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Introduction and Notes to Garfinkel-Syropoulos: "Trading with the Enemy"[1]

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OUTLINE

1. Introduction

As I listened to Michelle Garfinkel's presentation of this paper under the watchful eye of Prof Syropoulos in a seminar last April, 23,2017, I was intrigued by the idea that the mixed motives and gray areas in international

[1] Version of March 4, 2017 as presented at GPACS Workshop, UC-Irvine April 23, 2017.

[2] The Garfinkel-Syropoulos analysis, being mathematical focuses attention on "results," that is behaviour in the neighborhood of solutions, or equilibria, whereas geometry doesn't necessarily draw the reader in toward solutions. Use of geometry to derive relationships (rather than merely illustrate conclusions already known from mathematics) has gone out of style in recent years. Still, there geomtry may produce insight that mathematics does not.

[3] I realize this may be tedious for some. To them I recommend just paging through the diagrams till you find something that adds to your understanding of Garfinkel-Syropoulos.

[4] **Effects of Unequal Disproportional Factor Endowments:** If the endowments of K and L were not of the same

systems may lead at once to armed conflict among states both of whom nevertheless simultaneously benefit from trading with each other. Moreover, the idea that such seemingly schizophrenic behavior can be derived from a simple extension of the most primitive economic explanation for why nations trade --- that is, David Ricardo's theory --- seemed excellent.

So, when I obtained a written copy of this paper from Professor Garfinkel my mind was already prejudiced in its favor by her presentation. Except for the fact that it did not seem so "simple." Nevertheless, because the subject is important, and the paper elegant, and I considered myself knowledgeable about Ricardo, I was hooked. To arrive at the conclusion that this was indeed simple, however, was not necessarily easy. I followed a somewhat winding path as will anyone who reads on. But since the topic is important, and the conclusion insightful, I share with the reader of these notes, my journey from complicated to simple.

The first purpose of this paper, therefore, is heuristic: to disassemble the components of GS, and present them in a traditional manner that may instruct the reader. These are steps I found necessary to understand Garfinkel-Syropolous (GS hereafter). Methodologically the steps are geometric/ diagrammatic with only ancillary attention to mathematics. I hope they may add to the reader's grasp of the components in the GS system (of which there are many) and how they fit together.[2].

But the message of this paper is not solely heuristic. In the final analysis two countries, A and B, may allocate resources to fight each other with resource depleting "guns," G_A and G_B. Each does so on the basis of maximizing its own utility. Accordingly, a crucial substantive (in contrast to methodological) question arises of how to derive and to represent those utility functions, say $U_A(G_A, G_B)$ and $U_B(G_A, G_B)$ and, therefore, the reaction functions of each country:

$$G_A = f(G_B) \quad (1)$$

and

$$G_B = g(G_A) \quad (2)$$

Of course, other factors than G_B influence A's choice of G_A, but these are all incorporated into the function f. My treatment of this question I believe to be new substantively. However, to find it one must wade through the

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geometry. My style consists of a few equations and more than a few diagrams[3]. To follow the argument, the reader must be comfortable with Ricardo's 2-commodity theory of comparative advantage, with offer curves to represent demand supply conditions in trade, and with Edgeworth box diagrams and their relation to production possibility frontiers.

Figs 1a and 1b

2. Preliminaries

My assumptions will be identical to Garfinkel-Syropoulos, with one or two tiny, hopefully insignificant, changes to be noted soon. My notation is ever so slightly different. There are two countries, A and B. Each has an initial fixed endowment of labor (L^i_A, L^i_B) and capital (K^i_A, K^i_B). Both countries can use their L and K to produce either guns, (G_A , G_B) or a general resource (Z_A , Z_B) "later" to be converted into consumable goods. Z is not consumed directly but represents general consumption opportunities. The technology for producing G or Z is the same within each country and the same across countries - - - assumed here to be linear homogeneous. Thus, the production box is identical (except for scale) across countries with a straight-line contract curve, CC. CC identifies allocations of L and K that would be achieved by perfect competition with factors rewarded according to factor productivity, coincident with competition in markets for goods.

Figure 1a shows this Edgeworth box, for either country, with origins for Guns at O_G, and for Z at O_Z. The division of L between G an Z given as L_G and L_Z with superscript referring to country A or B to be added later if/as the context demands. Similarly, the division of K between G and Z is given as K_G, K_Z. The maximum of Z that endowed resources can produce is given by isoquant $Z_{\mathbf{max}}$, measured from O_Z , and the maximum of G that could be produced is given by isoquant $G_{\mathbf{MAX}}$ measured from O_G. In Fig 1b the straight line between $G_{\mathbf{MAX}}$ and $Z_{\mathbf{MAX}}$ gives the production possibilities (PP) for Z and G. Also shown for later use is what happens if the initial capital endowment capital K^i is increased to K^{ii} , then K^{iii} by equal increments and <u>free of charge</u>, so that $K^{ii} = K^i + \Delta$, and capital $K^{iii} = K^i + 2\Delta$. The production possibility curve shifts outward in parallel but by <u>diminishing</u> increments, so that the vertical or horizontal distance between PP^{iii} and PP^{ii} is less than that between

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PP^{ii} and PP^{i}.[4]

3. Single Country Autarky Equilibrium: No Guns

Considering a country say country A as a monolithic entity with no internal distribution issues. It is obvious such a country will or should take all the "free" capital there is, as this will enlarge its PP Frontier ever outwards though at diminishing rates. However, if inside of A the ownership of K and L is unequally divided among diverse groups, not everyone would benefit from such endless expansion; some will lose but this is an issue ignored in this paper[5]. Obvious also, with no other countries in the universe and no internal dissent A will and should allocate all its L-K resources to Z to achieve highest Z_{max} it can. There is no need for guns!

Next, following GS, with its Z_{max} country A can produce two goods, here called X and Y, with a constant unit-cost Ricardian technology --- using Z as the sole input such that a lot of X can be produced and rather little of Y. We draw the resulting production possibilities using curve ππ in Figure 3, so that the internal price of X is cheap and of Y is dear, as shown by the slope of ππ which is p_x/p_y. In autarky, country A settles at a tangency point yielding utility level U* where its (assumed to be uniform and homothetic among A's consumers) indifference map is tangent with the same consumer prices $p^* = p_x/p_y$ in consumption as in production[6].

Figs 2a and 2b

The dependency of point U* upon the influence of supply and consumer demand conditions then can be derived and illustrated by the offer curve (OC) construction. With point X_{max} as a starting point, supply demand equilibria at various prices p^k (k = 1, 2, 3 etc.) are traced out by the tangency points at U^1, U^2 etc. Also shown is the unambiguous benefit of an overall increase in resources, Z. With an increase in Z, the curve ππ

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4 **Effects of Unequal Disproportional Factor Endowments:** If the endowments of K and L were not of the same proportion in all countries, then even though production functions for Z and for G were identical all around, disproportional endowment would lead to total world production inside the true world production possibilities frontier, or PP^W.
This is easily seen in Fig 2a depicting a world Edgeworth box, and individual countries A and B within it. If endowments are rearranged in Fig 2a to be along the main diagonal CC^{W*} then world PP^W is PP^{W*} the straight line (45^O because of the assumption of identical technologies for Z and G) in Fig 2b. But with disproportional endowments, and no transfer of factors of production the across countries the contract curve in A is CC^A and in B is CC^B giving World Production Possibilities of PP^{AB} .

5 For a detailed analysis of this issue, the reader is referred to McGuire (1978)
6 GS are astute in their assumption that it is capital that can be added to the initial endowment, since the formal analysis could apply to any factor of production including labor. Leaving the overall population constant however limits confusion over the meaning of utility as applied to an entire country.

shifts out; the X-intercept moves to the right to say X^{**}_{max}, the offer curve begins at this new "greater" origin, and resulting welfare is unambiguously greater at U**, emphasizing again that international trade is absent.

4. Ricardian Competitive Equilibrium:
Ordinary Case And No Guns

Figs 3 and 4

Now Fig 4 adds a second "Ricardian" Country, say B. With the division of factor endowments among countries fixed, with no need for guns, and consequently with its resources Z^{B}_{max} B can also produce X, and Y except it can produce a lot of Y relatively speaking and not so much X. We show B's production possibilities $\pi\pi^{B}$ together with A's in Fig 4. Note the upside-down orientation of B's origin and $\pi\pi^{B}$ curve. B has a comparative advantage in good Y, while A has such an advantage in X. Moreover, this comparative advantage relationship does not depend on scale; however, A or B might expand or contract, the relative internal production prices do not change --- such is Ricardo. Since A can produce X relatively cheaply, and B can produce Y relative cheaply, trade can benefit both.

The variety of two-country competitive trading equilibrium is also illustrated in Fig 4 by the second offer curve B's, OC_B. A competitive trading equilibrium exists at the intersection of OC_A with OC_B. And where is this intersection? It varies with the shape and character of demand/supply in each country --- that is on the shape and position of OC_A and OC_B. Fig 4 shows three paradigm alternatives:

(a) OC_A^0 and OC_B^0 intersect at e^{00} giving an equilibrium price between p_A and p_B shown as p*. In this case trade causes both A's and B's welfare to increase over U* (Indifference curve hidden).

(b) OC_A and OC_B intersect at e^{01} giving an equilibrium price of p_A so that A gains nothing from trade and B captures all the benefits.

(c) OC_A and OC_B intersect at e^{10} giving an equilibrium price of p_B so that B gains nothing from trade and A captures all the benefits.

Fig 5a and 5b

5. Effects of Country Size on Ricardo's Equilibrium:
How Benefits of Trade Are Shared

The range of outcomes that are possible between Ricardo trading partners as was illustrated in Fig. 4, suggest a well-established principle of Ricardo trading. When a very small country and a very large country trade goods, the small country will capture all the benefits of trade, and the large country none of them. This was pictured in Fig 5, where at first --- indicated by origin O_A^{0} --- A is small and B is large. A's internal price

$p^A = p^A_x/p^A_y$ is lower than B's internal price $p^B = p^B_x/p^B_y$. A's offer curve $OC_A^{\mathbf{0}}$ --- intersects B's $\pi\pi^B$ either short of the intersection of B's offer curve, OC_B, with its own $\pi\pi^B$ or at the same point, at $e^{\mathbf{0}}$. In the former instance, a part of B's consumption of X comes from importing A's exports, but only in part. That is, B continues to produce both goods, and in consequence, gains nothing from trade. If B consumes at $e^{\mathbf{0}}$, it still gains nothing from trade, leaving all the gains to A.

Now let A grow a lot in size in Fig 5a with no change in comparative advantage $p^A = p^A_x/p^A_y$; its origin $O_A^{\mathbf{0}}$ shifts to the left to $O_A^{\mathbf{1}}$. Consequently, a new offer curve $OC_A^{\mathbf{1}}$ must be entered. The diagram shows so great a leftward shift in origin $O_A^{\mathbf{0}}$ that now B's (unchanged) offer curve $OC_{\mathbf{B}}$ intersects $\pi\pi^A$ at point e^1 either to the right of or exactly at $OC^{\mathbf{1}}_{\mathbf{A}}$, so that A --- now the larger country --- obtains none of the benefits of trade. We can generalize this relationship schematically with Fig 5b showing how relative country size determines regions where gains from trade go entirely to A, to B, or are shared by both regions. This picture will be used again presently in the paper[7]. The two rays from the origin in Fig 5b ideally should be derived from the underlying Ricardian Trade "boxes" depending on indifference maps and production possibility curves of each trading partner. I think that presenting this laborious result before its heuristic use is obvious would be an

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[7] This raises the interesting, and in our context, relevant question of what exactly is Country B's optimal size, just supposing its growth was obtained by a free gift-transfer of K from its trading partner? Just for the sake of argument suppose B started out as a small country compared to A. The answer to this question will bear on our later appraisal of conflict where countries fight to secure such a transfer. The problem on consideration becomes more complicated than expected.

Fig 6a

Imagine to begin that Z ---the one fungible intermediate good used in production of X and of Y in both countries – can be lump sum transferred from A to B. We picture this in Fig 6a. An initial position is given by A's origin at O_A and B's at O_B^{o}. B enjoys utility at U_B^{o} and A enjoys U_A^{o}. A's internal prices determine international prices. Now, reduce Z_A^{o} to Z_A^{1} by shifting A's PPF to the left by the difference between the two, and give that difference to B. This will shift B's origin from at $O_B^{\mathbf{o}}$ to $O_B^{\mathbf{1}}$ <u>maybe along a 45^O line</u> as shown. However, the move from at O_B^{o} to O_B^{1} depends on the absolute advantages of A vs. B. If the overall productivities of Z_A and Z_B are equal as assumed in the drawing, then as pictured, U_A declines along A's income expansion path $U_A^{\mathbf{0}} > U_A^{\mathbf{1}} > U_A^{\mathbf{2}}$, while B's utility increases along its income expansion path $U_B^{\mathbf{o}} < U_B^{\mathbf{1}} < U_B^{\mathbf{2}}$, (adjusted and redrawn to incorporate the movements of at $O_B^{\mathbf{0}}$ to $O_B^{\mathbf{1}}$ to $O_B^{\mathbf{2}}$). These transfers bring B to the margin where it is no longer "small" and A to the margin where it is no longer "big." Transfers beyond to $O_B^{\mathbf{2}}$ will force B to compete on world markets by lowering its price for its exports and raising the price for its imports. This effect will be inflated or diminished if the transfer sent from A of $-\Delta Z_A$ is less than or greater than the transfer received by B $+\Delta Z_B$. We have shown above that the PPF expands in response to increases in K, but at a diminishing marginal rate. Accordingly, depending on which side of an equal proportion K/L ray each country is positioned on, $\Delta Z_A <=> \Delta Z_B$. This effect will modify the validity of any argument that with respect to contested capital "more is better."

unwise demand on the reader's attention. I therefore relegate the derivation to a footnote.[8]

6. Extraordinary Case of Immiserizing Growth And Its Implications

Figs 7a and 7b

In addition to the foregoing generic possibilities, one particular feature of Ricardian production combined with offer-curve-competitive-trade that has special relevance to GS was noticed by Bhagwati in 1958. This result, known as "immiserizing growth," is that depending on the offer curves an increase in a country's overall resources may lead to it lower levels of utility, provided the country trades with other competitors. This is easily pictured in Figure 7a. Country A originally with resources Z^0_A, produces at X^0_A, the corner of $\pi\pi^0_A$, then exports good X to and imports good Y from B to enjoy utility level U^0_A. Next let Z_A increase to $Z^1_{\mathbf{A}}$. Now A will produce on $\pi\pi_{\mathbf{A}}^{\mathbf{1}}$ at $X^1_{\mathbf{A}}$, export equilibrium price and welfare outcome is seen at the intersection of $OC^1_{\mathbf{A}}$ and $OC^1_{\mathbf{B}}$ --- both drawn from the displaced x-intercept at X^1_A.

A more schematic picture of the effects of trade is given in Figures 7b and 7c, which plot a country's utility vs. its resource size Z, for the two cases where immiserization is absent Fig 7b and second where it is present Fig 7c. Without immiserization as a country grows while others do not, it will benefit from trade, uniformly; fast in the beginning, and slower as it grows into being a "large country" where it captures none of the benefits from trade. On the other hand, a country that experiences immiserization over a range would have a utility vs. resource profile more like fig 7b where after a range of normal benefit from trade, a point is reached where further growth reduces utility. Figure 7b shows this, with an assumption that once a stage of immiserization is reached, it gets worse until Z reaches the size where it trades as if under autarky. The trajectory of the utility

Figs 6b and 6c

[8] Figures 6a and 6b derive the rays p_A and p_{AB} and on the assumption the underlying utility functions are linear homogeneous. First consider 6 b. It erects several upward shifting origins for B, ie $O^0_{\mathbf{B}}, O^1_{\mathbf{B}} - O^2_{\mathbf{B}}$ along the upright through $X^{\mathbf{A}}_{\mathbf{MAX}}$. Each of these shifting origins corresponds to its own value of $Z_{\mathbf{B}}$. B's resource constraint $\pi\pi_{\mathbf{B}}$ is drawn from $X^{\mathbf{A}}_{\mathbf{MAX}}$. The same line applies to the upside-down B-opportunities irrespectvie of the B-origin. From each B-origin then draw the income expenditure path for $p_{\mathbf{B}}$ to its intersection with $\pi\pi_{\mathbf{B}}$. Then draw in the A offer curve that intersects each of these junctures of $IEP_{\mathbf{B}}$ and $\pi\pi_{\mathbf{B}}$ each OC_A drawn from the point $X^{\mathbf{A}}_{\mathbf{MAX}}$. Each OC_A curve implies its own A-origin, O_A and its own value of Z_A. Enter corresponding values of $Z_{\mathbf{A}}$ and $Z_{\mathbf{B}}$ on the x- and y-axes to the right, connect the dots, and label the locus p_B. Then, using the same procedure, build the $Z_{\mathbf{A}}$-$Z_{\mathbf{B}}$ locus for $p_{\mathbf{A}}$ as depicted in Fig 6c.

curve could follow many less extreme paths with immiserization merely a down sloping wiggle, so that pictured is solely for illustration.

The crucial point about the phenomenon of immiserizing growth for purposes of our problem is that without it, a country always benefits from owning more resources whether it trades or not. This means that a positive 1:1 relation exists between Z and utility. So, absent immiserization and absent competition for new capital seizures, we know a country will and should strive to maximize Z. As we will soon see, however, this is no longer true when immiserization lurks in trade.

7. Trade as the Source of Interdependence Among Countries And It Crucial Function in Determining a Measure of Utility

Fig. 8a and 8b

The foregoing argument highlights an interdependence between countries caused by trade in a manner especially useful for when they also might compete for external resources. But to make the connection useful to us we have to elaborate the analysis to allow Z_B to vary systematically. Usually one thinks of a country's welfare (when considered as a unified agent) as dependent on its own resources, plus market conditions. But here we see that those "market conditions" will include the resource level enjoyed by a trading partner. Thus, rather than being able to write $U_A = f(Z_A)$ as Fig 7 might suggest, where the Z_A represents A's owned resources, we will benefit from explicit recognition that $U_A = h(Z_A, Z_B)$. In other words, for some problems the foregoing diagram 7b is insufficient or misleading. Therefore, let us consider how to construct $h(Z_A, Z_B)$.

This construction will bridge the gap between Z and U, and it is a crucial input on our journey to discern, if, when, how, and why trade induced prosperity will cause Ricardian countries to fight with guns more intensely or less so. Here is a less diagrammatic, less technical behind the constant-utility contours. [9]

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[9] Suppose I want to draw in constant value contours for Country B. We can agree that in the wedge to the left of the ray $p_W = p_B$ through the origin, B's welfare is unaffected by increase or decrease in Z_A^N. B trades as if in autarky. So any value or effective utility contour is a horizontal line; and as Z_B^N increases, one crosses higher and higher constant value contours. Now consider extending one of those value contours - - - say Z_B^*.

Imagine that the line Z_B^* continues into the middle wedge and then beyond that into the wedge where A's size governs, and B obtains all the gains from trade. Will that extended horizontal line, represent an unchanging level of Value for B as it did before it crossed the delimiting ray $p_W = p_B$? Well obviously, not! In the middle region B benefits not only from the autarchic value of Z_B^* but in addition B realizes the benefits of trade - - - some of them as it shares them with A. The share going to B gradually increases as line Z_B^* continues across the middle wedge approaching the

The argument which used Fig 5 above about relative country size in effect stated that roughly or approximately for some ratios of resources --- on or above $[Z_B/Z_A]^*$ for example in Fig 8a --- that B's internal price ratio establishes world price, that A captures all the benefits from trade, and that further increases in Z_B will have no effect on A's utility. Fig 8a repeats Fig 5b but <u>tentatively</u> adds three versions of a plot of $U_A = h(Z_A, Z_B)$, showing one utility contour say $U_A^{\mathbf{o}}$. To the left of the critical ratio the contour is vertical; increases in Z_B are of no benefit to A. We have labeled these alternative, "candidate" utility contours $U_A^{\mathbf{o}_\alpha}$, $U_A^{\mathbf{o}_\beta}$, and $U_A^{\mathbf{o}_\gamma}$. They all pass through point "s" on the ray $p_{\mathbf{B}}$.

Now consider the combination at point "s" in Fig 8a where A is small, B is large, and world price $p^W = p^B$ --- B's internal cost/ratio. The equivalent point in a Ricardo trading box, with corresponding values of Z_A and Z_B is also given at "s" in Fig 8b, together with the relevant offer curves. Now ask, what configuration of the Ricardo trading box would yield the same utility for A, i.e. $U_A^{\mathbf{o}}$ if, starting from point s, it grew to become the "large" country and B the small one, and world prices came to be governed by A's interior price ($p^A = p^A_{\mathbf{x}}/p^A_{\mathbf{y}} = p^W$= world price) rather than by the internal price of B? That is, where is the point "t" on ray p^A in Fig 8a that provides to country A the same level of utility that was provided at point "s" on p^B ? Fig 8b constructs an analysis/answer.

Figs 8c

For $U_A^{\mathbf{o}}$ to co-exist with $p^A = p^W$, A's resources must increase, shifting $\pi\pi_{\mathbf{A}}$ to $\pi\pi^1_{\mathbf{A}}$ ie the Z_A line to the right so that A's offer curve drawn from the new $X_{\mathbf{max}}$ intersects A's $U_A^{\mathbf{o}}$ at the slope of $\pi\pi^1_{\mathbf{A}}$. This intersection is labeled "t" in Fig 8b. Fig 8b using the same origin for A, O_A, shows A's required $Z_A^{\mathbf{1}}$ with x-intercept $X^1_{\mathbf{max}}$. Next, the diagram shifts $O_{\mathbf{B}}$ parallel to the right so that the old $Y_{\mathbf{max}}^{\mathbf{B}}$ (now relabeled $Y_{\mathbf{max}}^{\mathbf{B1}}$) intersects the x-axis as before, but now at point $X_{\mathbf{max}}^{\mathbf{A1}}$. Now, for "t" to represent a point on ray $p_W = p_A$ in Fig 8a, it must represent an equilibrium in Fig 8b. B's offer curve now beginning at $X_{\mathbf{max}}^{\mathbf{A1}}$ (and coinciding with point $Y_{\mathbf{max}}^{\mathbf{B1}}$)

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region where A's size dominates. So let <u>line</u> Z_B^* cross over that next ray where $p_W = p_A$ defining the wedge where B obtains all the benefits from trade. Accordingly to draw a constant value contour for B - - - call it Z_{BV}^* - - - equivalent in value to <u>line</u> Z_B^* (to the left of ray $p_W = p_B$) we cannot extend line Z_B^* across that ray. The equal value line descends till ray $p_W = p_A$ is reached, and then levels off horizontal again. Following this logic, I have drawn in a few equal value contours for B - - - Z_{BV}, and have drawn in some equal value contours for A, following the same reasoning.

has been shifted parallel along with the shift of O_B^0 to O_B^1. Therefore, for equilibrium that offer curve (passing through $\pi\pi_B$ as before, now at point s^1) must intersect $\pi\pi^1_A$ at "t". To explore or confirm this we need to extend B's offer curve OC_B^1 past s^1.

Suppose it did just happen that OC_B^1 did intersect $\pi\pi^1_A$ at "t" after passing through point s^1. That version of B's offer curve is labeled $OC_B^{1\alpha}$. In this case the correct placement of "t" in Fig 8b is shown as t^α. However, there is no necessity for OC_B^1 to go through t on $\pi\pi^1_A$; its trajectory depends on B's indifference map. Fig 8b shows two other possibilities for OC_B^1 , namely $OC_B^{1\beta}$, and $OC_B^{1\gamma}$. If B offer curve is $OC_B^{1\beta}$, which intersects $\pi\pi^1_A$ to the left of "t" then Y_{max}^{B1} is too great (as B's demand will drive the price of A's exports, X, up above p_A) and therefore the origin O_B^1 must be lowered, reflecting a lower value of $\pi\pi_B$ and of Z_B . This lower value of Z_B is shown in Fig 8a at t^β. If on the other hand B's offer curve is $OC_B^{1\gamma}$, which intersects $\pi\pi^1_A$ to the right of "t" at t^γ then B is not supplying all of A's consumption of good Y; so, in this case O_B^1 can be raised, so that $\pi\pi_B$ and Z_B are increased. This configuration is shown as point t^γ in Fig 8a. Suppose we connect up points "s" and "t" with straight lines or simple curves. Immiserization potentials lurking in the preference maps of A and/or B would probably cause wiggles and slope reversals in such curves, but we leave that aside for now. This procedure allowed us to combine the effects of comparative size and trade in a Ricardian world into a single measure along any single contour in Fig 8a called "utility" although maybe "resource equivalence" is a better term. Obviously, by repeating the foregoing procedure, we can build a map of utility contours both for A and for B, so one could think of Fig 8a as filled with such contours for A and for B.

Omission of immiserization notwithstanding we still have a rich banquet of possible combination of utility maps for our two countries. Figure 8c in each of its panels depicts four seemingly generic configurations, with radically diverse implications for countries' incentives to fight and/or trade.

The first implication of our logic is that if starting as a large, price-setting trader, a country declines in magnitude, its losses from diminished size will be offset entirely or partially by gains in trade generated from its adversary/partner's relative growth. The top two panels, I and II of Fig 8c illustrate this logic. In panel II a declining country only "requires" its partner to grow at a fraction of its own decline to stay even (stay on the same utility contour). But in Panel I of 8c for the benefits of trade to balance out A's loss from a lesser size, B must grow by more than A's decline. That this potential can create a dynamic for interactions between trading and fighting seems obvious, and I will return to examine this potential in a later section.

A second implication of Fig 8 is displayed in panels III and IV of 8c, and seems as remarkable as it looks logical. To see this, imagine a country (A) so small that it has no impact on world price, which is determined entirely by the internal prices of its partner (B). Next let this country grow to the point that it now dominates, determines international prices.[10] Then for some configurations of preference maps, B also may be allowed or required to grow without diluting A's world dominance. This seems to be the message, encrypted in indifference maps like U^{γ} of Fig 8a which have been extended or applied in Fig 8c-III and 8c-IV. We will find that a grasp on these relationships will help as we try to figure out how a country's quest for gain from trade, and from productive size meld together.

8. Single Country's Allocation to Guns: Absent International Trade

Next suppose contrary to the foregoing assumptions of Section 4, and now comporting with GS, that additional capital beyond the initial endowment is no longer freely available. Imagine "Guns" or equivalent resources are needed to acquire additional capital $\Delta K = \kappa$ in excess of the secure endowment. Then following 1 their line of thinking and suppressing country A, B indices:

$$\kappa = \kappa\,[G(L_G,K_G)]. \tag{3}$$

Here resources available to A remain L^i (as before) but now K is augmented by whatever is captured so that the balance between capital availability and capital allocation becomes:

$$(K_Z + K_G) = K_T = (K^i + \kappa) \tag{4}$$

where K_T indicates total capital both that captured "by force" and the initial endowment. Probably $\kappa[G]$ should show diminishing marginal returns, for example

$$\kappa = a[G]^{1/2} \tag{5}$$

with "a" some constant. But this is not essential. Rather, an assumption of constant marginal returns and average productivity will prove useful presently in which case we will write simply:

$$\kappa_A = a(G) \tag{6}$$

And note, reflecting GS, that production functions, Z and G, i.e.

[10] The reader is reminded that we are dealing with an abstract Ricardian world here with only two countries. The language of this hypothetical world will seem hyperbolic if applied to the real world of many countries, many goods, many resources, and a close to infinite number of combinations of trade and/or fight.

$$G = G(L_G, K_G) \quad (7)$$

and

$$Z = Z(L_Z, K_Z) \quad (8)$$

are identical functions.

What then is the optimal allocation of resources away from production of Z and into capturing capital, considering that to make guns itself requires capital[11] ? Let us consider the problem this way preliminary to asking: how much K and L should Country A allocate to guns and divert away from Z, <u>when those guns compete with another country</u>. That is, as an introduction to the problem, first we assume no such competition. Accordingly, we ask: what is the solution to the maximization problem:

$$\underset{L_G,K_G}{Max}\, Z\{[(L^i - L_G),(K^i - K_G + \kappa[G(L_G,K_G)]\} \quad (9)$$

The Lagrangian is straightforward.

$$\underset{L_Z,K_Z,L_G,K_G}{Max} \; Z(L_Z,K_Z) + \lambda_1(L^i - L_G - L_Z) + \lambda_2(K^i + \kappa - K_Z - K_G) + \lambda_3\{\kappa - \kappa[G(L_G,K_G)]\} \quad (10)$$

FOCs give:

$$\partial/\partial L_Z : \quad Z_L - \lambda_1 = 0 \quad (11)$$

$$\partial/\partial K_Z : Z_K - \lambda_2 = 0 \quad (12)$$

$$\partial / L_G : \; - \lambda_1 + \lambda_3[\kappa_G\, G_L] = 0 \quad (13)$$

$$\partial / K_G : - \lambda_2 + \lambda_3[\kappa_G\, G_K] = 0 \quad (14)$$

$$\partial / \kappa : \lambda_2 + \lambda_3 = 0 \quad (15)$$

$$(11 \,\&\, 12) \rightarrow \quad Z_L/Z_K = \lambda_1/\lambda_2 \quad (16)$$

$$(13 \,\&\, 14) \rightarrow \quad G_L/G_K = \lambda_1/\lambda_2 \quad (17)$$

$$(14 \,\&\, 15) \rightarrow \quad \kappa_G \cdot G_K = 1 \quad (18)$$

An Edgeworth Box, Fig 9a, with width and height of the given endowments of L and K respectively as repeated from Fig 3 helps to understand the problem. The payoff to guns, G, in terms of additional capital is shown as a vertical extension of the K-axis with the relation between G and κ shown to the right. As K is

[11] Here is one tiny difference with Garfinkel and Syropolous. They assume that production of G only uses factors in the fixed, endowed proportions K^i / L^i, whereas we are assuming that both Z and G are produced with the same technology using factors in the same proportions, and that those proportions are post capture proportions $(K^i + \kappa)/ L^i$,

enlarged through conquest, the vertical dimension of the box increases, while its width remains unchanged so that the straight-line contract-curve between Z and G rotates to intersect the extended $K + \kappa$ axis. Isoquant Z^i gives the available Z when nothing is allocated to guns. To enjoy κ, Z must be given up and allocated to G by moving southwest along the CC^j contract curve that pertains to κ^j. The amount of Z needed to pay for G and thereby capture K is obtained from the function

Fig 9a

$$\kappa = \kappa[G(L_G, K_G)] \qquad \text{(3 repeated)}$$

which function will be geometrically derived presently. For the moment, I just mark off for each, κ^j (j= 1, 2, 3) a cost assumed for that amount, measured by the relevant distance from point κ^j along the relevant CC^j curve. Connecting the points so constructed shows the opportunities for Z in the space, L_Z - K_Z. --- opportunities created by allocating productive factors to capture of κ "by force." This opportunity curve is labeled

$$K_Z = \phi[L_Z]. \qquad (19)$$

The opportunity curve crosses various Z-isoquants,

$$Z\{[(L^i - L_G),(K^i - K_G + \kappa[G(L_G, K_G)]\} \qquad (20)$$

and illustrates the solution to the optimization problem at a tangency with Z-isoquant $Z^A_{\mathbf{OPT}}$. Of course, if the z-isoquant Z^i is so steep or the curve $K_Z = \phi[L_G]$ is so shallowly sloped at the start so that ϕ does not cross Z^i then it is optimal to acquire no extra capital. But assuming it exists, the tangency in Fig 9a merely illustrates a familiar equality between marginal rates of technical substitution in Z, $MRTS_{\mathbf{Z}}$ and in G, $MRTS_{\mathbf{G}}$ [12]. **Fig. 9b**

We can also select key information from 9a and transfer it to Figure 9b. This includes (a) the gross benefits of procuring guns or "gross" productivity of G labeled as Z^{max} [κ(G)] from the values of Z-isoquants along the vertical extended of Figure 9a, (b) the opportunity costs of so doing taken from the values of G (or foregone Z) along the various CC curves and labeling these amounts "C_Z(G)" for "Cost of G in terms of Z" and (c) the net benefits by subtracting (b) from (a). Doing this identifies the net benefit optimum Z^N at $G_{\mathbf{OPT}}$ in a conventional graphic Fig 9b.

[12] One advantage of building G = γ(Z) ie the opportunity cost of guns, in this way is that the effects of diversity in Z and G production-functions is easily introduced. For instance, to analyze the consequences if G relative to Z is the more labor intensive idustry, or vice-versa.

9. Construction of Opportunity Curve $K_Z = \phi[L_Z]$
Resulting From Possibilities for Armed Capture of Resources.

Here I provide a construction of the opportunity frontier, $K_Z = \phi[L_G]$, to be drawn in the space of the inputs to the intermediate product or fungible good, (L_Z, G_Z). This construction derives ϕ rather than merely illustrating as Fig 9a does. To do this consider a geometric construction.

Fig. 10

Figure 10 shows a useful variation on Fig 9a. Dimensions of the Edgeworth box give the original endowments L^i - K^i. Isoquants for provision of G are entered relative to origin O_G and are increasing to the south-west as shown by their numbering along the box diagonal, CC_0 that obtains before any acquisition of κ. Note that the values of G so displayed also show the opportunity cost of G in terms of Z foregone for various K_Z / L_Z ratios[13]. For that reason we label the G-isoquants also as $C_Z(G)$ --- opportunity "cost" of G in terms of Z.

The "capital capture opportunities" as a function of G, i.e. the productivity of G in terms of increasing the amount of K captured, or $\kappa = f(G)$ is as drawn to the right of the Edgeworth box. Here, the function is linear. In this case as G increases along the right x-axis, κ is shown to increase measuring downwards. Accordingly the new Edgeworth box expands downward in its vertical dimension so that new required ratios of $(K^i + \kappa)/ L^i$ are shown as the new box-diagonals, one for each value of κ and, therefore, of G.

Now derive $K_Z = \phi[L_Z]$, that was merely illustrated in Figs 9a and 9b, consider the functions in parametric form $L_G(\kappa)$ and $K_G(\kappa)$. Choose some value of $G = G_1$ and, therefore, of $\kappa = \kappa_1$; and for this value extend the box downward by that amount, generating a new (more "southern") origin for Z and a new diagonal CC_1. Along CC_0 find the point for G_1 and its associated G-isoquant, and mark the intersection of that isoquant with that diagonal line CC_1. Repeat for all values of K, G, and κ, then connect the dots to obtain $L_G = \mu[K_G]$. Figure 10 shows the curve so produced[14]. Next for any value on the curve $L_G = \mu[K_G]$ --- for instance the point on CC_1 --- trace vertically up for that value of L_G to intersect with CC_1 as copied/shifted to the diagram's second quadrant to originate at κ_1 and labeled there as CC_1^*. This intersection gives values of $L^1_Z = (L^i - L^1_G)$ and of

[13] Our diagrams imply that since the functions $G(L_G, K_G)$ and $Z(L_Z, K_Z)$ and are identical, the contract curve must bea straight line connecting the origin O_Z with O_G and here the same conclusion is derived.
[14] Another benefit of geometric analysis like ours is its flexibility. As an example of this how does the opportunity curve $K_Z = \phi[L_G]$, that is derived for given initial endowments of L and K, change if those endowments are increased proportionally while the capital capture function $\kappa = \kappa[G(L_G, K_G)]$ is unchanged.

$K^{1}_{Z} = (K^{i} + \kappa_{1} - K^{1}_{G})$ on the promised function $K_Z = \phi[L_Z]$. Fig 10 (2nd quadrant) shows this construction for 3 points 1, 2,and 3.

10. The Effect of A Country's Size on its Allocation to Guns

The constructions of Figs 9a and 10 can be applied to answer questions important to grasp the structure of our problem. But first, note how Fig 10 illustrates that constant marginal returns to "investment" in guns in no way undermines the incentives to fight for capital. And the same figure also implies how greater efficacy of guns for capturing capital will re-works the curves curve $L_G = \mu[K_G]$, and $K_Z = \phi[L_Z]$. Rotating κ(G) clockwise in the 4th quadrant will produce a counterclockwise rotation of $\mu[K_G]$ in the 3rd quadrant, and a clockwise rotation of $K_Z = \phi[L_Z]$ in the 2nd quadrant. The progression clearly leads to a higher value for Z_{OPT}.

The relevance of country-size on its allocation of resources to guns for purpose of capturing "extra" capital seems obvious. Does a big country compared to a small one have a greater or lesser incentive to capture additional capital? Does it matter if the country in question is already relatively richly endowed in capital vs labor? And if so how? And why? The answers to these questions will prove important, even crucial, in the upcoming analysis of how trade between rivals influences allocations to guns, and vice versa.

Fig. 11

Happily, the answer to these questions is obtained from our geometry. Re-draw Figures 10 as Fig. 11 to compare two countries with identical fighting potentials i.e. identical κ = f(G) functions, but of different size. For the case depicted the two countries have the same endowed proportions of L and K, but one is of a large scale, the larger country has stronger incentive to fight. In Fig 11 the crucial function $K_Z = \phi^B[L_Z]$ for the larger country (superscript "B" is for "big") lies above/outside of that for the small (superscript "S") country. The opportunity cost measured in terms of good Z foregone is less for the large country because in paying for G it is better endowed in the relatively scarce resource, L. Let the overall height and width of the original Edgeworth box say double, while the payoff schedule κ = f(G) remains unchanged. Then, as before, connect up the dots to obtain a new locus of points $K_G = \mu^{B}[L_G]$. How does this locus $\mu^{B}[L_G]$ compare with the old locus $\mu^{S}[L_G]$? It is uniformly less steep and rotated clockwise about origin O_G to the left as obvious in Fig 11.

11. An Example Calculated:

The foregoing text and especially diagram 9a suggest diminishing marginal returns to Guns, say

$$\kappa = a[G]^{1/2} \quad \text{(5 repeated)}$$

where to repeat G and Z would have the same production function, say the CRS functions

$$G = \varpi L_G^{1/2} K_G^{1/2}) \quad (21)$$

$$Z = \varpi L_Z^{1/2} K_Z^{1/2}) \quad (22)$$

Here, comporting with Fig 10, I produce a simpler example where $\kappa[G]$ is linear, first to show that in utilization of force for resource capture, diminishing marginal returns are not necessary to support incentives to allocate resources to arms. A second reason for the exercise is that calculating optimal allocations is easier and more transparent with $\kappa[G]$ linear. Specifically, I assume

$$\kappa[G] = \alpha(L_G)^{1/2}(K_G)^{1/2} \quad (23)$$

to be inserted directly into the objective function, that function being the net production of Good Z.

$$\underset{L_G, K_G}{\text{Max}}: Z = [(L^i - L_G)^{1/2}][K^i - K_G + \alpha(L_G)^{1/2}(K_G]^{1/2}]^{1/2} \quad (24)$$

$$\partial Z/\partial K_G : -1 + \alpha/2(L_G)^{1/2}(K_G)^{-1/2} = 0 \rightarrow \sqrt{K_G / L_G} = \alpha/2 \quad (25)$$

$$\partial Z/\partial L_G : \quad [K^i - K_G + \alpha(L_G)^{1/2}(K_G)^{1/2}] = (\alpha/2)[K_G/L_G]^{1/2}[(L^i - L_G)] \quad (26)$$

Then, substituting from (25) into (26) gives:

$$(K^i + K_G) = (L^i - L_G)(\alpha^2/4) : or \; 2K_G = \{[(\alpha^2/4)(L^i)] - K^i\} \quad (27)$$

The following Table gives values for variables of interest, values which maximize Eqn. (24).

Thus, this example confirms the elements derived from the earlier geometry. In particular it illustrates how an optimal extension of endowments (captured at an expense of foregone intermediate good Z) requires factor intensity equalization across Z(L,K) and G(L, K), and importantly how diminishing marginal returns in the productivity of G as an instrument of capital acquisition are not necessary for an interior solution. Noteworthy also is that just two calculated examples suggest that incentives to appropriate new capital through force may be rather sensitive to the efficacy of force. In the above example, merely doubling productivity coefficient α (from 2 to 4) increases the optimal capture of new capital by a factor of 7 ((224/32) = 7).

Table 1

Outcomes from Optimal Allocation of Resources to Capture Additional Capital, κ:
When Objective is Maximization of Net Product $Z[(L_G]^{1/2}(K_G)^{1/2}]$ and $\kappa[G] = \alpha(L_G)^{1/2}(K_G)^{1/2}$

α: Productivity of Guns	2	4	
L^i : Endowment of Labor	64	64	
K^i : Endowment of Capital	32	32	
L_G : Allocation of Labor to Guns	16	28	
K_G: Allocation of Capital to Guns	16	112	
κ: Amount of Capital Captured	32	224	
$K^i + \kappa$: Total Capital Available	64	256	
L_Z : Final Labor Allocated to Production of Z	48	36	
K_Z : Final Capital Allocated to Production of Z	48	144	
Z^{max} : Final Max Z	48	72	
Z^i : Initial Max Z with no Guns and $\kappa = 0$	45.25	45.25	
K^i/L^i	½	½	
$(K^i + \kappa) / L^i$	1/1	4/1	

12. The Effect of Trade On the Allocation to Guns And on the Measure of Benefit

Our lengthy analysis demonstrates that the issues Garfinkel and Syropolous have addressed possess numerous "moving parts." We are now close to being able to assemble them to understand better how they fit together. The next component in the assembly is to understand how a country's incentive to allocate to guns depends on its trading opportunities considered as a parameter, irrespective of the source of those opportunities, whether a rival or not. Specifically, how does the opportunity to trade at some assumed constant world price influence the incentive to arm, to capture extra capital? To answer this question we must extend the analysis surrounding 8a, 8b and 9a to the case where the country trades.

There are two issues:

First, how does the introduction of trade alter the construction of the returns to investment in guns?

Second: how does the introduction of trade affect the relations between net resources, and utility or welfare?

In the absence of trade, resources and utility (excluding distribution issues) are essentially interchangeable. This has allowed us to avoid problems of utility measurement. Does this continue to obtain in a trade plus guns environment? The answer involves the connections between a country's net resources, its trade opportunities, and its welfare or utility. First we plot A's welfare or utility as a function of its resources, Z^N netted out for expense of G needed to acquire κ. Figure 12a consolidates information from Fig (7) into the space of Z-G. It summarizes with the curve $G = \gamma(Z)$ showing how the ability to capture new capital allows various higher levels of output-Z. Only the area to the left of $G = \gamma(Z)$, that is excluding area to the right is feasible. The tangency point of Figure 8a translates in G-Z space into the vertical slope of $G = \gamma(Z)$ where Z is maximized.

Now consider the case where A has rather ordinary preferences, and therefore displays no "immiserizing growth" and where growth enters exogenously. Then as Z^i increases (and with it the value of Z^N achieved by fighting) --- and as the lower panel of Fig 12a displays --- A's welfare first increases faster than Z^N because A benefits from trade. However, eventually the rate of increase of benefits from growth slows to match the rate of increase in net resources Z^N; this happens once A has become a large country whose internal prices set world price.

Figs. 12a and 12b

We commented on this relationship above in Fig 7b and 7c. Here we use and extend this relationship in Fig 12a combining the best choice of guns from Fig 8b in the upper half of 12a, with the utility payoff vs. Z^N as in Fig 7b in the lower part of 12a. The diagram constructs a locus for how G_{OPT} and Z^N vary with Z^i. Just what the source of the growth of Z^i is may make a difference to the function $\gamma(G)$ and to the locus of tangency points all connected up and labeled $G = \rho(Z^i)$. If A is growing all its productive factors, then allocation to Z will increase (see Fig 11), but if the source of growth were exclusively augmentation of endowed capital, K^i, then as Z^i increases G will decline. In either case the utility welfare path derived in Fig 12a gives a valid if rough picture. As Z^i increases and G is optimized, Z^N increases as well and importantly with Z^N country A's utility will increase also.

Things change however if A is vulnerable to experiencing "immiserizing" growth, as pictured in Fig 12b, combining Fig 7c and 8b. Now the benevolent and informed decision maker for A will see that over a range of values of Z^N, growth combined with trade is not in A's interest as A's welfare declines. Over some stretch of the x-axis, therefore, A will first, (once $Z^i = Z^i_\alpha$) is reached begin to reduce allocations to G progressively down to nil at $Z^i = Z^i_\beta$. This choice maintains and Z^N at Z^i_β. A will then continue with G = 0 till its endowed production reaches $Z^i = Z^i_\gamma$ and it will allow welfare to decline to the kink in the utility curve --- at which point it will progressively return[15] to fighting for capital up to $Z^i = Z^i_\Delta$. So in this interval the positive 1:1 connection between resources and welfare is broken. A's best choice after $Z^i = Z^i_\gamma$ is to wait until Z^i_τ is reached. Once past $Z^i = Z^i_\Delta$, expenditures on Guns resume their ordinary upward (or downward) trend.

13. Introduction of Adversarial Conflict

It may seem odd to have traversed so many topics in this paper before arrival at the subject implied by the title. Adding Country B as an adversary with incentive to fight over κ is straight forward at the first level of analysis. G-S employ the conflict success function of Gordon Tullock[16]. Here, I think, less specificity is better given the reliance on geometry in this paper. Accordingly, to include an adversary, instead of writing

$$\kappa_A = \kappa_A[G^A] = a[G(L^A_G, K^A_G)] \qquad \text{(7 repeated)}$$

as we did earlier, we can directly add B to A's capital acquisition function writing:

$$\kappa_A = \kappa_A[G^A, G^B] \quad \text{; with } \partial K_B / \partial G_B > 0;\ \partial^2 K_B / \partial G_B^2 < 0;\ \partial^2 K_B / \partial G_A G_B \lesseqgtr 0 \qquad (28)$$

and symmetrically for Country B. A disadvantage here is that describing conflict our way is open ended, in the sense that the total amount of κ to be divided is not specified. G-S insure that for any adjustment in forces (Guns) by either adversary the net loss in κ by one is just balanced by the other's gain. I hope this divergence from G-S mathematics is not fatal, but rather is a compromise worth making for its simplicity in depicting basic adversarial and trade relations in this context trade combined with fight.

Figure 13

Introducing $\kappa_A = \kappa_A[G^A, G^B]$ now, as a first approximation, allows Fig 8b to be elaborated as shown in Fig 13. A separate κ_A function is drawn for each value of G^B, which yields a locus of tangencies giving A's choice of G_A and therefore of Z_A for various values of G_B. We assume here, for a start, that both rivals desire to maximize net resources, Z^N, although the arguments presented above seem to undercut the assumption, if immiserization threatens. We will comment more on the inadequacy of Z^N as a proxy for utility later. Figure 13 is derivable from Fig 9 which would have to be adjusted or extended to add several $\kappa(G_A, G_B)$ curves

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[15] The model proffered her is not dynamic. There should be no inference drawn that a country will allocate to guns, later reduce the allocation, and later still return to capturing capital again.
[16] So successfully employed by Hirshleifer, Skaperdas, Garfinkel and Skaperdas, and others.

producing a pencil of $\phi(L_G)$ curves, one for each value of G_B. Even assuming the resources-vs-utility problem can be finessed we will see that the equilibrium of rivalry may not be simple. But first, for A's "reaction function" we will write

$$G_A^* = g_A(G_B) \tag{29}$$

or sometimes

$$Z_A^* = h_A(G_B). \tag{30}$$

This locus for A was drawn in Fig 12 with rising and a falling section. Both $h_A(G_B)$ and $g_A(G_B)$ are to be derived directly from Fig 8 or 9. This feature is likely not a necessary characteristic of the maximization trajectory but it seems possible, and suggests a source of multiple equilibria and instabilities in Nash solutions. With Fig 13 as a base then we can turn to the next section to examine the implications for two such competitors. Nash-Cournot behavior, of course, admits no collusion o collaboration between A and B. But this may not entail that A will ignore entirely the effect of its choice of G_A on G_B. Between total shortsightedness and subtle leader follower decision protocol may exist numerous mixed cases. For our purposes here though I will cleave to the simplest of Nash protocols, both here and when trade between rivals figures in.

14. Nash-Cournot Competition for κ in Absence of Trade Between Adversaries

The next diagram, Fig 14, puts the two reaction functions together in the space of $(G_A$-$G_B)$. If the two nations do/can not trade with each other, then we would also know it is rational for each to maximize its own **net** resources, $Z^{NET}(\kappa) = Z^{max}(\kappa) - C[G(\kappa)]$, as a 1:1 correspondence exists between $Z^{NET}(\kappa)$ and a country's utility (assumed to be monolithic, consensual, or uniform). Thus maximizing $Z^{NET}(\kappa)$ maximizes utility if autarky prevails. The question relevant to this paper then becomes is there anything special about competition for capturing extra capital that might produce unusual reaction curves? With reaction curves as drawn in Fig 14a a standard stable, interior outcome is expected; but an arrangement like Fig 14b is not necessarily inconsistent with the underlying maximizations, and presents several stable and unstable outcomes.

Figs 14a and 14b

This issue, to be delayed, for another day is how underlying structure might induce multiple equilibria and instabilities as in Fig 14b. By underlying structure I would include introducing difference in production functions for G and for Z --- different within and between traders/adversaries, as well as different effectiveness of capture, $\kappa[(G)]$. I will not pursue the conditions that generate such problems since this is not a focus of G-S, but it would be worth the time to do so in the future. Maybe some particular configuration of initial endowments, for instance, is more likely to lead to instabilities and multiple equilibria.

15. Incentives for Adversaries to Fight for κ Or to Give-Up Despite or Because of Their Free Trade

Our construction of the resource acquisition opportunity curve --- $\kappa = \phi[L_G]$, assumes that the sole rationale for fighting to acquire new additional capital is that it increases production possibilities, or net resources, --- labeled Z^N. This seems highly plausible under autarchy when a country is isolated, and can consume only what it produces. But what ought to be the objective when resources are valuable also because the trade they may support, and the imports they permit. Ideally one would like to attach utility levels for A for each resource combination, of Z_A^N and Z_B^N. That is $U_A = V^A(Z_A^N, Z_B)$ and for B $U_B = V^B(Z_B^N, Z_A)$ where functions V^A and V^B also depend, of course, upon the underlying preference structures in A and B. Garfinkel and Syropoulos utilization of indirect utility may finesse this locally, but cannot show the contours of the proper V-objective functions, so the working parts of their model are under the hood, hidden. Which is particularly unwelcome because the most interesting insights require comparisons of discreetly different allocation outcomes, and global welfare differences.

Nevertheless, the course of this paper suggests that for large range parameters in the GS model we can deduce some important implications. As long as one can take intermediate good Z as a stand-in for utility, certain results seem definite. For the purposes of this paper this caveat means, I have not yet seen how to deal with immiserizing preference structures. But dismissing that wrinkle let me return to Figs 5b and 8a and 8b which represent schematically what we know in a general way about benefit sharing under Ricardo, and how shares can depend decisively on relative size. We know that disproportionally large countries benefit not at all from trade, while disproportionately small countries derive the entire benefit of trade. This principal was mentioned above, and illustrated with Figure 8. A loose generalization or extension of this idea is repeated in Fig 15 for one particular configuration of Fig 8c, namely, panel I.

Size of Countries A and B are given by net resources after expenditure for Guns, i.e. Z_A^N for A and Z_B^N for

B. As before, rays labeled $p_W = p_A$ and $p_W = p_B$ divide the space into three regions, viz. (a) where A is large and A's internal price p_A determines world price, p_W ; (b) where B is large and B's internal price p_B determines world price, p_W ; and (c) where A and B are closer to the same size and world price is at an equilibrium depending on demand and supply conditions in both countries, so $p_B > p_W > p_A$. In the border regions where one country or the other dominates, Bhagwati's phenomenon of immiserizing growth (discussed above) is not relevant but in the central pie-slice of the quarter, immiserizing growth is possible, but explicit analysis will be excluded here . Implicit immiserization structures may underlie some of our conclusions, but analysis of this will also be set aside.

Now consider the following "thought experiment." Without trade but after fighting over capital acquisitions A and B find themselves with Z_A^0 and Z_B^0. Plot that point t^0 in Fig 15. Next through point t^0 draw two utility contours for A and B, as derived in Fig 8a and 8b. Fig 8a showed these contours as curved-scalloped lines in fact, but they could be of almost any shape. Fig 15 shows them as straight line-segments. If they trade, from point t^0 we can show their welfare gains, measured in "autarky-equivalent" values of Z. Those become $Z_{\mathbf{A}}^{\tau}$ and $Z_{\mathbf{B}}^{\tau}$, showing that both have gained from trade.

Now, ask from that t^0-equilibrium are further mutual gains possible? Well, movements along the darker, thicker, outer frontier seem to suggest $U_A = V^A(Z_A^N, G_B)$ might increase while $U_B = V^B(Z_B^N, G_A)$ remains constant, or that $U_B = V^B(Z_B^N, G_A)$ that might increase while $U_A = V^A(Z_A^N, G_B)$ remains constant[17]. **Fig 15**

However, either of these shifts would appear to require an increase in the overall total $(Z_A + Z_B)$. So it seems the possibility of Pareto gains merely reflects the sub-optimality of Nash-Cournot competition for $\boldsymbol{\kappa}$. Thus to answer the question we must fall back and consult our earlier analysis concerning the effect of size on a country's choice of Guns vs. the intermediate resource Z. The answer would seem to depend on the shape of $G_A = f(Z_A, G_B)$ and $G_B = g(Z_B, G_A)$ from Fig 13 or similar exercises. But for trade to stimulate a unilateral or a mutual reduction in allocations to guns without collusive or collaborative coordination, revised Nash decisions

[17] In the figure contour $U_{\mathbf{B}}$ is flatter than $U_{\mathbf{A}}$ is, but the statement remains valid if contour $U_{\mathbf{B}}$ is steper than contour $U_{\mathbf{A}}$.

about G_A and/or G_B must be able to move point t^0 to a higher utility curve for both in Fig 15. But for the utility curves of Fig 15 this is not possible[18]. So, we must return to ask if from t^0 by changing Z_A only, and thus by moving t^0 horizontally, can A move to a higher Z_A-curve? I delay my answer to this to section 17.

16. Putting It All Together: Simultaneous Equilibrium in Trade and Conflict.

All the ingredients to show a simultaneous interaction between fighting and trading have now been constructed. Figure 16 assembles them. The reader who has understood each component of the diagram can see in the 3rd quadrant at the intersection of reaction functions the interactions in choices of guns are stable, giving a Nash-Cournot outcome of (G^*_A, G^*_B) and resulting distribution of world production at (Z^*_A, Z^*_B). The 1st quadrant shows utility of this outcome for A and B including the benefits of mutual trade. Note that to enjoy the same utility --- (U^*_A, U^*_B) --- that trade at (Z^*_A, Z^*_B) allows would require at (Z^e_A, Z^e_B) in isolation. Conceivably Z^e_A and Z^e_B are feasible if A and B employ some other decision protocol. However, even if we can assume informally that (G^*_A, G^*_B) has exhaustively distributed a fixed total of κ, as in G-S, then (Z^e_A, Z^e_B) might be feasible under partial disarmament. But with a Nash-Cournot set-up, from (Z^*_A, Z^*_B) any increase in Z_A, requires a reduction in Z_B. I emphasize that while Fig 16 illustrates an overall equilibrium involving both trade and conflict "simultaneously," it does not derive that equilibrium and hence does not show how a perturbation from those equilibrium choices of G_A and G_B produces lower utility levels U_A and U_B and opens a path back to the U^*_A-U^*_B optimum. I leave this for further work.

Fig 16

17. When Will Trade Cause Fighting to Intensify or Diminish? (Provisional/Unfinished)

I believe that the answer to this question can be found using the diagrams provided here, once they are extended to include immiserization. Absent immiserization, national welfare is 1:1 correlated with "Net

[18] But it could be true if A, realizing that a change in G_A will affect not only of Z_A directly but also Z_B directly or indirectly. If A understood this relationship then he would recognize his unilateral power to move point t^0 both north-south as well as east-west in come combination. However, imputing foresight and subtlety like this to our State-Actors exceeds the primitive Nash-Cournot posture we adopted in Section 14 above. So, here I reject the more complex protocol. It could be relaxed though and would add insight to this paper and dramatically change how the various configurations of Fig 8c affect incentives and outcomes.

Resources," Z^N including those captured by fighting net of their cost. This gets around direct measurement of utility or welfare problems. We have also developed a method to compare those resources under isolation and autarky with resources in a trade regime. This was pictured in Figures 8, 14, 15 and 16.

The question can be answered by:

(1) Deriving the equilibrium values of Z^N and G for A and B which would arise in absence of trade,

(2) Identifying in Fig 14 or 8 the utility contours of A and B that said combination produces when A and B are assumed to trade --- given those values of Z^N,

(3) Determine the "autarky equivalent" values of Z that (Z^N + Trade) generates. We have labeled those values Z_τ.

(4) If $Z_\tau < Z^N$ then the outcome of trade is to effectively reduce resources available to A and/or B. In this case the country for which this is true (I conjecture it cannot be true simultaneously for both countries) has an incentive to reduce its expenditure on Z-enhancing Guns. This answer assumes that a country cannot refrain from trade when the opportunity exists. Competitive and decentralized markets not the country as an entirety controlled by its government, make the decision to trade.

(5) Although If $Z_\tau > Z^N$ still, depending on the exact shape and positioning of the utility contours such as $U_A = U_A(Z_A, Z_B)$ when one side can move toward a higher contour by reducing G and therefore reducing Z^N it will have an incentive to do so. (Since Z^N has already been maximized the only possible move is to reduce it.)

Fig 17a and 17b

Figures 17a and 17b illustrate this argument. At this writing I am not sure whether the underlying preference functions must contain the seeds of immiserization but I conjecture that that that must be the case.

18. Idea of "Trade Equivalent" Resources

My question here is how to write the formula for $U_A = U_A(Z_A, Z_B)$ to show A's welfare is a linear (or more complicated than linear) function of both its own (Z_A) and it trading partner's resources (Z_B). The formula for these "Equivalent Resource Contours" would seem to be approximately the three piecewise linear components.

19. Conclusions

20. References

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Hirshleifer, Jack, (1989), "Conflict and Rent-Seeking Success Functions: Ratio vs Difference Models of Relative Success," *Public Choice* 63(2), 101-112.

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Tullock, Gordon,

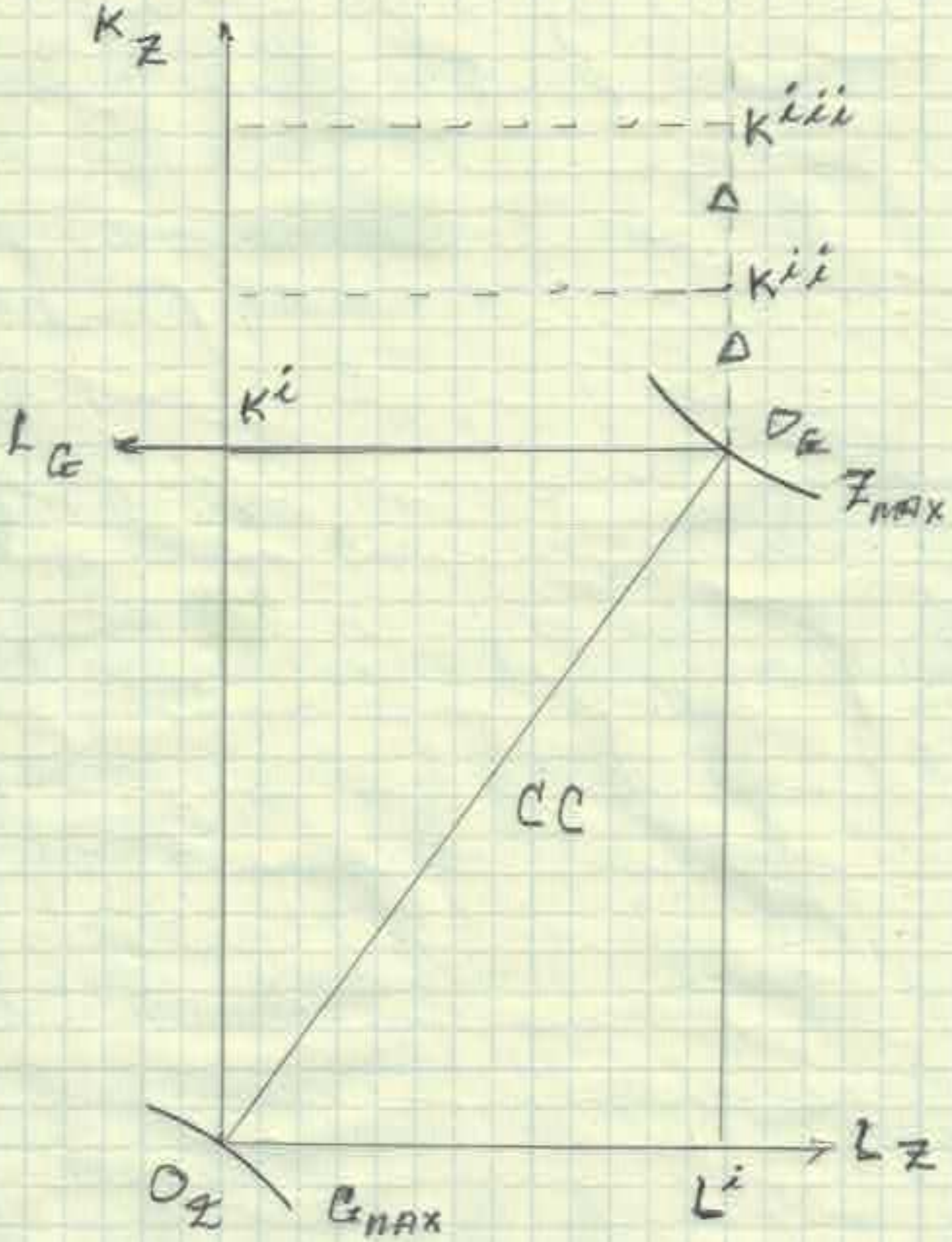


Fig 1a

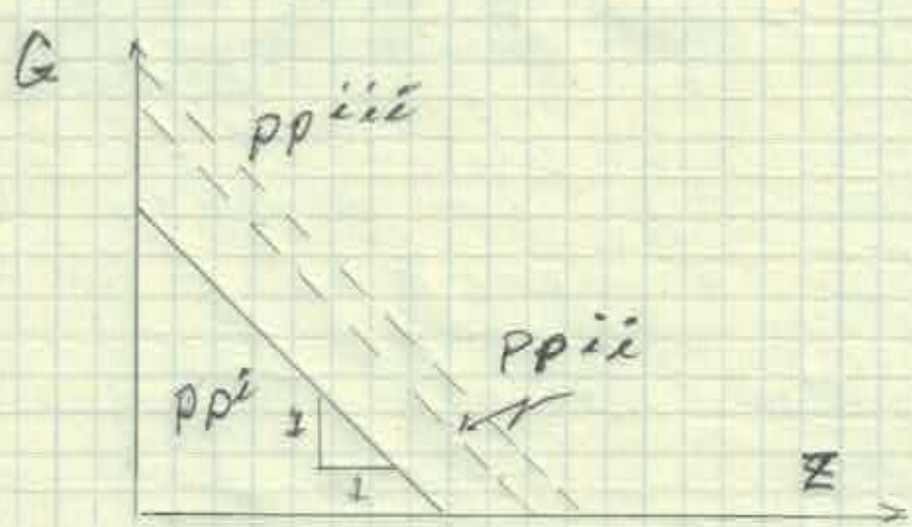


Fig 1b

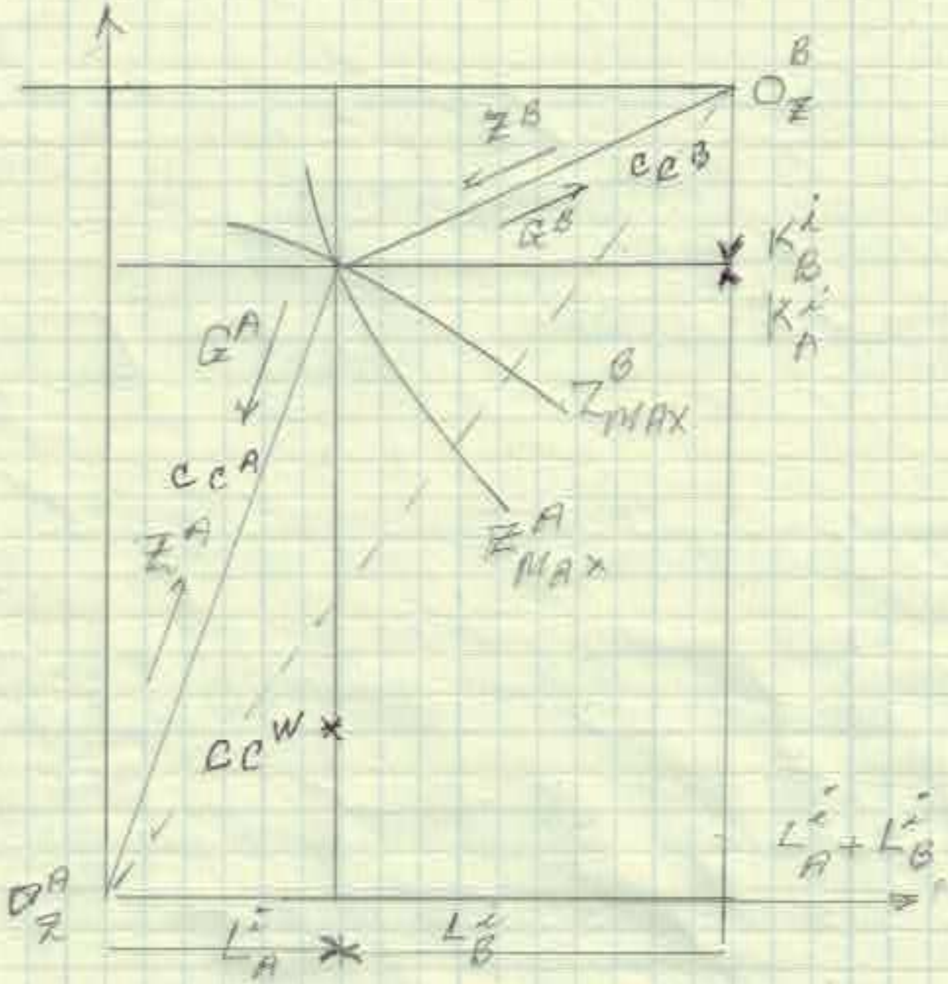

O^B_Z
Z^B
CC^B
G^B
K^i_B, K^i_A
G^A
Z^B_{MAX}
CC^A
Z^A_{MAX}
Z^A
CC^W
$L^i_A + L^i_B$
O^A_Z
L^i_A
L^i_B

Fig 2a

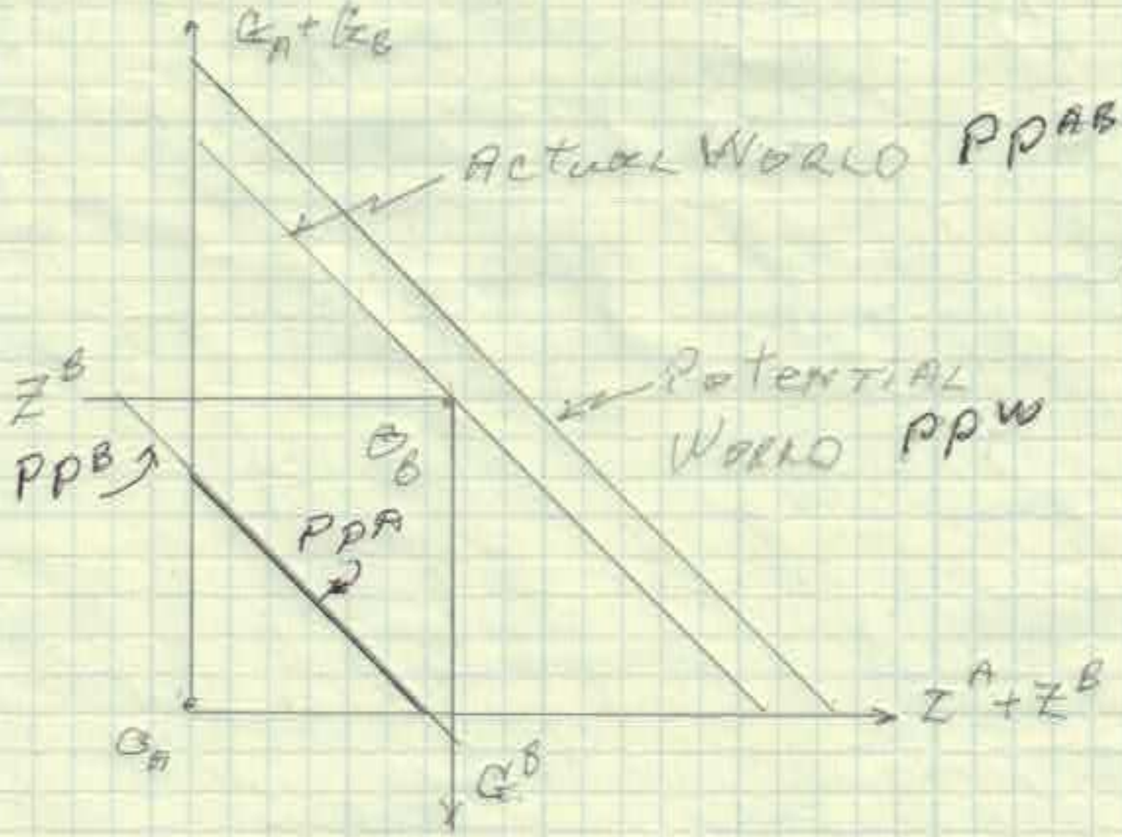

$G_A + G_B$
ACTUAL WORLD PP^{AB}
POTENTIAL WORLD PP^W
Z^B
PP^B
O_B
PP^A
$Z^A + Z^B$
O_A
G^B

Fig 2b

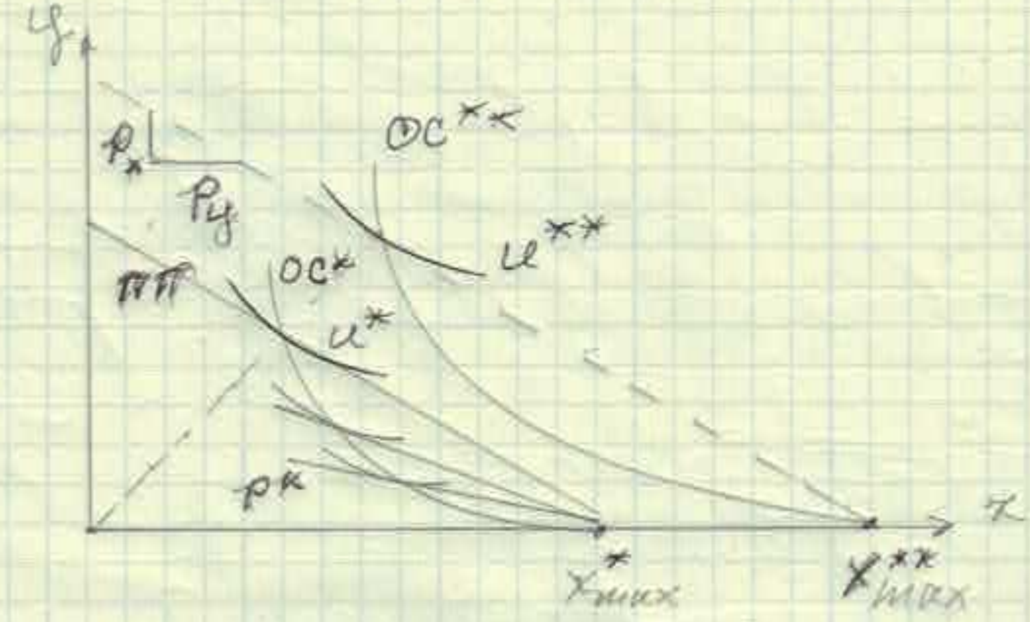

y
p_x
p_y
OC^{**}
u^{**}
$\pi\pi$
OC^*
u^*
p^*
x
x^*_{max}
x^{**}_{max}

Fig 3

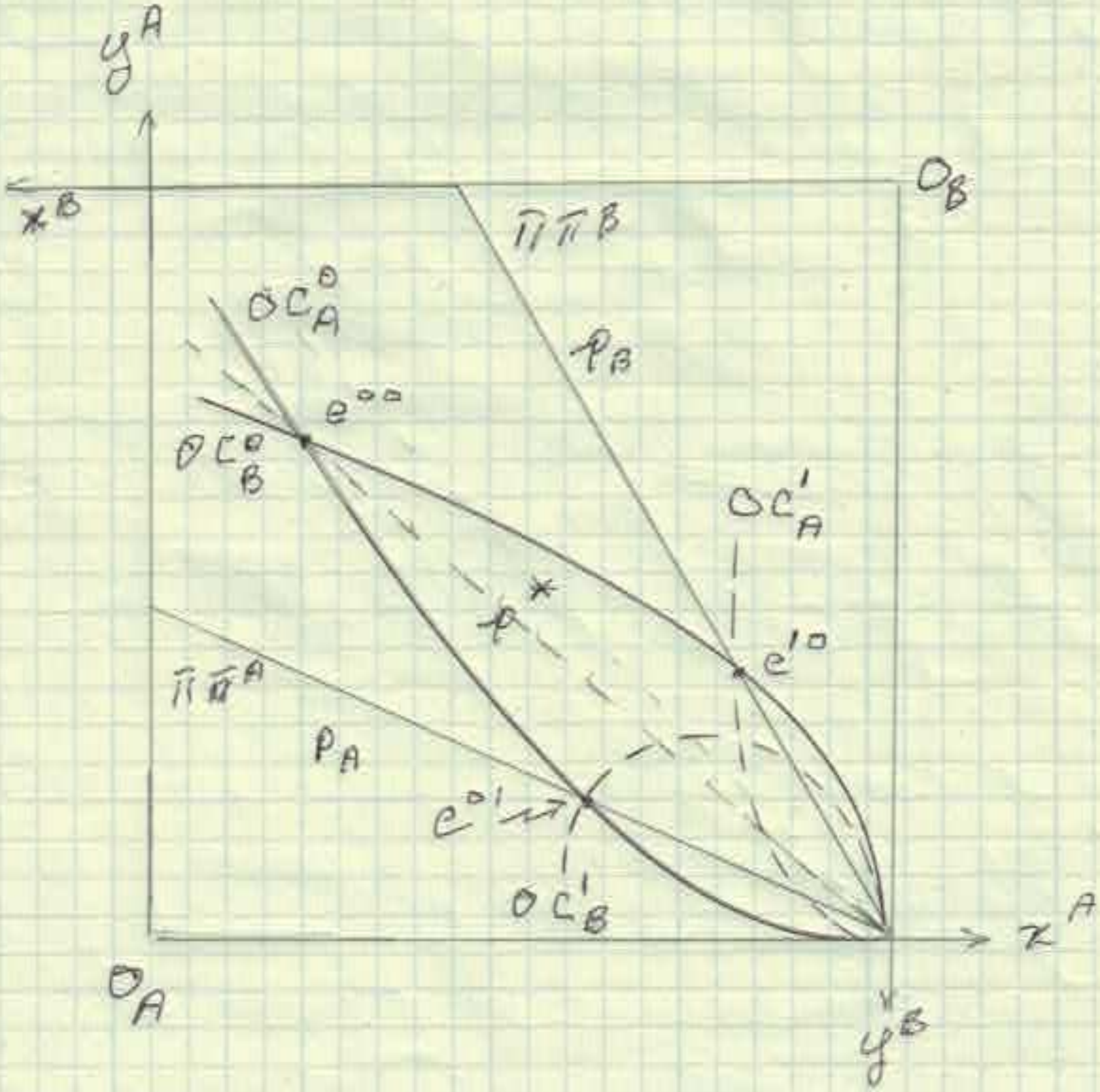

y^A
x^B
O_B
$\pi\pi B$
OC^0_A
p_B
e^{00}
OC^0_B
OC'_A
p^*
e'^0
$\pi\pi^A$
p_A
$e^{0'}$
OC'_B
x^A
O_A
y^B

Fig 4

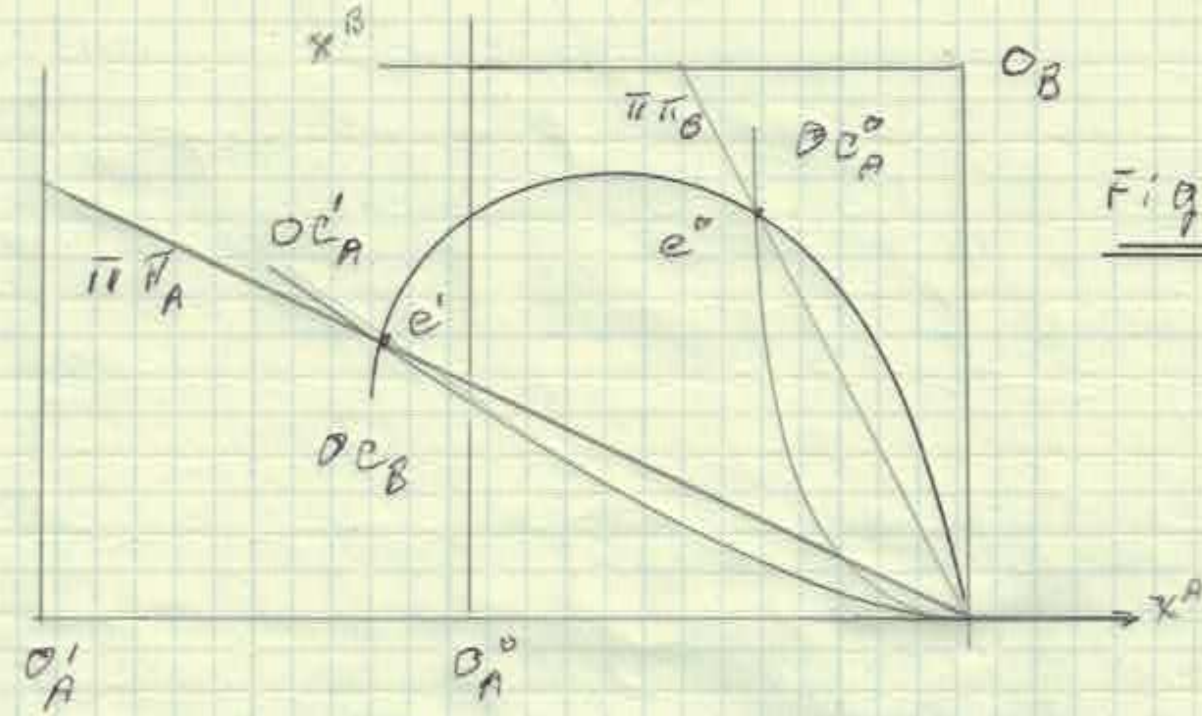

x^B
O_B
$\pi\pi_B$
OC_A^0
e^0
OC_A^1
$\pi\pi_A$
e^1
OC_B
x^A
O_A^1
O_A^0

Fig5a

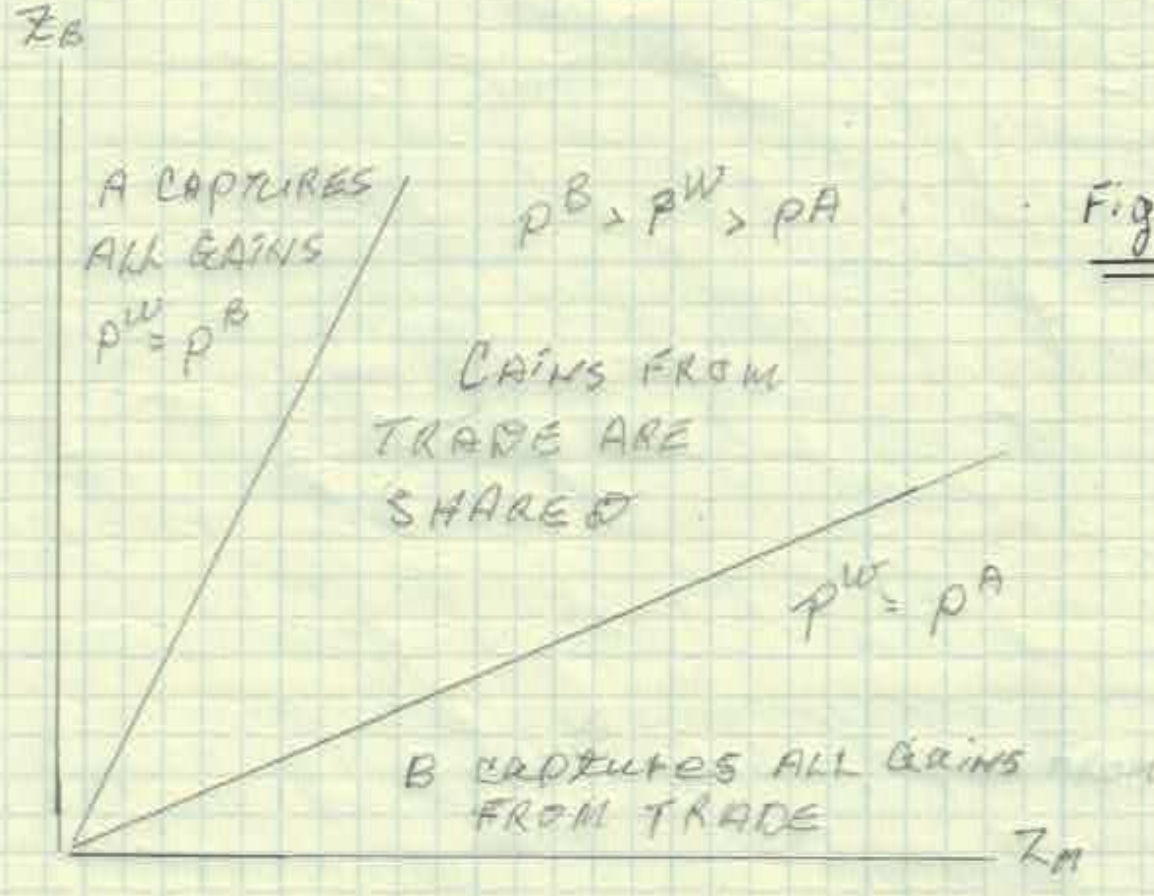

Z_B
A CAPTURES ALL GAINS
$p^W = p^B$
$p^B > p^W > p^A$
GAINS FROM TRADE ARE SHARED
$p^W = p^A$
B CAPTURES ALL GAINS FROM TRADE
Z_M

Fig 5b

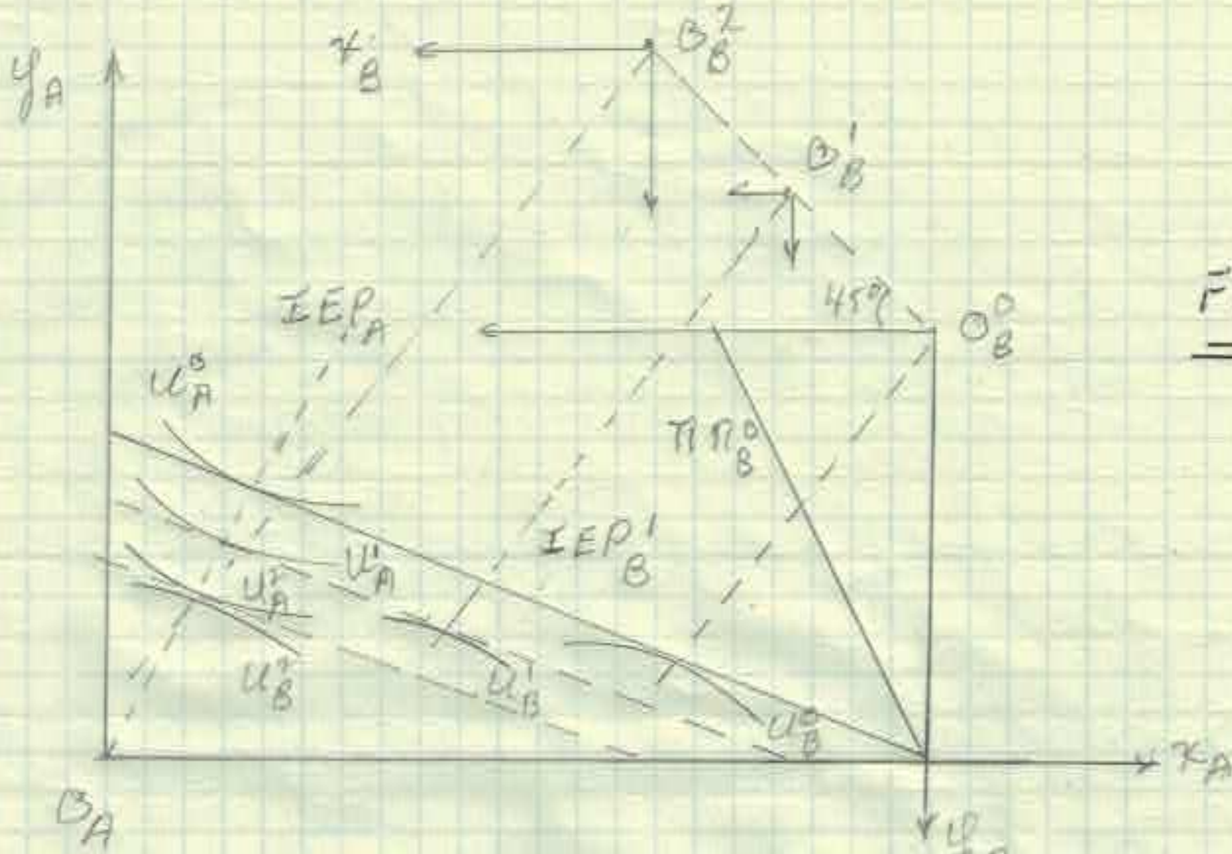

y_A
x_B
O_B^2
O_B^1
45°
O_B^0
IEP_A
u_A^0
$\pi\pi_B^0$
IEP_B^1
u_A^1
u_B^1
u_B^0
O_A
x_A
y_B

Fig 6a

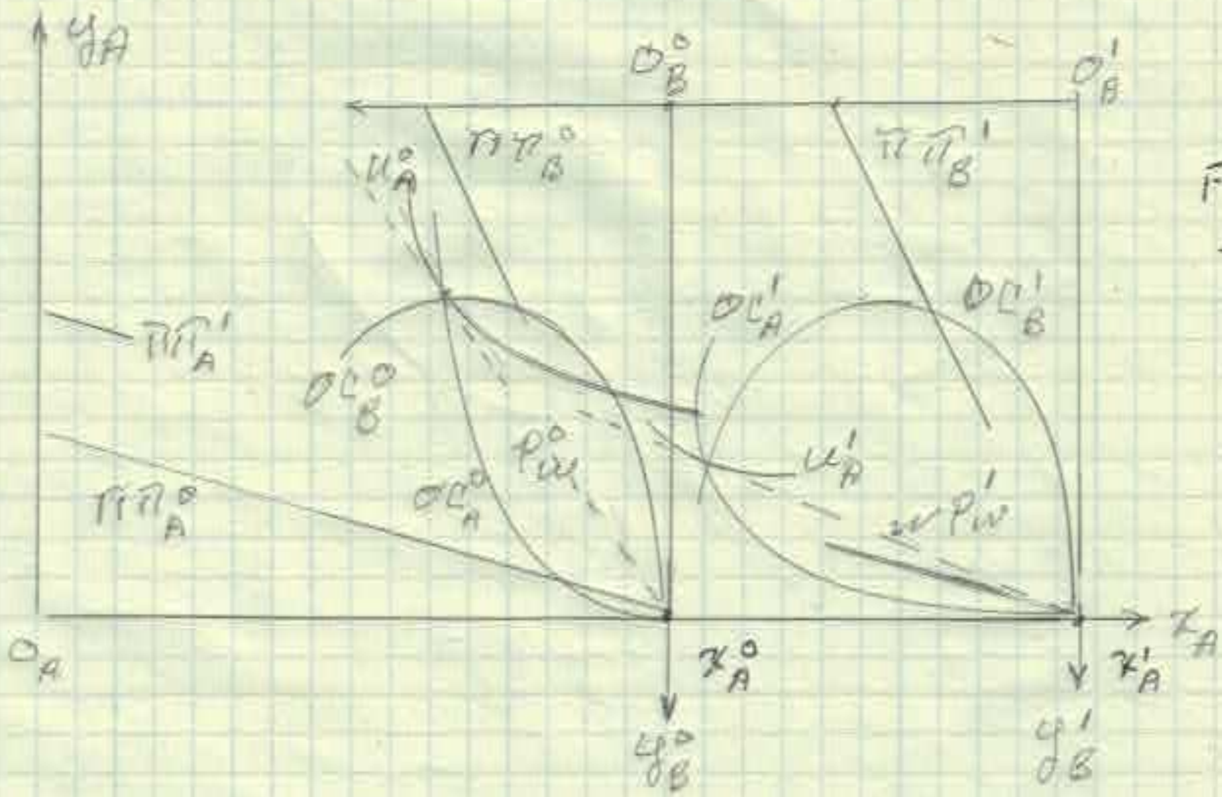
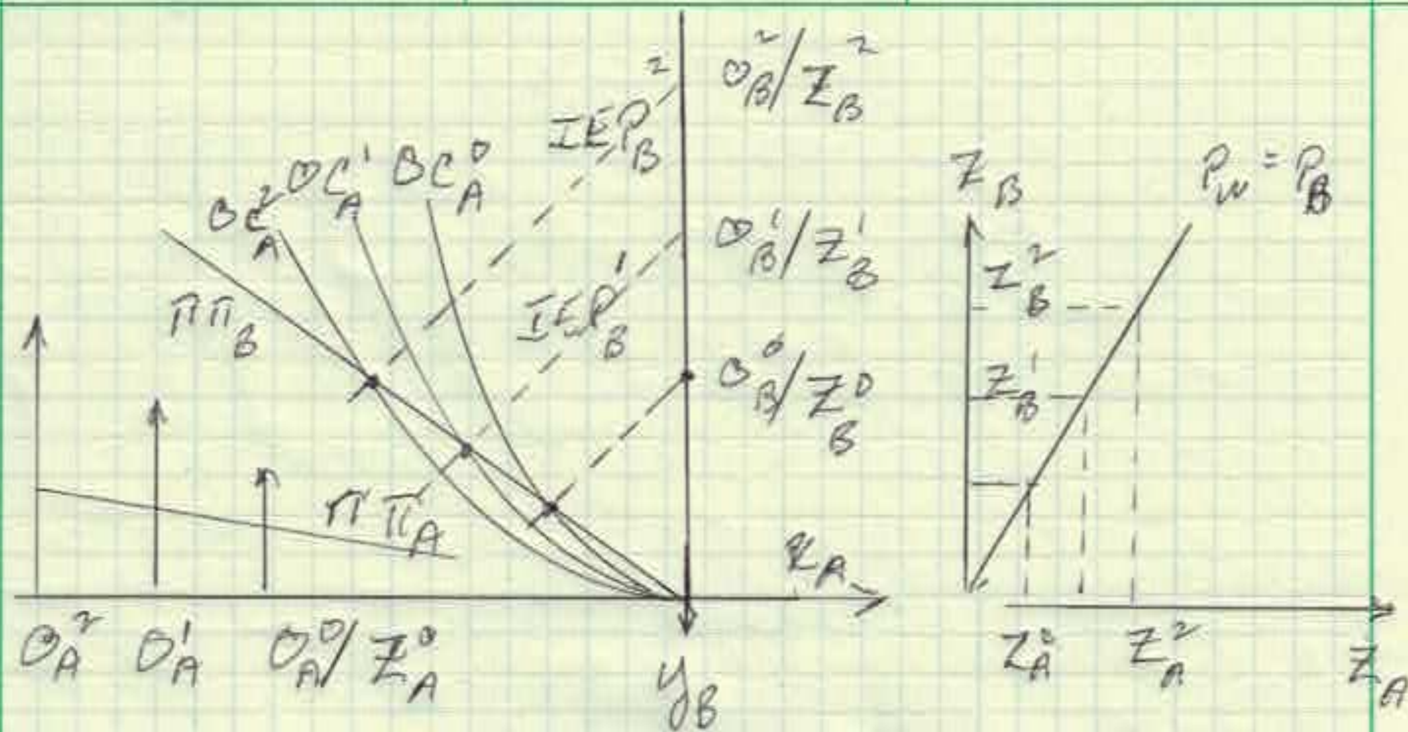
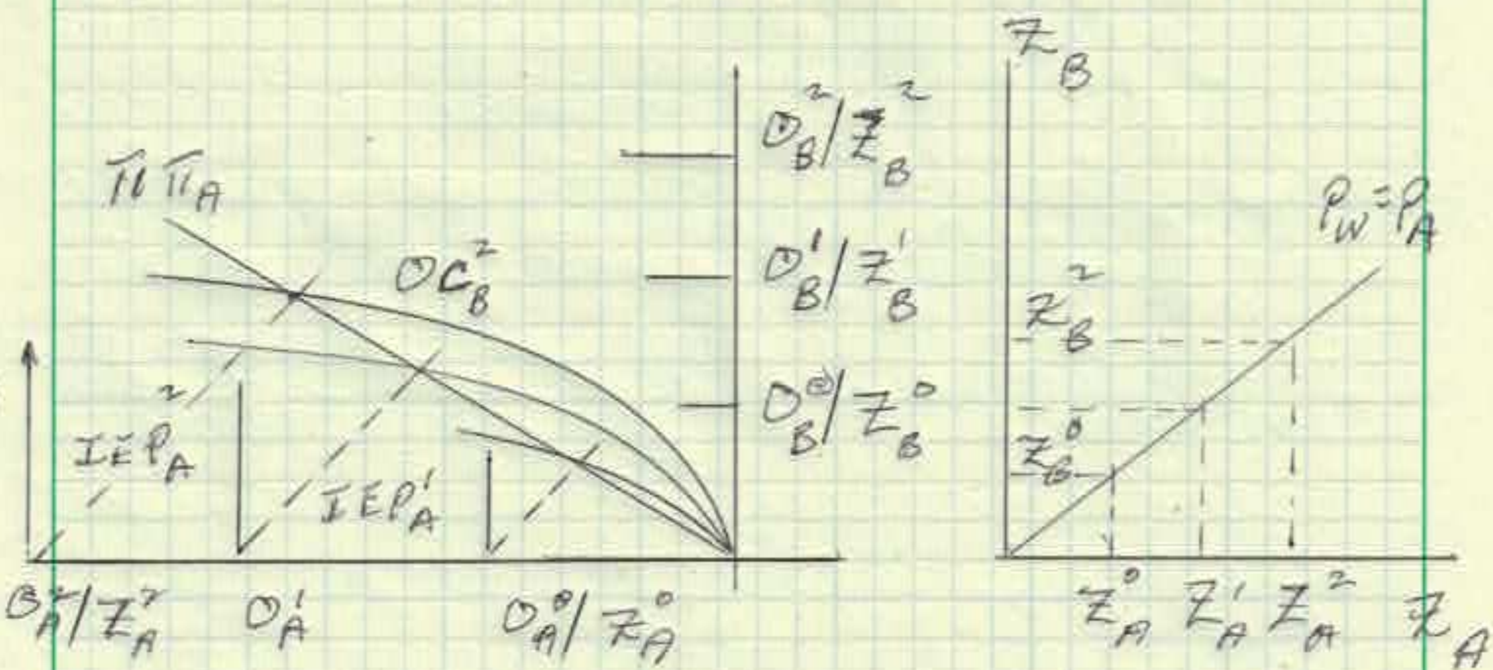

y_A
O_B^0
O_B^1
u_A^0
$\pi\pi_B^0$
$\pi\pi_B^1$
OC_A^1
OC_B^1
$\pi\pi_A^1$
OC_B^0
P_W^0
u_A^1
$\pi\pi_A^0$
OC_A^0
P_W^1
O_A
x_A^0
x_A^1
x_A
y_B^0
y_B^1

Fig 7a



IEP_B^2
O_B^2/Z_B^2
BC_A^2
OC_A^1
BC_A^0
IEP_B^1
O_B^1/Z_B^1
ππ_B
O_B^0/Z_B^0
ππ_A
x_A
O_A^2
O_A^1
O_A^0/Z_A^0
y_B
Z_B
P_W = P_B
Z_B^2
Z_B^1
Z_A^0
Z_A^2
Z_A

Fig 6b



O_B^2/Z_B^2
ππ_A
O_B^1/Z_B^1
OC_B^2
Z_A
O_B^0/Z_B^0
IEP_A^2
IEP_A^1
O_A^2/Z_A^2
O_A^1
O_A^0/Z_A^0
Z_B
P_W = P_A
Z_B^2
Z_B^0
Z_A^0
Z_A^1
Z_A^2
Z_A

Fig 6c

Fig 7b

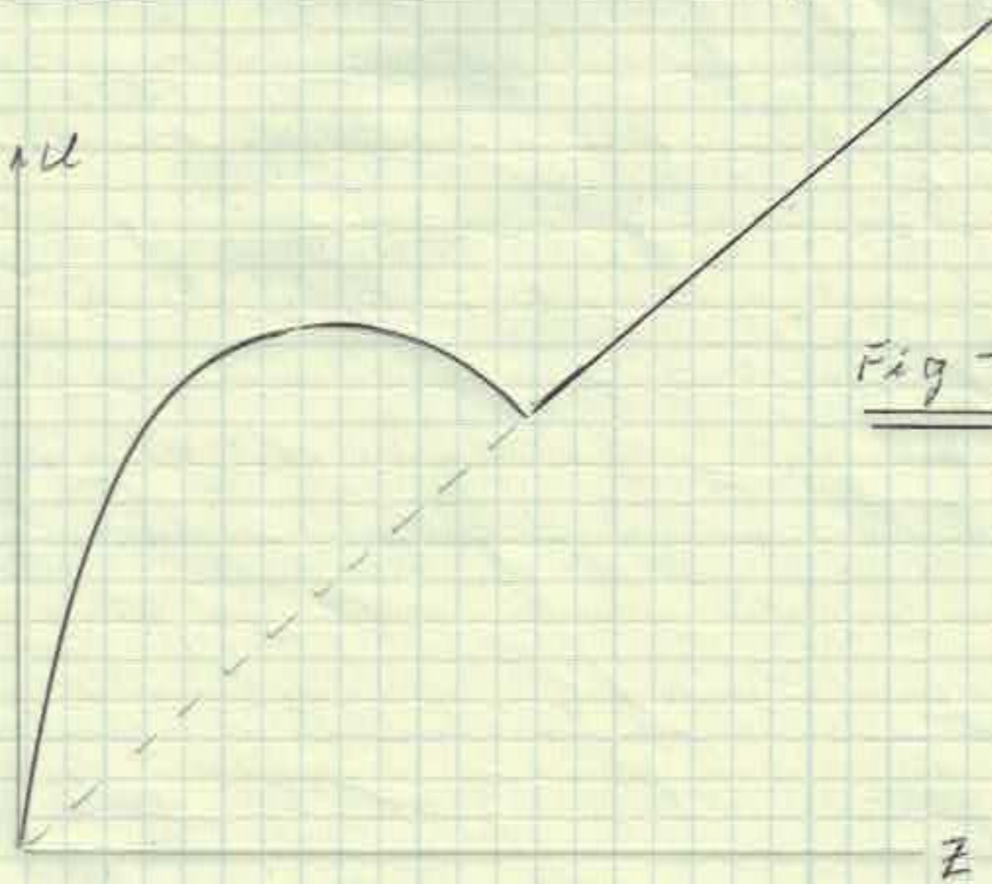


Fig 7c

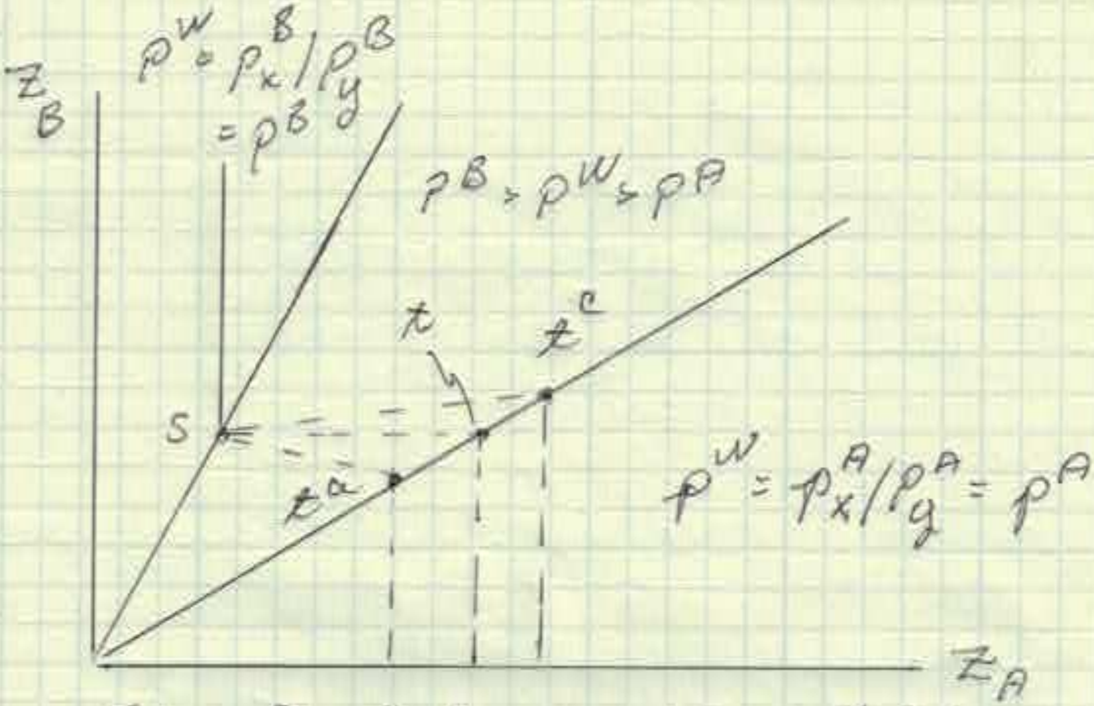
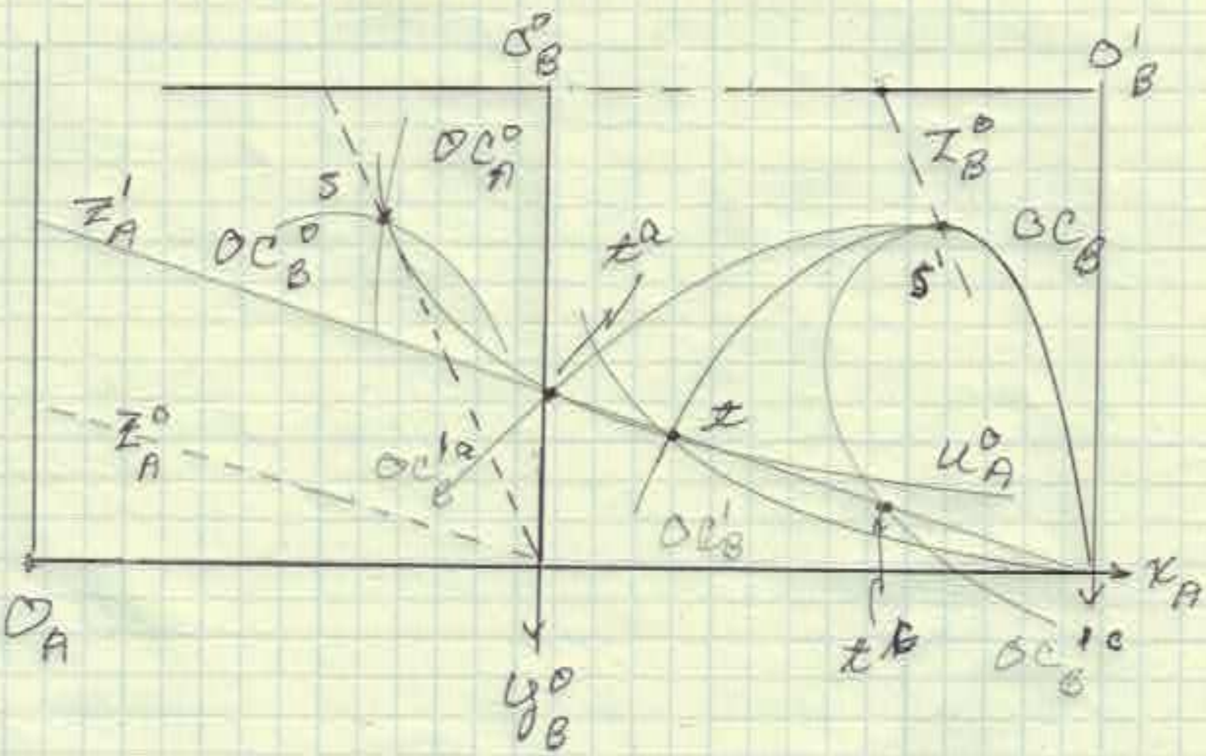


Fig 8a: ALTERNATE UTILITY CONTOURS for A

Fig 8b



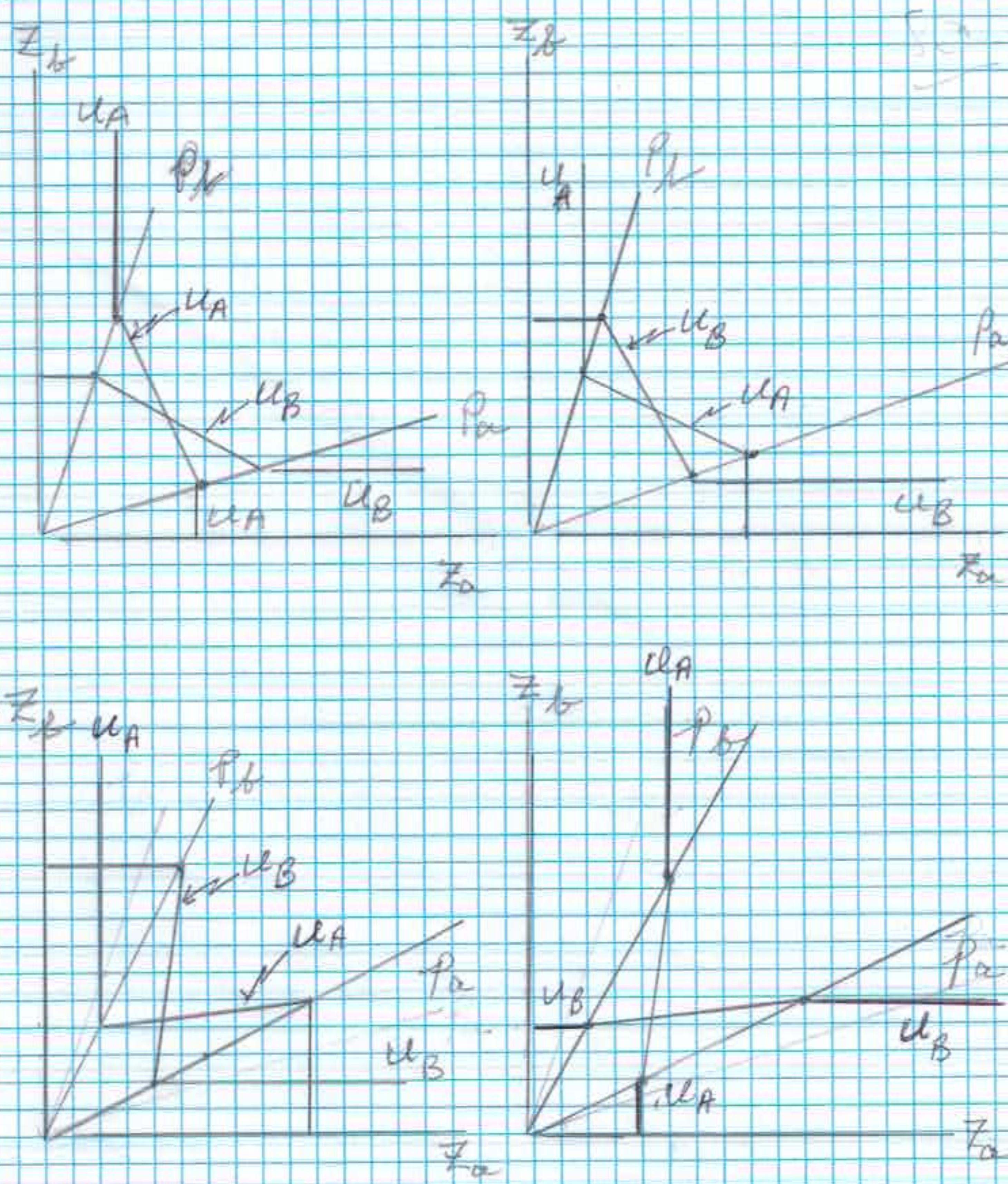

Zb
uA
Pb
uA
uB
Pa
uA
uB
Za
Zb
uA
Pb
uB
Pa
uA
uB
Za
Zb
uA
Pb
uB
uA
Pa
uB
Za
Zb
uA
Pb
uB
Pa
uB
uA
Za

FIG 8C

Fig 9a

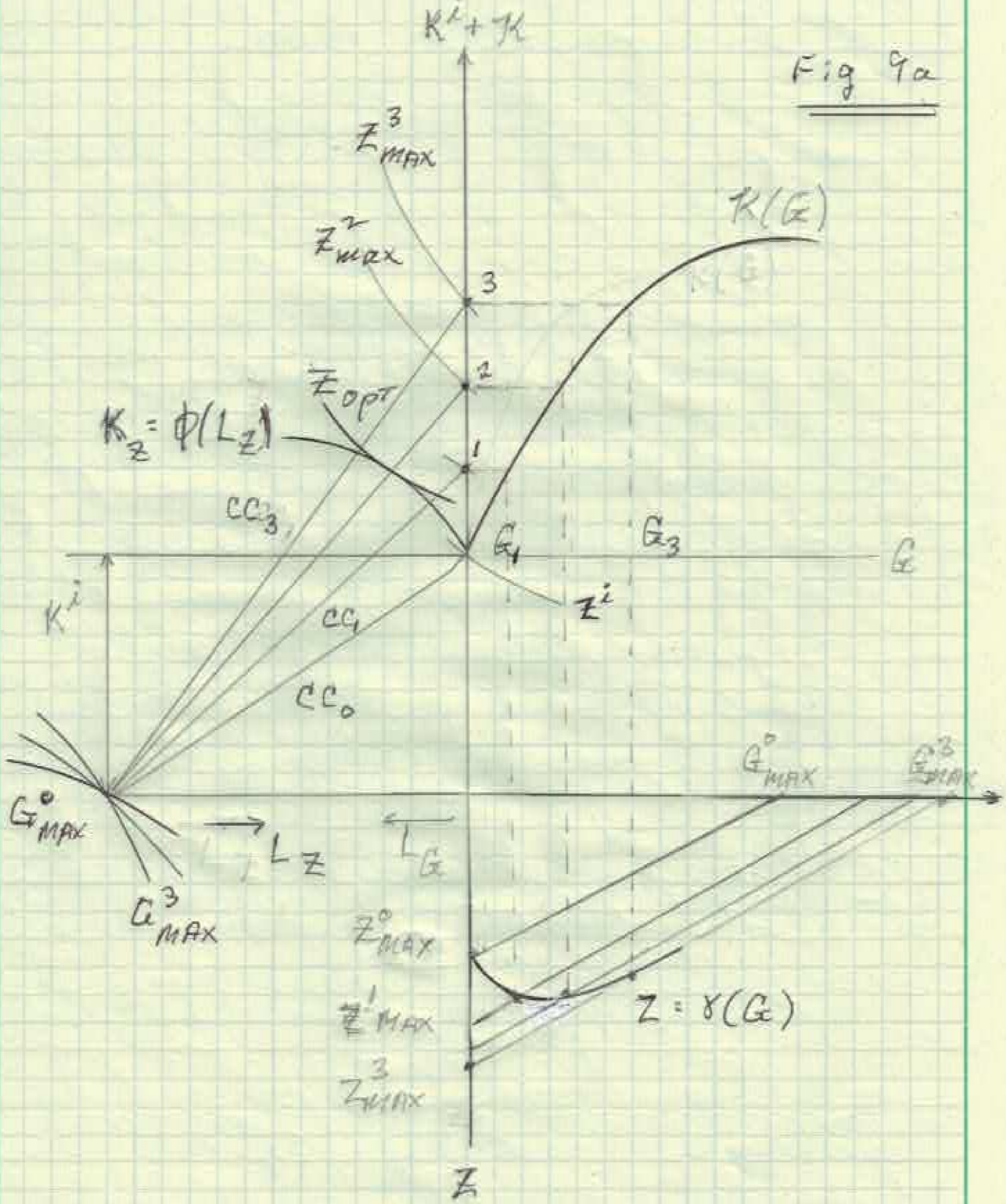
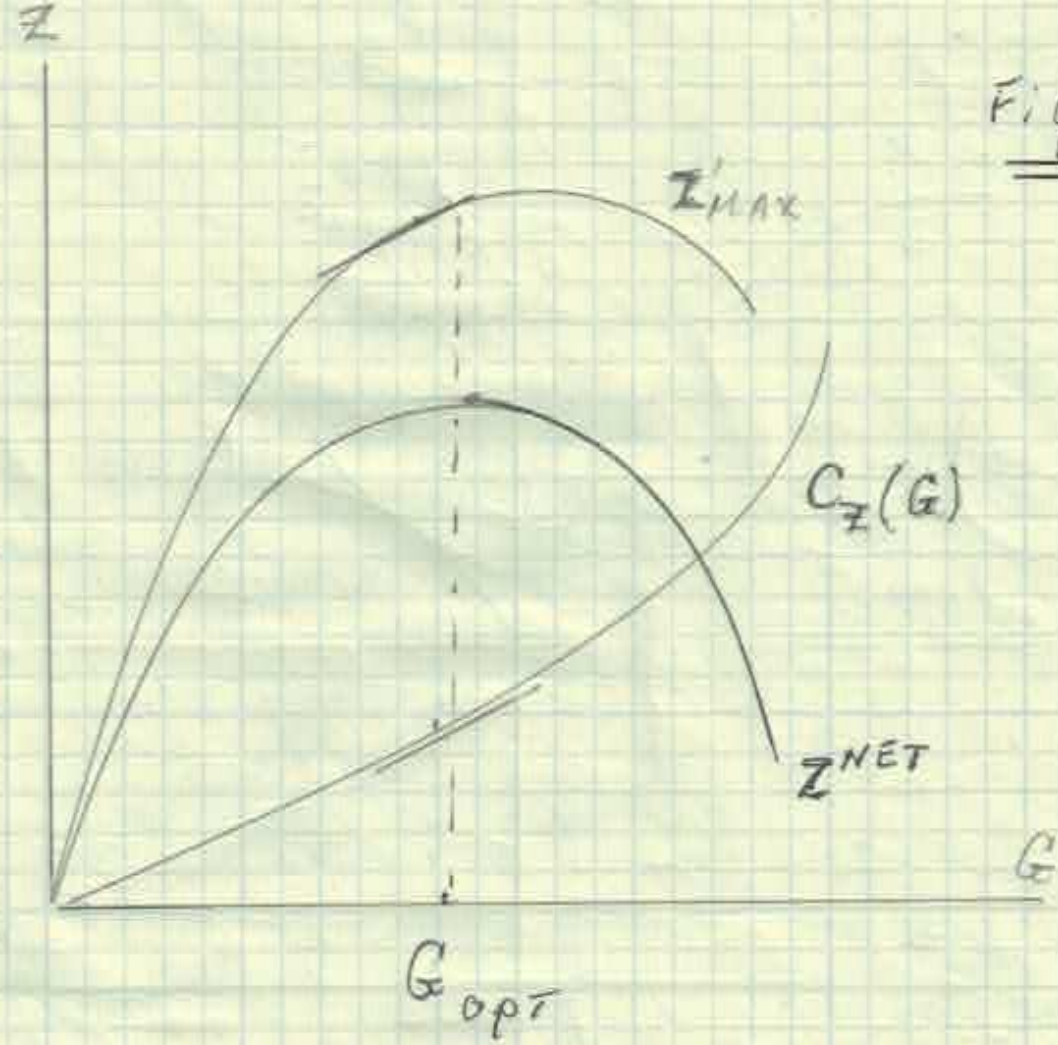

K^i + K
Z^3 MAX
Z^2 MAX
K(G)
3
2
Z OPT
K_z = φ(L_z)
1
CC_3
G_1
G_3
G
K^i
Z^i
CC_1
CC_0
G^0 MAX
G^3 MAX
L_z
L_G
Z^0 MAX
Z^1 MAX
Z = γ(G)
Z^3 MAX
Z

Fig 9b



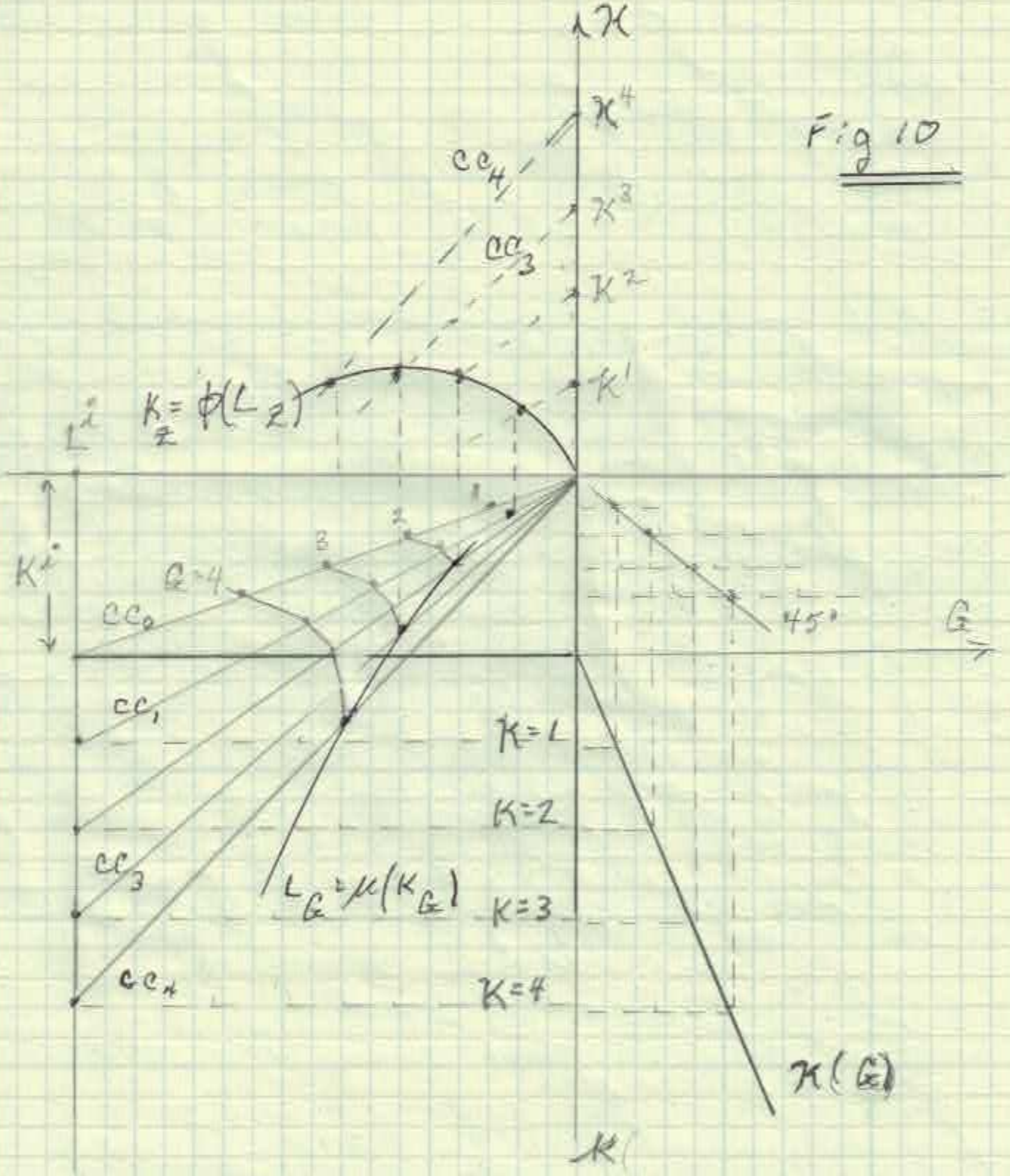
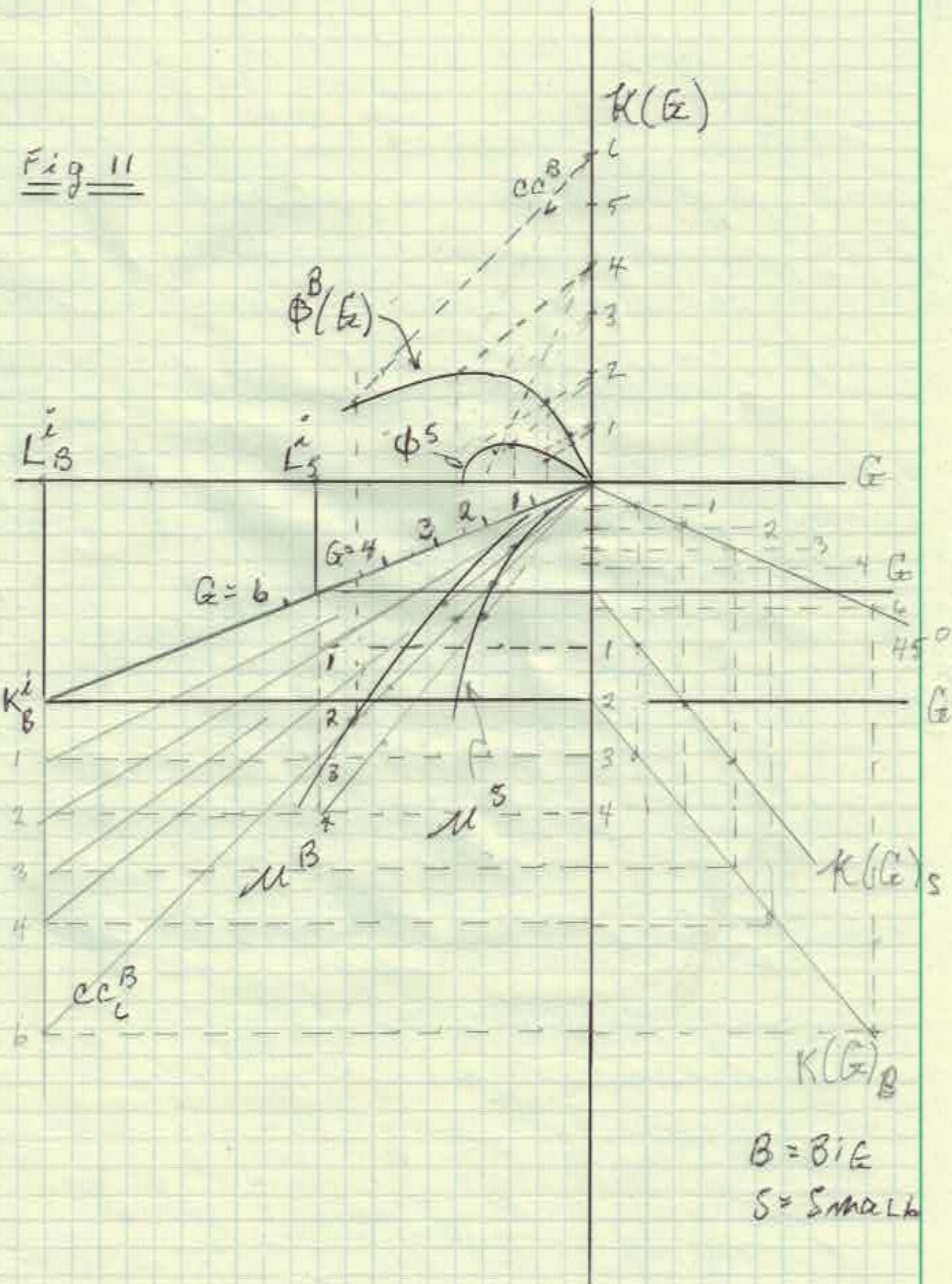


Fig 10
$\mathcal{K}$
$\mathcal{K}^4$
$\mathcal{K}^3$
$\mathcal{K}^2$
$\mathcal{K}^1$
cc_4
cc_3
$K_z = \phi(L_z)$
L^i
K^i
G=4
3
2
1
cc_0
cc_1
cc_3
cc_4
45°
G
K=1
K=2
K=3
K=4
$L_G = \mu(K_G)$
$\mathcal{K}(G)$
K

Fig 11



B = BiG
S = Small

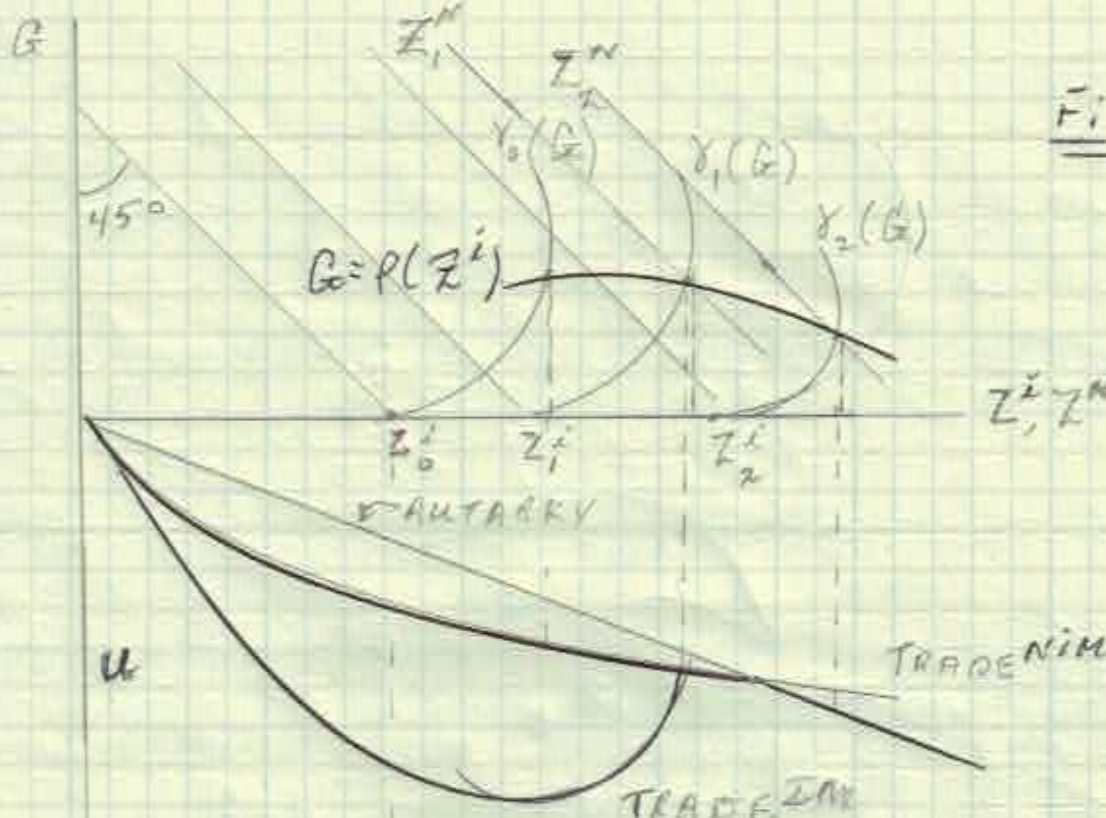


Fig 12a

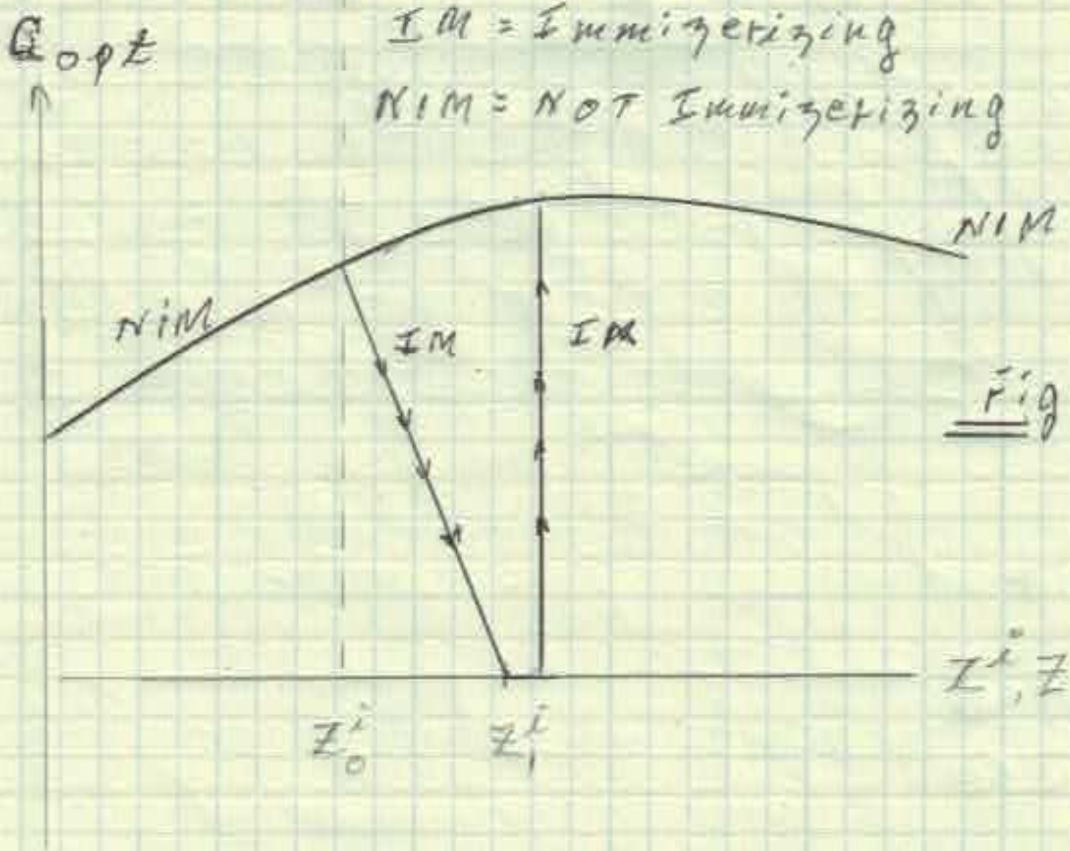


Fig 12b

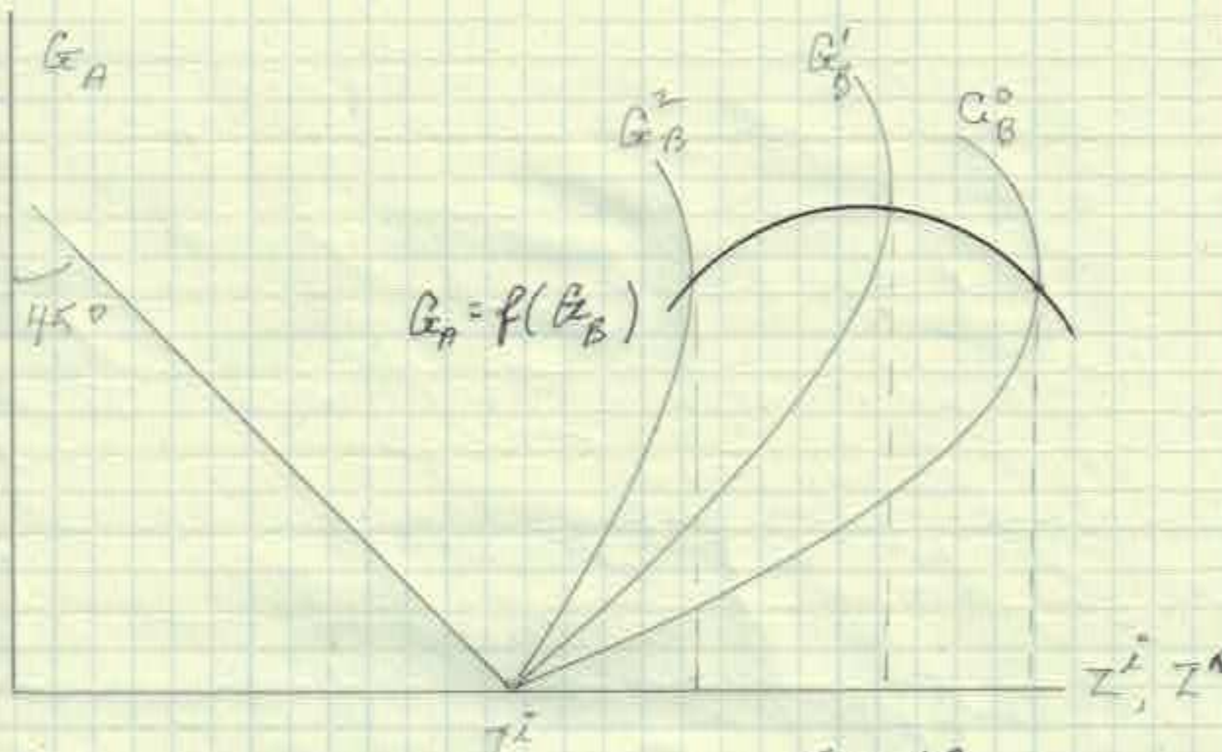


Fig 13

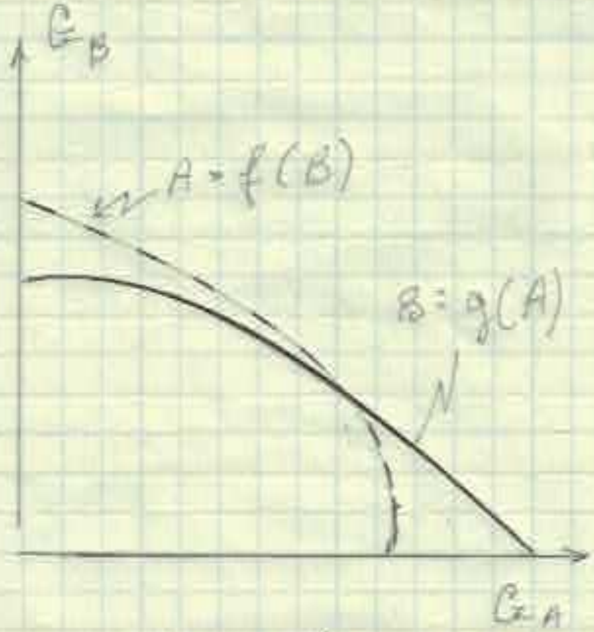


Fig 14a

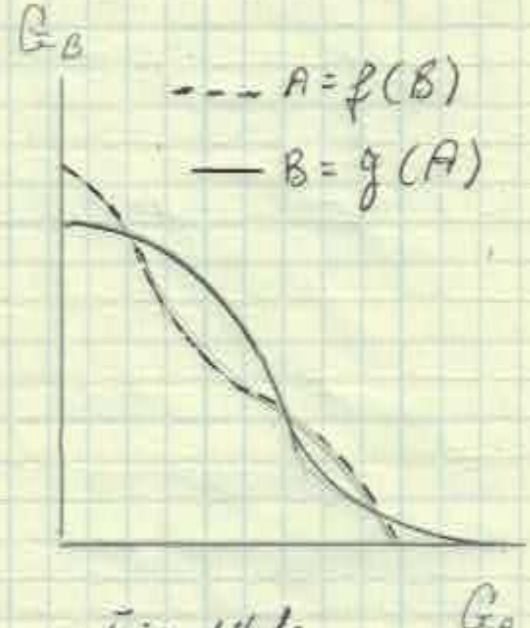


Fig 14b

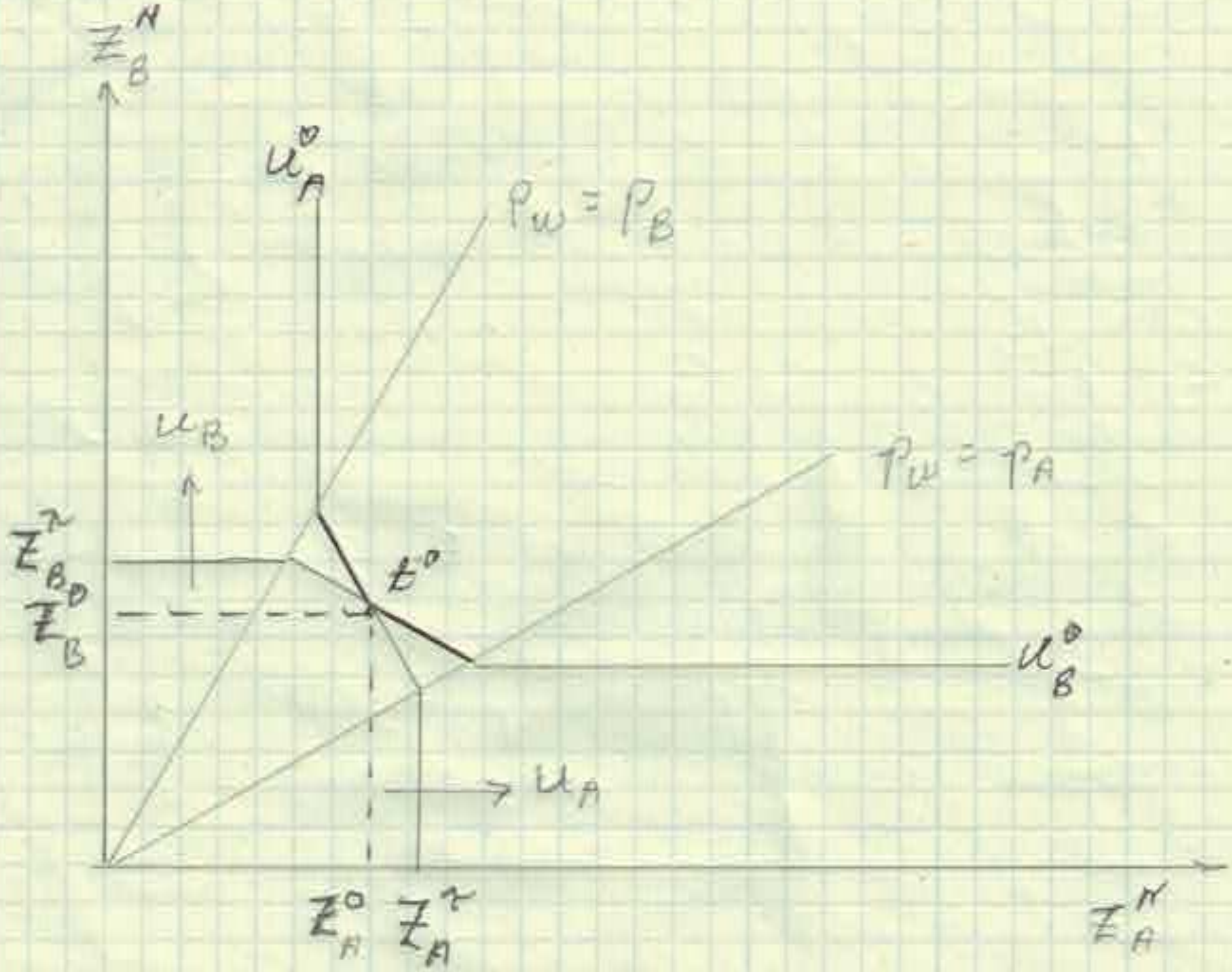
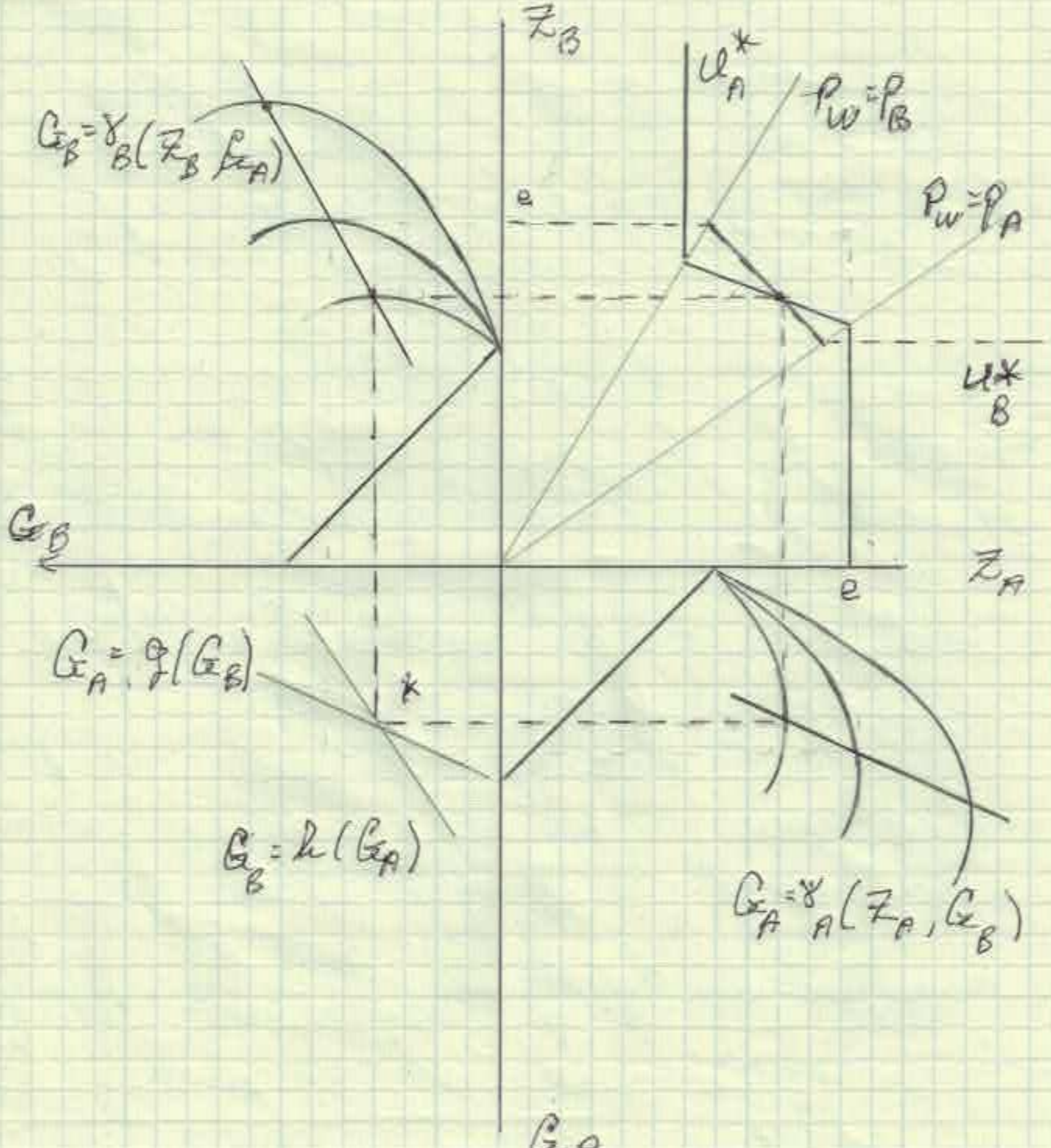


FIG 15



Z_B
U_A^*
$P_W = P_B$
$G_B = \gamma_B(Z_B, G_A)$
e
$P_W = P_A$
U_B^*
G_B
Z_A
e
$G_A = g(G_B)$
*
$G_B = h(G_A)$
$G_A = \gamma_A(Z_A, G_B)$
G_A

FIG 16

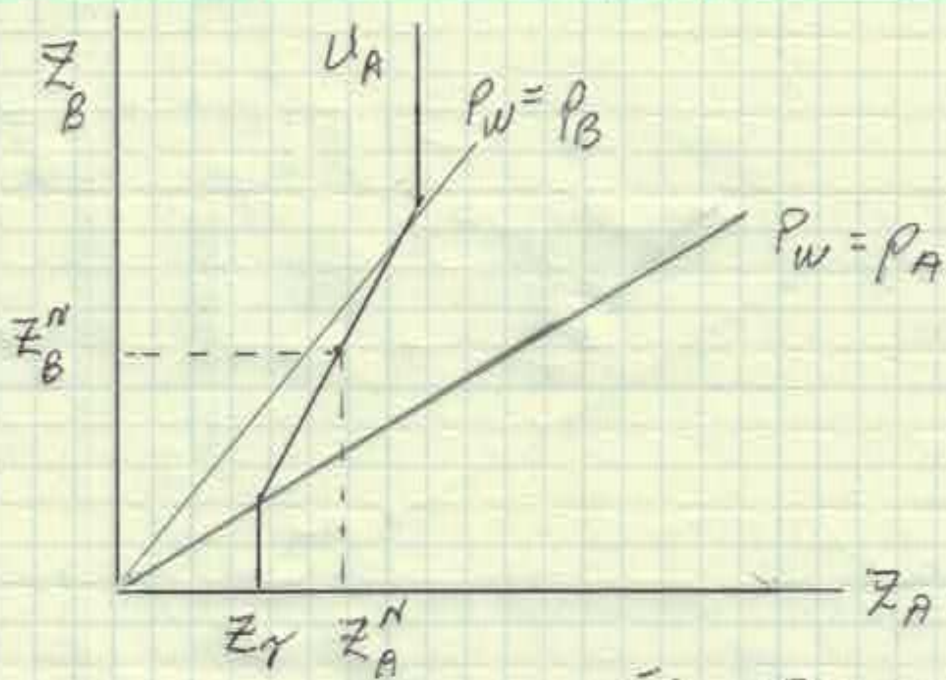


Fig 17a

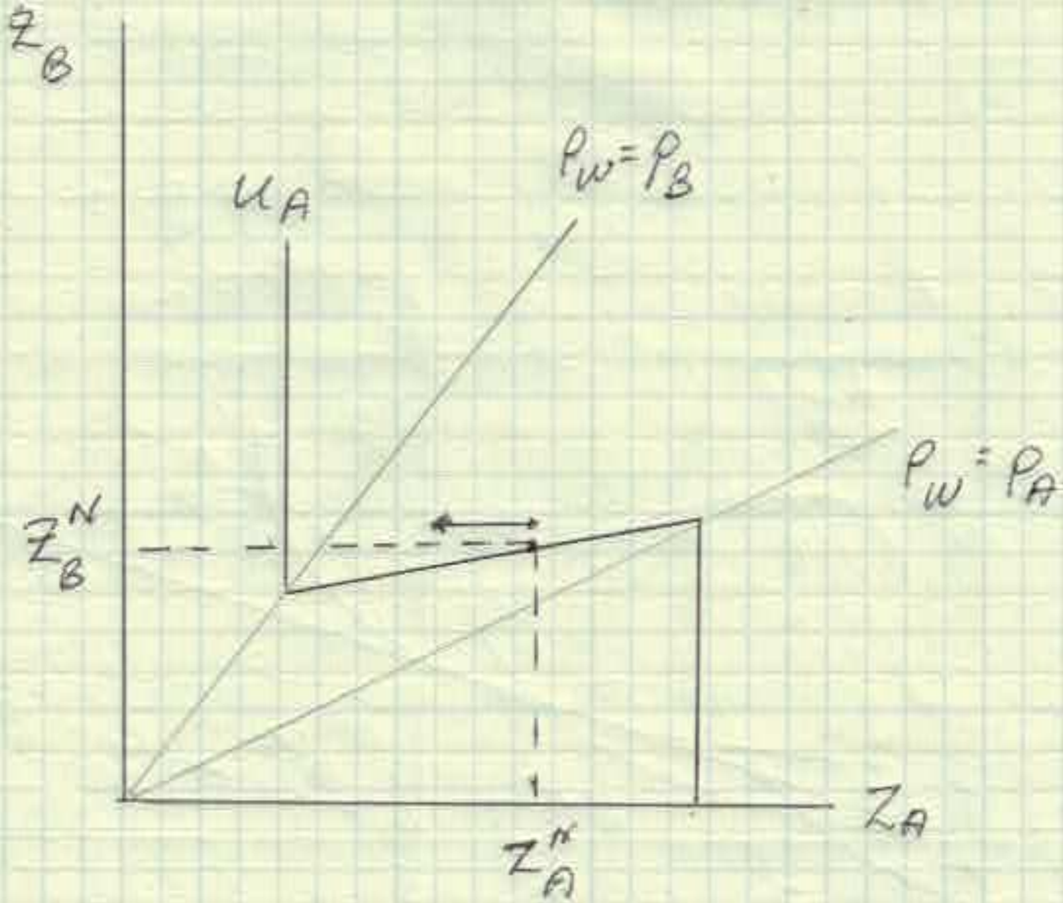


Fig 17b