameter ξ changes. This is important because parametric changes may create slack in the resource constraint of one country and conversely.

From our discussion above in connection with (A.25b), the direct effects of increases in the trade parameters $\xi \in \{\sigma, \tau\}$ are negative for the constrained country; and, by Proposition 3(c), both the indirect effects of these parameters are negative as well, thus giving us $dV_s^i/d\xi < 0$ for $\xi \in \{\tau, \sigma\}$ and completing the proof of part (b.i).

Consider next the unconstrained country j, for whom only the direct effect matters. The expression in (A.25a) shows that, since $G_s^j > G_s^i$ and thus $\phi^j > \phi^i$, $V_\beta^j > 0$, which implies $dV_s^j/d\beta > 0$. Our previous discussion in connection with (A.25b) indicates further that $V_\xi^j < 0$ for $\xi \in \{\tau, \sigma\}$. This proves part (b.ii).

Part (c). Since the value of settlement is relatively clear in the presence of destruction, it suffices for this part to consider the less obvious case of positive gains from trade. Letting $\beta = 1$, suppose $R^j = \bar{R}$ initially, so that country i is inconsequential. Since $\omega^j = 1$ in this case and there is no need for arming we have $V_s^j = \bar{R}$. Now suppose $R^i = \varepsilon > 0$ is infinitesimal, so that $R^j = \bar{R} - \varepsilon$ is smaller but arbitrarily close to $\bar{R}$. Suppose, in addition, that country i arms: $G_s^i = R^i$. Because country i produces an infinitesimal quantity of guns, country j will be able to induce a division (with a larger but also infinitesimal G^j) that brings it arbitrarily close to its optimum λ_T^j which satisfies $\lambda_T^j < 1$ (by Proposition 3(b)). But this division generates a payoff $V_s^j(\lambda_T^j, \cdot) > \bar{R}$ because there are gains from trade for country j. Likewise, the smaller country realizes some gains from trade such that $V_s^i > 0$. ||

Proof of Proposition 5. We take as our starting point, the benchmark case where $\delta = 0$, $\mu^i = 1$ and $\beta = 1$ and thus $[R_L^s, R_H^s] = [R_L^c, R_H^c]$ and $G_c^i = G_s^i$, such that $\lambda_s^i = \phi_c^i = \frac{1}{2}$, for $i = 1, 2$.

Part (a). One can confirm this part of the proposition by differentiating the threshold values in (11) and (17) with respect to δ, μ and β and making the relevant comparisons.

Part (b). Suppose country i's arming is unconstrained by its endowment under conflict as well as under settlement. Then, $\widetilde{B}_c^i(G^j)$ is defined implicitly by country i's FOC under conflict, which can written from (9) as $\phi_{G^i}^i \bar{X}(1 + \delta) - \phi^i = 0$. Country i's FOC under settlement can be written from (16) as $\widetilde{v}_{G^i}^i = \widetilde{V}_{G^i}^i$. The concavity of ω^i in λ^i assuming $\sigma < \infty$ and $\tau < \infty$ (by Lemma A.4(b)) together with our assumption that country i is unconstrained and our focus on $G^j \leq G_s$(where G_s is the symmetric equilibrium) imply $\omega_{\lambda^i}^i \leq \omega_{\lambda^j}^j$ holds, and with strict inequality for $\lambda^i > \lambda^j$. Thus,

$$\frac{2\omega_{\lambda^i}^i}{\omega_{\lambda^i}^i + \omega_{\lambda^j}^j} \leq 1,$$

with strict inequality for $\lambda^i < \lambda^j$. Evaluating $\widetilde{V}^i_{G^i}$ at $\widetilde{B}^i_c$, using the above observations, gives:

$$\widetilde{V}^i_{G^i}\big|_{\widetilde{B}^i_c(G^j)} \leq \beta \bar{X} \phi^i_{G^i} - \omega^i = \frac{\beta \phi^i}{1+\delta} - \omega^i.$$

Assuming $R^j > 0$ such that $\phi^i < 1$ means that the first term in the far right expression is less than 1 for any $\beta \leq 1$ and $\delta \geq 0$. Furthermore, our assumptions that $\sigma < \infty$ and $\tau < \infty$ imply that $\omega^i > 1$. Thus, the expression above is negative, and by the concavity of $\widetilde{V}^i$ in G^i, we have $G^i_s < G^i_c$ for the unconstrained country. If either $\sigma = \infty$ or $\tau = \sigma$, such that there are no gains from trade to either country (i.e., $\mu^i = 1$ for $i = 1, 2$ and thus $\omega^i = \lambda^i$), the condition for splitting the surplus under settlement (13) implies

$$\lambda^i = \beta \phi^i + \tfrac{1}{2}(1 - \beta),$$

which implies the unconstrained country's best-response function under settlement is implicitly defined by

$$\widetilde{V}^i_{G^i} = \beta \bar{X} \phi^i_{G^i} - \phi^i = 0.$$

Evaluating this expression at $\widetilde{B}^i_c$ gives

$$\widetilde{V}^i_{G^i}\big|_{\widetilde{B}^i_c(G^j)} = \frac{\beta \phi^i}{1+\delta} - \phi^i.$$

Provided that either $\beta < 1$ or $\delta > 0$ (or both), the expression above is negative, thereby completing the proof that the unconstrained country's arming under settlement is less than that under war, provided that $\beta < 1$, $\delta > 0$ or $\sigma < \infty$ and $\tau < \infty$.

Let us now turn to the possibility that country i is constrained in its arming choice. In the case that it is resource constrained under both conflict and settlement, $G^i_s = G^i_c = R^i$. In the case that country i is resource constrained under conflict only, we have $G^i_s < G^i_c = R^i$, which completes this part of the proof. ||

Proof of Proposition 6. Based on the results shown in Propositions 2 and 4, our discussion in the text indicates it is the smaller of the two countries that has a strict preference for settlement provided $\beta < 1$ and/or $\mu > 1$ for any $\delta \in [0, 1]$. Accordingly, to establish the dominance of settlement over war for both countries, it suffices to focus on the larger country. Let us suppose that is country i (i.e., $R^i \geq \bar{R}/2$).

To demonstrate parts (a) and (b) of this proposition, we show that there exists a value of $m \equiv \mu/\beta$, call it m_0, that solves $V^i_s(R^s_H(m), m) = U^i_c(R^s_H(m))$, where $R^s_H = \frac{1}{2}(1 + \frac{m}{1+m})\bar{R}$. This point of equality is represented by point C in Fig. 2(b) where the functions V^i_s and U^i_c meet. However, we do not restrict β and δ to equal 1 here.

To find m_0, first recall that for $R^i \geq R^s_H$ we have $G^j_c = R^j$ and $G^i_c = \widetilde{B}^i_c\left(R^j\right) = -R^j + \sqrt{(1+\delta)\,R^j\bar{R}}$. As such, $\bar{X}_c = \bar{R} - \bar{G}_c = \sqrt{(1+\delta)\,R^j\bar{R}}$ and $\phi^i_c = 1 - R^j/\sqrt{(1+\delta)\,R^j\bar{R}}$. Using these results in (8) gives

$$U^i_c(R^j)\big|_{R^i \geq R^s_H} = \frac{\beta}{1+\delta}\phi^i_c\left[\bar{X}_c + \delta\bar{R}\right] = \beta\bar{R}\left[1 - \sqrt{\frac{R^j}{(1+\delta)\,\bar{R}}}\right]^2. \tag{A.26}$$

Evaluating the above expression at $R^i = R^s_H$, which implies $R^j = R^s_L = \frac{1}{2}\left(1 - \frac{m}{1+m}\right)\bar{R} = \frac{\bar{R}}{2(1+m)}$, in turn, yields

$$U^i_c\left(R^s_L\right) = \beta\bar{R}\left[1 - \sqrt{\frac{1}{2\,(1+\delta)\,(1+m)}}\right]^2. \tag{A.27}$$

Recall the stationarity of the setup under settlement implies $V^i_s(R^i) = v^i_s(R^i)$. Then, using (14), the general expression for country i's payoff when $R^i \geq R^s_H$ and thus $G^j_s = R^j$ is

$$V^i_s(R^j)\big|_{R^i \geq R^s_H} = \omega^i\bar{X}_s = \omega^i\left[R^i - \widetilde{B}^i_s(R^j)\right]. \tag{A.28}$$

While we do not have an explicit solution for $\widetilde{B}^i_s(R^j)$ when $R^j < R^s_L$ (or equivalently when $R^i > R^s_H$), Proposition 4(a) shows when we evaluate the above expression at $R^i = R^s_H$ (which again implies $R^j = R^s_L$), we get

$$V^i_s(R^s_L) = \frac{\beta m^2}{1+m}(\bar{R}/2). \tag{A.29}$$

Setting $U^i_c(R^s_L) = V^i_s(R^s_L)$ respectively from (A.27) and (A.29), after rearranging, implies

$$\left[\sqrt{2\,(1+\delta)\,(1+m)} - 1\right]^2 = (1+\delta)\,m^2,$$

which can be solved for m to obtain the positive root

$$m_0(\delta) = 1 - \frac{1}{\sqrt{1+\delta}} + \sqrt{3 - \frac{2}{\sqrt{1+\delta}}}. \tag{A.30}$$

Notice that $dm_0/d\delta > 0$, with $m_0\,(0) = 1$ and $m_0\,(1) = 1 - \frac{1}{\sqrt{2}} + \sqrt{3 - \sqrt{2}} \simeq 1.552$.

Suppose this value of m_0 is unique, such that there is no other value of R^i for which $V^i_s(R^i) = U^i_c(R^i)$.[73] By our definition $m \equiv \mu/\beta$, the above solution implies that the critical value of β, called $\beta_0(\delta)$ in the proposition, is given by $\beta_0(\delta) = 1/m_0(\delta) \leq 1$, with $d\beta_0/d\delta < 0$. When $\beta < \beta_0(\delta)$, war is sufficiently destructive such that both countries strictly prefer

[73] Although we have not shown analytically that the value of m_0 which implies $V^i_s(R^i) = U^i_c(R^i)$ is unique, extensive numerical analysis suggests that it is and occurs at the kink of $V^i_s(R^i)$ where $R^i = R^s_H$.

settlement, regardless of how $\bar{R}$ is initially distributed and even if there are no gains from trade ($\mu^i = 1$ for $i = 1, 2$). For $\beta > \beta_0(\delta)$, the critical value of μ, called $\mu_0 = \mu_0(\delta, \beta) > 1$ in the proposition, is given by $\mu_0(\delta, \beta) = \beta m_0(\delta)$, with $\partial\mu_0/\partial\beta > 0$ and $\partial\mu_0/\partial\delta > 0$. For $\mu > \mu_0(\delta, \beta)$, settlement again dominates war even though war is at most only moderately destructive.

Now consider a value of $m \leq m_0(\delta)$, which implies $V_s^i(R_H^s) < U_c^i(R_H^s)$. By Proposition 2(b.ii), we know that $\lim_{R^i \to \bar{R}} U_c^i(R^i) = \beta\bar{R}$; and, by Proposition 4(b)-(c), we know that $dV_s^i/dR^i > 0$ and $V_s^i > \bar{R}$ (since $\mu > 1$) for R^i close to $\bar{R}$. Thus, if either $\beta < 1$ or $\mu > 1$, $V_s^i(R^i) > U_c^i(R^i)$ for R^i close to $\bar{R}$, and there must exist two intersections between $V_s^i(\cdot)$ and $U_c^i(\cdot)$, one at an allocation, call it $R_A \in (\bar{R}/2, R_H^s)$ such the one associated with point A in Fig. 2(b) and another at an allocation $R_B \in (R_H^s, \bar{R})$ such as the allocation associated with point B in the figure. Then, for allocations $R^i \in (R_A, R_B)$, war dominates settlement for country i, while for all other allocations R^i ($\geq \bar{R}/2$), both country i and country j strictly prefer settlement over war.

If the value of m for which $V_s^i(R^i) = U_c(R^i)$ as identified at $R^i = R_H^s$ above is not unique, there will exist at least two other values of m, call them m_{01} and m_{02} where $m_0 < m_{01} < m_{02}$, for which $U_c^i(R^i) = V_s^i(R^i)$ at other values of R^i. However, we know that $V_s^i(R_H^s(m), m)$ is increasing and continuous in m, where for a sufficiently high m, $V_s^i(R_H^s) > \bar{R}$ holds, whereas $\max\{U_c^i(R^i)\} \leq \bar{R}$, with strict inequality when $\beta < 1$. $\quad ||$

Proof of Proposition 7. With the benchmark case where $\delta = 1$, $\beta = 1$ and $\mu = \mu_0$ characterized in the main text, we now consider what happens when μ falls below μ_0 such that, from Proposition 6, $V_s^i(R_H^s) < U^i(R_H^s)$, as illustrated for country $i = 1$ in Fig. A.3(b). Fig. A.3(a) shows equilibrium arming for country $i = 1$ under conflict, settlement and the optimizing unilateral deviation for all distributions of initial resource ownership $R^1 \in (0, \bar{R})$.[74] To see the implications of $\mu < \mu_0$ for the profitability of a unilateral deviation depending on the distribution of initial ownership claims to $\bar{R}$, recall that, when $R^i \geq R_H^s$, a unilateral deviation by country i means it operates on its unconstrained best-response function under conflict $G_d^i = \widetilde{B}_c^i(G_s^j)$, with country j being resource constrained, $G_s^j = R^j$. Thus, we have again $W_d^i(R^i) = U_c^i(R^i)$ for all $R^i \geq R_H^s$. Since $U_c^i(R^i)$ is independent of μ, this segment of $W_d^i(R^i)$ is also independent of μ.[75]

Next consider allocations of $R^i \in (R_L^d, R_H^s)$. Such allocations imply that country i, in its unilateral deviation, continues to operate on its best-response function under open conflict; however, country j is now unconstrained and, by Proposition 3(a), produces $G_s^j = R_L^s$, im-

[74] Again, we use green for values under settlement, pink for values under conflict and blue for values under a unilateral deviation by country $i = 1$.

[75] However, as noted below, $W_d^i(R^i)$ does depend indirectly on μ for $R^i < R_H^s$.

plying $G_d^i = \widetilde{B}_c^i(R_L^s) = R_L^d$ $(< \bar{R}/2)$, meaning that $W_d^i(R^i) = U_c^i(R_H^s)$ for such allocations.[76] But, at the same time, our assumption that $\mu < \mu_0$ implies $V_s^i(R^i) = V_s^i(R_H^s) < U_c^i(R_H^s)$ for $R^i \in (R_L^d, R_H^s)$.[77] Accordingly, for $R^i \in (R_L^d, R_H^s)$, $W_d^i(R^i) > V_s^i(R^i)$ holds, and at least one of the countries and possibly both (specifically, when $R^i \in (R_L^d, R_H^d)$) will have an incentive to deviate from settlement, thereby precluding settlement as a stable outcome for sufficiently symmetric countries.

As R^i falls below R_L^d, $W_d^i(R^i)$ falls too due to the tightening of country i's resource constraint whose effect dominates any favorable strategic effect (for $R^i \leq R_s^L$), and approaches 0 as $R^i \to 0$. Although $V_s^i(R^i)$ also eventually falls with decreases in $R^i < R_L^s$, this payoff approaches some positive amount (by Proposition 4) as $R^i \to 0$. Thus, there exists at least one intersection where $W_d^i(R^i) = V_s^i(R^i)$ for $R^i < R_L^d$. Let us denote that point by R_A.[78] As R^i falls below R_A, $W_d^i(R^i)$ falls below $V_s^i(R^i)$, such that for sufficiently uneven initial distributions of $\bar{R}$ the smaller country has no incentive to deviate from settlement.

Considering the larger country's perspective, we also know (from the proof of Proposition 6) that there exists an intersection between $V_s^i(R^i)$ and $U_c^i(R^i)$ at some $R^i > R_H^s$ when $\mu < \mu_0$. Let R_B denote that value of R^i. Then $W_d^i(R^i) = U_c^i(R^i) \geq V_s^i(R^i)$ for all $R^i \in (R_H^s, R_B)$ and $W_d^i(R^i) < V_s^i(R^i)$ for all $R^i \in (R_B, \bar{R})$. Thus, for sufficiently large values of R^i implying sufficiently low values of R^j, neither country has an incentive to deviate unilaterally from settlement.

If there were multiple intersections between $V_s^i(R^i)$ and $U_c^i(R^i)$ for $R^i > R_B$, then there would exist multiple intervals of R^i for which $V_s^i(R^i) > W_d^i(R^i)$. Nonetheless, since $\lim_{R^i \to \bar{R}} V_s^i(R^i) \geq \bar{R}$ holding as a strict inequality for $\mu > 1$ and $\lim_{R^i \to \bar{R}} U_c^i(R^i) = \beta\bar{R}$, provided $\mu > 1$ and/or $\beta < 1$ there must exist an interval just adjacent to $\bar{R}$ that has this property. ||

[76] Notice a lower μ implies increased arming under settlement by the opponent: $G_s^j = R_L^s = \frac{\bar{R}/2}{1+m}$ where as previously defined $m = \mu/\beta$. Of course, we also have that $R_H^s = \bar{R} - R_L^s$ is negatively related μ. Therefore, a decrease in μ implies a downward shift in $W_d^i(R^i)$ for $R^i < R_H^s$, with the flat segment meeting $U_c^i(R^i)$ at a new, lower value of R_H^s, illustrated by the black dot at that resource allocation in Fig. A.3(b). The blue dot to the right of that on $U_c^i(R^i)$ at the intersection of that payoff and the dashed black line represents the point where $W_d^i(R^i)$ converges to $U_c^i(R^i)$ at a higher value of μ such as μ_0.

[77] A lower μ adversely affects V_s^i, as fully described in Proposition 4. In addition, one can show that, for a given decrease in μ, the flat segment of $V_s^i(R^i)$ (for $R^i \in (R_L^s, R_H^s)$) falls by more than the flat segment of $W_d^i(R^i)$ (for $R^i \in (R_L^d, R_H^s)$).

[78] At that point, since arming for both countries under the deviation are identical to that under settlement $(R^i, \widetilde{B}_s^j(R^i))$, the gains realized under settlement equal the savings by not having to arm in the next period. That this intersection occurs at R_L^s in Fig. A.3 is merely a coincidence. It could occur above or below R_L^s.

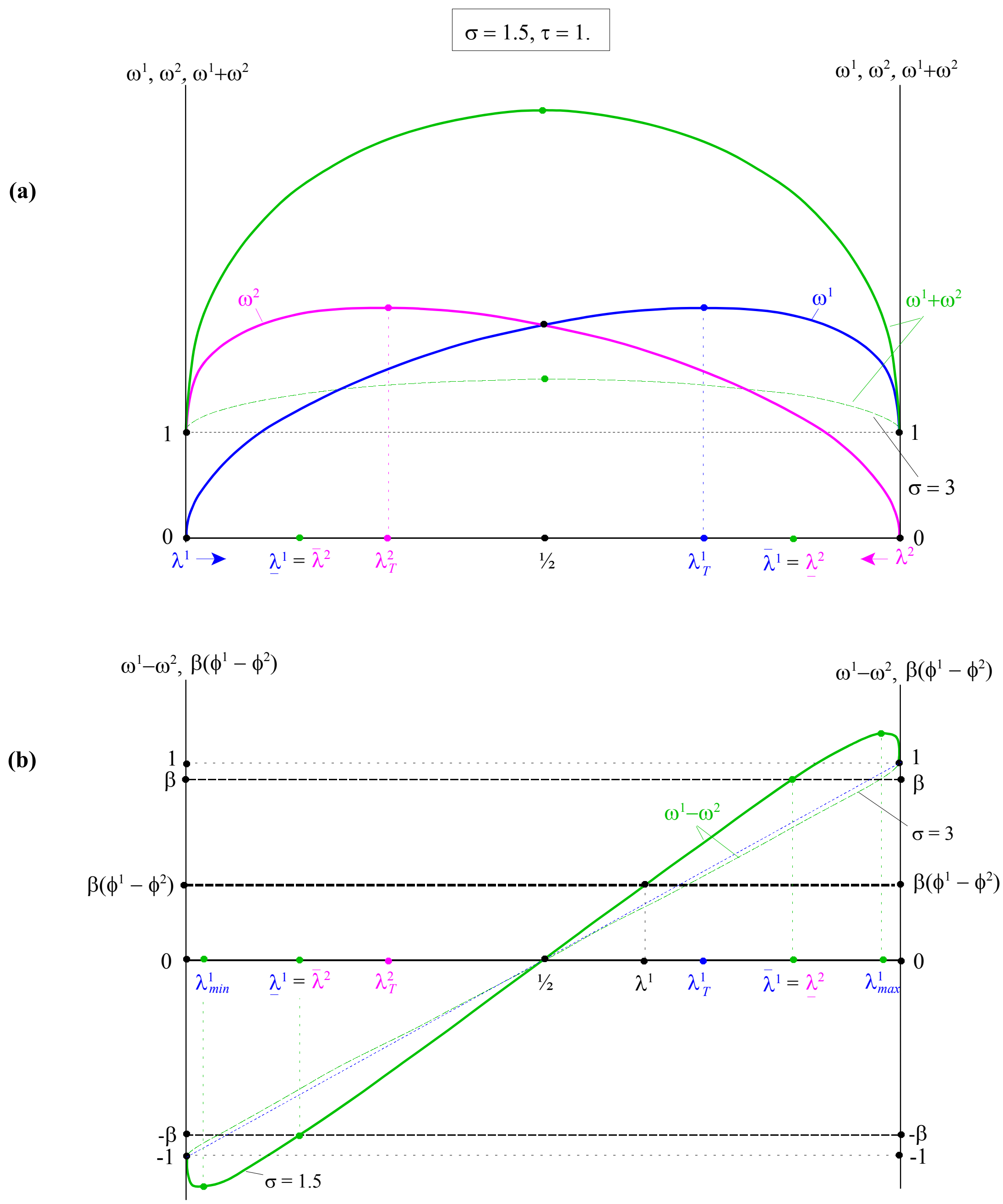

Figure A.1: Payoffs and the Determination of the Division of the Common Pool under Splitting of the Surplus

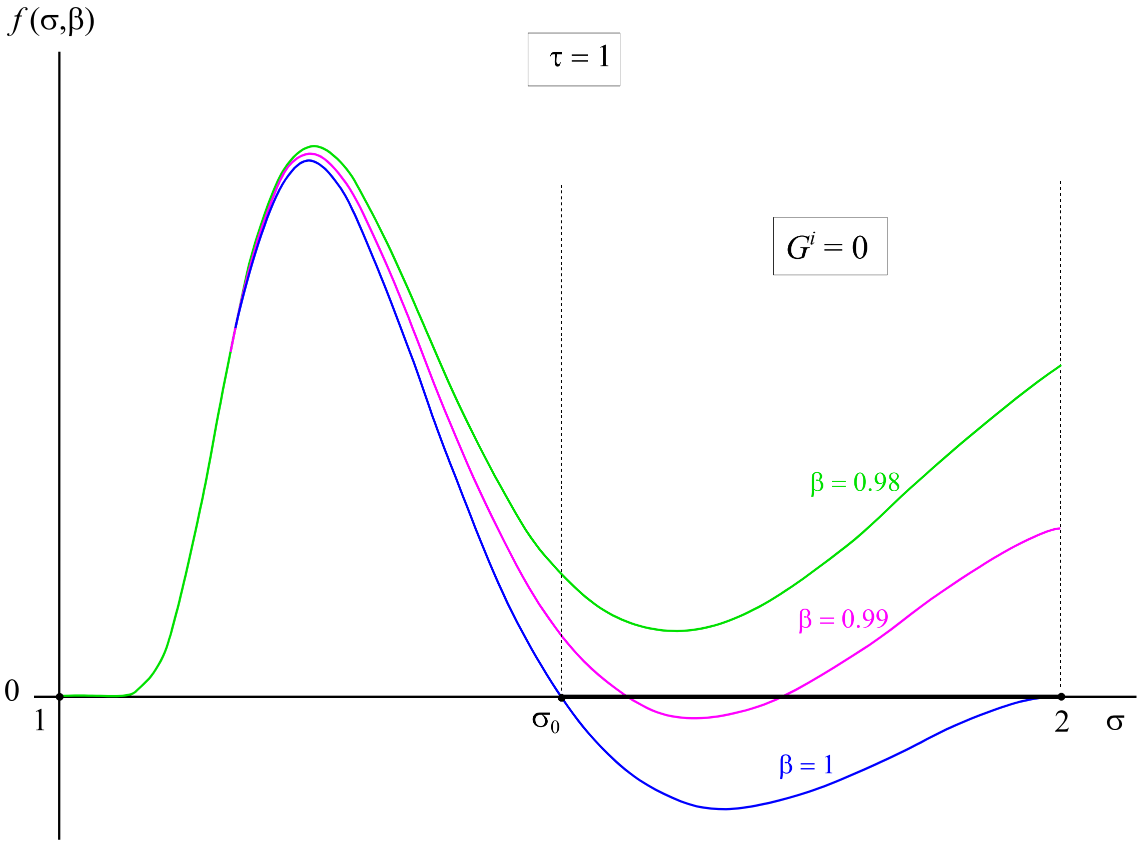

Figure A.2: Identifying the “No-Arming” Equilibrium Under Settlement

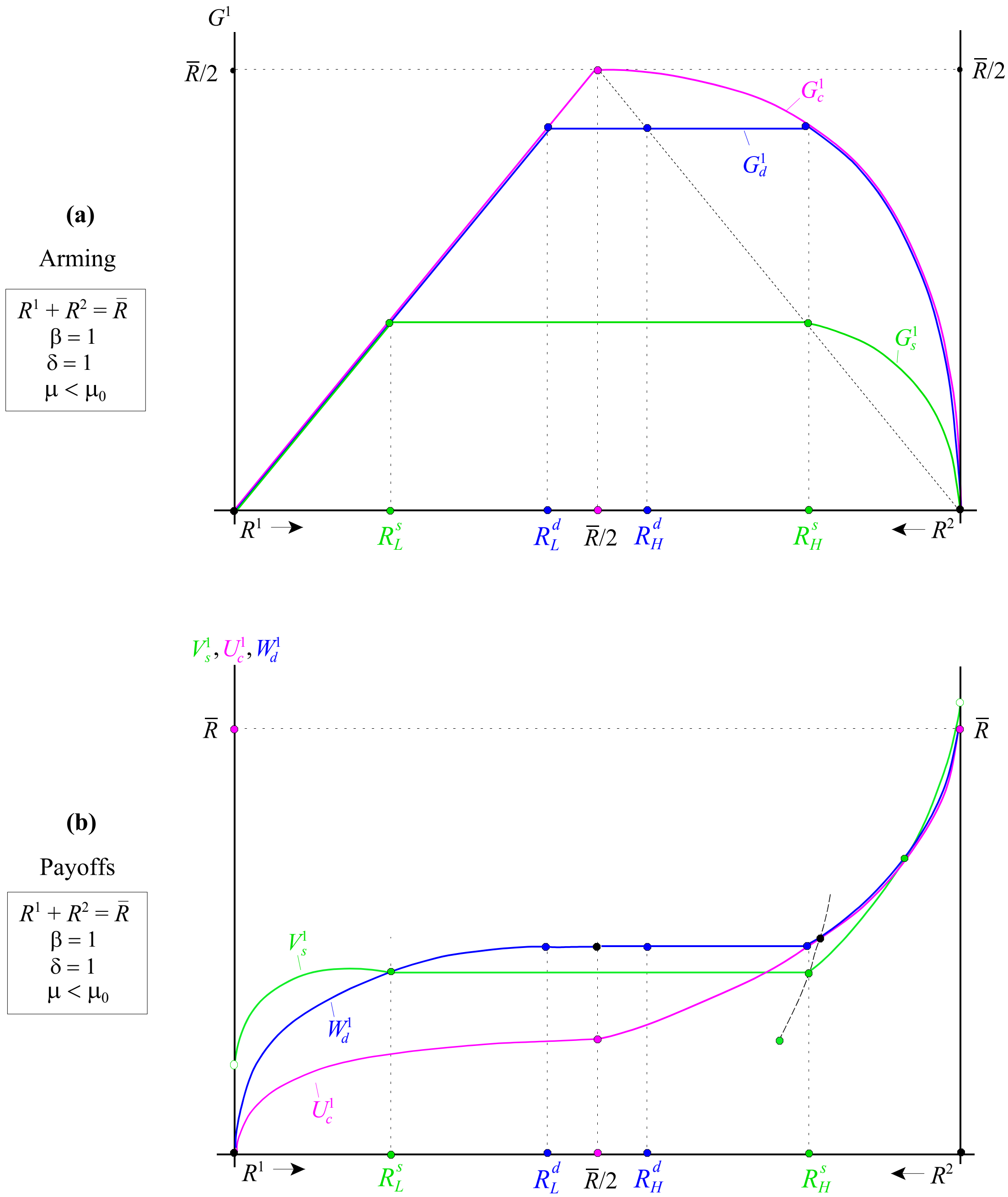

Figure A.3: Arming and Payoffs under Conflict, Settlement, and Unilateral Deviations for Alternative Distributions of Initial Resource Ownership and Smaller Gains from Trade