Trading with the Enemy[†]

Michelle R. Garfinkel[‡]
University of California, Irvine

Constantinos Syropoulos[§]
Drexel University

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Abstract: We analyze how trade openness matters for interstate conflict over productive resources. Our analysis features a terms-of-trade channel that makes security policies trade-regime dependent. Specifically, trade between adversarial countries reduces their incentives to arm given the opponent's arming. If they have a sufficiently similar mix of initial resource endowments, a move to trade brings with it a reduction in resources diverted to conflict and thus wasted, as well as the familiar gains from trade. Otherwise, a move to trade can induce greater arming by one of them and thus need not be welfare improving for both. Moreover, when the two adversarial countries do not trade with each other but instead trade with a third (friendly) country, a move from autarky to trade intensifies conflict between the two adversaries. Building on the welfare implications, we also analyze the endogenous choice of trade regimes.

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[‡]Email: mrgarfin@uci.edu
[§]Email: c.syropoulos@drexel.edu

1 Introduction

International trade takes place within an anarchic setting. Absent an ultimate adjudicator and enforcer, countries inevitably have unresolved disputes and nearly all expend resources on defense to prepare for the possibility of outright conflict or to improve their bargaining positions under the threat of conflict. Despite such anarchy prevailing in international relations, the classical liberal perspective views greater trade openness among potential adversaries as reducing or even eliminating conflict (e.g., Polachek, 1980). One argument in support of this perspective is that conflict precludes the realization of at least some of the gains from trade; then, countries acting collectively and wanting to reap those gains would have a greater interest under trade in maintaining a peaceful order and avoiding war. However, it is well-known—with the prisoners' dilemma being a stark and simple example—that in many economic, social and political interactions, collective rationality need not and often does not lead to a stable, equilibrium outcome. Instead, interactions guided by individual rationality more often lead to "bad" outcomes. Indeed, the realist/neo-realist perspective, emphasizing self-interest and individual rationality as well as the anarchic nature of international relations, argues contrary to the classical liberal view that trade can aggravate conflict between nations (e.g., Waltz, 1979). Specifically, the benefits from freer trade can fuel frictions and conflict, as some states perceive that they (or their rivals) will develop a military edge in security competition.[1]

Our central objective in this paper is to identify and explore some key mechanisms at play in determining how the expansion of international trade affects the intensity of conflict. The analysis is based on a variant of the Ricardian model of trade, suitably extended to allow for international disputes. As in the canonical Ricardian model, there are two large countries that possess different (but constant returns to scale) technologies for producing two consumption goods from a single input. The difference in technologies generates comparative advantage that provides the rationale for mutually advantageous trade, even if one country is more efficient in producing both goods. However, in contrast to the standard Ricardian model, ours supposes that consumption goods are produced with an intermediate input, whose level depends on two primary factor inputs. Moreover, at least some portion of one of these factors is contestable. It could be oil, minerals, timber, land, or water resources.[2] The division of the contested resource depends on the countries'

[1]Pushing this logic one step further, one could argue that actual or potential rivals would not trade with each other (e.g., Grieco, 1990; Gowa, 1995). See Copeland (2015) for a recent survey of the theoretical and empirical literature in international relations regarding trade and conflict.

[2]Although we have seen a sharp drop in interstate wars since WWII, interstate conflict over resources remains relevant in international affairs. One ongoing dispute involves the countries surrounding the Caspian Sea (Russia, Kazakhstan, Turkmenistan, Iran, and Azerbaijan) over how to divide the rights of exploration and exploitation for oil. Another dispute involves China, Taiwan, Vietnam, the Philippines, Indonesia, Malaysia, and Brunei for control over the Spratly and Paracel islands in the South China Sea, where there

"arms" or "guns," which are also produced domestically. Since arming is endogenous, so too are the resources available to produce consumption goods. As arming rises, reflecting more intense conflict, production of consumption goods falls and hence security costs rise.

A key feature of our analysis, distinguishing it from earlier treatments of conflict and trade in the economics literature, is the endogeneity of world prices.[3] While not all countries are sufficiently large to influence world prices and thus the terms of trade, those considered "super-powers" in geopolitics are. Even smaller countries have market power in some industries (such as oil, gas and minerals), and accordingly can influence the terms of trade that, in turn, influence market access by other countries to imports of such goods (Bagwell and Staiger, 2001).[4] If trade policies can influence world prices as suggested by the empirical work of Broda, et al. (2008) and Bagwell and Staiger (2011), world prices likely depend on residual resources and thus on arming as well. It stands to reason, then, that policymakers take that influence into account when choosing their security policies.

To highlight this mechanism, we employ a simple model that abstracts from many salient features of today's world economy, such as the presence of increasing returns, heterogeneous firms, foreign investment, and endogenous growth. In addition, the model does not differentiate between the mobilization of resources for conflict and the potentially destructive deployment of those resources, nor does it consider explicitly the disruptive effect of conflict to shutdown trade between warring nations.[5] Instead, motivated by the empirical relevance of military expenditures, we focus on arming.[6] Insofar as resources are absorbed into the

are suspected oil reserves. Caselli et al. (2015) provide an interesting analysis that examines the importance of the proximity of oil fields to countries' shared border in the escalation of conflict between them. Also, see Klare (2012), who provides many examples where the competition for scarce resources (including but not limited to oil), for which property rights are not well defined or enforceable, has turned or can turn violent.

[3]The few papers that do exist is this literature consider small-country settings where world prices are treated as exogenously determined (e.g., Skaperdas and Syropoulos, 2001; Garfinkel et al., 2015). One notable exception is Skaperdas and Syropoulos (2002), who study the interactions between countries in the context of an exchange model. That analysis differs from the present one in that it focuses primarily on how the anticipation of exchange affects arming incentives via its impact on the division of resources based on Nash bargaining.

[4]To gauge the importance of such influence, consider, for example, the United States' concern about having access to oil in world markets, China's and Japan's concerns about having access to primary materials, and the overall concern in Europe about having access to natural gas from Russia.

[5]This is not to deny the importance of conflict's disruptive effect. To the contrary, this effect is empirically relevant and can be viewed as an opportunity cost of conflict that serves as the basis for classical liberal view as described above. While some have found that conflict has little to no significant effect on trade (e.g., Barbieri and Levy, 1999), Glick and Taylor (2010) find compelling evidence of a negative and significant effect that persists over time. In related research, we explore the adversarial countries' arming choices and their subsequent decision to either fight that would preclude trade between them or negotiate a peaceful settlement (Garfinkel and Syropoulos, 2017).

[6]SIPRI researchers estimate that, in 2016, global military expenditures were $1,686 billion, accounting for 2.2 percent of global GDP. The five biggest spenders that year (in current U.S. dollar terms) were the United States ($611 billion or 3.3 percent of GDP), China ($215 billion or 1.9 percent), Russia ($69.2 billion or 5.3 percent), Saudi Arabia ($63.7 billion or 10 percent), and India ($55.9 billion or 2.5 percent). See Tian et al. (2017) for more details.

production of arms and thus are unavailable for the production of goods traded in world markets, these expenditures represent an important additional opportunity cost of conflict. Given our focus, this paper can be thought of as offering an analysis of a modified version of the classical liberal view, one that applies to cold wars.[7]

Within the setting of our model, we compare the outcomes under two polar trade regimes, autarky and free trade. As one would expect, given the amount of resources allocated to arming, a shift from autarky to free trade unambiguously results in higher payoffs to both countries. However, such a switch also influences arming incentives.[8]

Under autarky, where countries can consume only what they produce, each one chooses its arming (or its security policy) so as to equate the marginal benefit of capturing the contested resource to the marginal cost of diverting resources from the country's own production and thus consumption. At the same time, each country's arming choice adversely affects the opponent by reducing its access to the contested resource. In equilibrium, where both countries ignore this negative security externality, arming is strictly positive.

Importantly, trade induces each country to internalize, at least partially, the negative externality of its security policy on the resources available to its rival. The result is to lower arming incentives given the rival's policy. To be more precise, as in the case of autarky, when the two countries trade with each other, each one chooses its arming to balance its marginal benefit with its marginal cost. In the case of trade, however, each country's payoff depends on the production of its adversary's exportable, which the adversary produces relatively more efficiently. Accordingly, an increase in one's own arms has an additional cost under trade: a reduction in the adversary's share of the contested resource and thus a reduction in the adversary's production of its exportable. This added cost, which is reflected in a deterioration of the importing country's terms of trade that lowers the marginal benefit of arming relative to the marginal cost, means that a country's incentive to arm, given the adversary's arming choice, is strictly lower under free trade than under autarky.

To be sure, a shift from autarky to free trade on *equilibrium* arming and welfare depends not only on this direct effect that can be viewed as an inward shift in each country's best-

[7] Acemoglu and Yared (2010) find empirically that military expenditures and the size of the military in terms of personnel are negatively related to trade volumes. Although that analysis treats the military variables (reflecting the "nationalist" or "militarist" sentiments) as exogenous, the negative relationship found can be viewed as preliminary evidence in support of the modified version of the classical liberal view we consider here, focusing on military expenditures. More recently, Seitz et al. (2015) link empirically trade, wars and military spending from 1993 through 2001, a period when inter-state wars were quite rare. They find that trade brings added benefits largely through its effect to dampen defense spending, not only by trading partners, but other countries as well. These added benefits are, according to their estimates, comparable to the direct welfare benefits of greater trade openness.

[8] Our welfare analysis and comparison of trade regimes resemble those in Arkolakis et al. (2012) and Costinot and Rodríguez-Clare (2014). The primary difference is that in our work real income is endogenously determined under non-cooperative interactions in arming. Nonetheless, as in those analyses, the elasticity of substitution in consumption plays a prominent role.

response function, but also on the indirect or strategic effect as the rival country's arming falls. However, we find that the strategic effect is, by and large, of second-order importance. That is to say, provided the distribution of the resource that is partly contested (what we call "capital") and the uncontested resource (what we call "labor") across the adversarial countries is not severely uneven, the strategic effect could reinforce the direct effect or, even if it moves in the opposite direction, is swamped by the direct effect. In such cases, since trade is no worse than autarky for a given level of arming, the reduction in arming due to trade renders trade unambiguously superior to autarky. And, the difference in payoffs exceeds what is predicted by traditional trade theory that admits no possibility of conflict at all. In particular, the reduction of arming relative to autarky is an added benefit of trade that is consistent with the spirit of classical liberalism and the writings of authors such as Angell (1933) who extolled the virtues of trade openness and globalization.[9]

Nevertheless, there do exist sufficiently asymmetric distributions of labor and capital resources such that one country—specifically, the country whose capital-to-labor ratio is sharply larger to imply a sufficiently higher opportunity cost of arming under autarky than its rival—is induced to increase its arms as its opponent reduces its arming in response to a shift from autarky to free trade. This possibility is consistent with the empirical finding of Morelli and Sonno (2017) that asymmetries in oil resource endowments across two countries reduce the pacifying effect of bilateral dependence between them in trade.[10] What is more, even when the country that is induced to arm by more remains less powerful under trade, the resulting adverse strategic effect realized by the rival could swamp its gains from trade to render trade unappealing. This finding is reminiscent of the view expressed by realists/neorealists in the international relations literature that highlights trade's effect to generate uneven gains to trading partners and thereby differentially influence their arming and thus the balance of power. But, in our analysis, the differential influence of trade on the two countries' arming choices and the implied influence on the balance of power hinges on sharp differences in the mix of their initial holdings of secure capital and labor, not simply on differences in the size of their economies.[11]

[9] Of course, Angell wrote the first edition of his book just before the outbreak of World War I, an event that can be considered a significant counterexample to the optimism of classical liberalism. However, see Gartzke and Lupu (2012), who in examining the interplay of the networks of economic interdependence and military alliances in the period leading up to World War I offer a more favorable interpretation of that major conflict in relation to the liberal theory.

[10] The result is also consistent with Beviá and Corchón's (2010) theoretical finding that sharp asymmetries in endowments can reduce the effectiveness of resource transfers between countries to avoid war.

[11] Also see Bonfatti and O'Rourke (2017), who consider the role of increasing trade dependence for two adversarial countries with the rest of the world, in a dynamic, leader-follower setting, to influence the likelihood of a preemptive war by the follower. In that analysis, similar to ours, understanding the emergence of conflict between the two countries does not hinge on differences in the size of their economies. But, in Bonfatti and O'Rourke's analysis, one fundamental sort of asymmetry is critical—that is, the leader's ability to block imports (necessary for arming) to the rival.

Our finding that the effect of trade openness to pacify international relations may be limited also complements the more recent theoretical work in the economics literature based on extensions of Heckscher-Ohlin models that emphasize differences in factor endowments instead of differences in technologies. In particular, Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015) study settings with two small countries that do not trade with each other; yet, they possibly trade with the rest of the world. A shift from autarky to trade in such settings changes product prices and thus relative factor prices, thereby altering the cost of combining resources in the production of arms.[12] They find that, depending on world prices, trade can intensify competition for resources used to produce tradables to such an extent that the added arming and the associated conflict costs outweigh the gains from trade and thus render autarky preferable to free trade.

Another possible limit to the notion that the expansion of trade can induce lower equilibrium arming follows from a modified version of our setting, consisting of three countries: two adversarial countries as before and a third, friendly country. Suppose, in particular, that the two adversarial countries are identical in every respect so that, even in the absence of barriers to trade, they would not trade with each other. Also, suppose that there exist technological differences between these two countries on the one hand and the third country on the other hand that, given the resources allocated to arming, make trade mutually advantageous. In this case where the two adversarial countries compete in the market for the same good exported to a third country, there is an added marginal benefit to arming under trade—namely, a positive terms-of-trade effect—implying increased arming incentives under trade relative to those under autarky. That expanded trade opportunities with another (friendly) country can intensify conflict between two adversaries is similar in spirit to Martin et al. (2008), who show in the context of a "new" trade model with product differentiation that increasing the opportunities for trade among all countries reduces the interdependency between any two and thus can make conflict between them more likely. However, while that analysis emphasizes the importance of the disruptive effects of conflict, ours highlights the importance of the endogeneity of arming and its trade-regime dependence. Furthermore, our analysis, like Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015), suggests that, in this case, trade between friends could make the adversaries worse off.[13]

[12] We abstract from this mechanism in the current model, to isolate the importance of the endogeneity of the countries' terms of trade for arming incentives, with trade patterns being determined by technology as emphasized in Eaton and Kortum (2002). Of course, in more general (and large-country) settings, both mechanisms could be present.

[13] It is important to note, though, that the result in this paper is due to the beneficial effect that a country's arming has on its own terms of trade to add to arming incentives, whereas the result in Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015) derives from the impact of world prices to increase arming incentives through factor prices. A similar result has been found in settings where there is conflict over some resource between groups within a single (small) country that trades with the rest of the world (see Garfinkel et al. (2008) and references cited therein).

Our analysis also has implications for the endogenous determination of trade regimes. To examine these implications, we extend the model to allow the contending countries to choose non-cooperatively and simultaneously autarky or free trade, before they choose security policies. As expected in the two-country setting, when a shift to free trade induces lower equilibrium arming by both countries, free trade is a subgame perfect equilibrium of the extended game. We cannot, however, rule out the possibility of autarky as another equilibrium outcome. In the three-country case, free trade will emerge as the unique subgame perfect equilibrium, provided that the added security costs are lower than the traditional gains from trade for the two contending countries with the third, friendly country. By contrast, if the added security costs exceed those gains, then the subgame perfect equilibrium is necessarily asymmetric. One contending country trades freely with the third, friendly country, whereas the rival chooses autarky.[14] Thus, similar to the findings of Matsuyama (2002), the possibility of trade with a third, friendly country is "symmetry-breaking." Although the adversaries arm identically, the one that engages in trade enjoys a higher payoff.

In what follows, the next section presents the basic model, focusing on just two countries that trade with one another. Section 3 derives the countries' payoff functions under the regimes of autarky and trade, showing that, for given arms, autarky is weakly dominated by trade. In Section 4, we characterize the countries' incentives to arm under each trade regime. Section 5 studies how equilibrium arming and payoffs compare across trade regimes, identifying conditions under which increased trade openness induces lower arming by both countries and those under which it can induce greater arming by one of them to possibly render trade unappealing to the other country. We extend the framework by introducing a third (non-adversarial) country, in Section 6, to explore the importance of the nature of the trading relationship between the contending nations in determining how increased trade openness can influence equilibrium arming and payoffs. Section 7 examines the countries' non-cooperative choice of trade regimes in both the two- and three-country cases. Concluding remarks follow in Section 8. All technical and supplementary details are relegated to appendices.

2 Contesting a Resource in a Ricardian Setting

Consider a world with two countries, indexed by a superscript $i = 1, 2$. Each country i holds secure endowments of two productive resources: labor denoted by L^i and capital denoted by K^i. Capital could be land, oil, minerals, timber or water resources. In contrast to standard

[14] Since the two adversaries are identical ex ante, which one trades freely and which one remains in autarky cannot be determined. The asymmetric nature of this equilibrium contrasts sharply with Garfinkel et al. (2015), who find in a different setting, where two small and identical adversaries potentially trade with the rest of the world, that the equilibrium is symmetric. For all world prices that ensure the Pareto dominance of autarky, free trade is a possible subgame perfect equilibrium; and, for a subset of those prices, it is the unique equilibrium.

trade models, we suppose that there is an additional amount of capital, denoted by K_0, that is insecure and contestable by the two countries.

Each country i has the capacity to produce two goods for final consumption, X^i_j ($j = 1, 2$), and "guns" (or arms), G^i.[15] Guns should be viewed as a composite good reflecting the country's military strength that is deployed in an effort to take control of the insecure resource K_0. Specifically, the share of K_0 captured by country i is given by

$$\phi^i = \phi(G^i, G^j) = \frac{f(G^i)}{f(G^i) + f(G^j)}, \quad i \neq j = 1, 2, \tag{1}$$

where $f(\cdot) > 0$, $f(0)$ is arbitrarily close to 0, $f'(\cdot) > 0$, and $f''(\cdot) \leq 0$. This specification of the conflict technology, also known as the "contest success function," is symmetric so that $G^1 = G^2 \geq 0$ implies $\phi^1 = \phi^2 = \frac{1}{2}$; it also implies ϕ^i is increasing in country i's own guns ($\phi^i_{G^i} > 0$) and decreasing in the guns of its adversary ($\phi^i_{G^j} < 0$, $j \neq i$).[16] The influence of guns on the division of K_0 between the two countries can be thought of as the result of either open conflict (without destruction) or a bargaining process with the countries' relative military strength playing a prominent role.[17]

Guns are produced with *secure* labor and/or capital endowments. Let w^i and r^i be the competitive rewards paid to labor and capital in country i, respectively. Now define $\psi(w^i, r^i)$ as the cost of producing one gun in country i. This unit cost function, which is identical across countries, is increasing, concave and homogeneous of degree one in factor prices. By Shephard's lemma, its partial derivatives ψ^i_w and ψ^i_r give the conditional demands respectively for labor and capital in the production of one gun. Thus, the quantities of labor and capital diverted to contesting K_0 in each country i are respectively $\psi^i_w G^i$ and $\psi^i_r G^i$, with ψ^i_r/ψ^i_w representing the corresponding capital-labor ratio. Throughout, we assume that the secure labor and capital resource constraints do not bind in the production of guns for either country i: $L^i - \psi^i_w G^i > 0$ and $K^i - \psi^i_r G^i > 0$.

Once guns have been produced and the disputed resource has been divided, the residual quantities of labor and capital available to country i are respectively $L^i_Z = L^i_g - \psi^i_w G^i$ and $K^i_Z = K^i_g - \psi^i_r G^i$, where $L^i_g \equiv L^i$ and $K^i_g \equiv K^i + \phi^i K_0$ denote "gross" quantities

[15] Our analysis focuses on adjustments at the intensive margin, but could be extended to consider a continuum of consumption goods, as in Dornbusch et al. (1977). We chose the two-good version of the model in view of economists' general familiarity with it and because it reveals possible discontinuities in the countries' best-response functions that do not arise in the continuum-good model and that can matter for equilibrium and payoffs.

[16] This specification differs modestly from the "ratio" or "power" form typically assumed (e.g., Hirshleifer, 1989; Skaperdas, 1996)—i.e., $f(G) = G^b$ with $b \in (0, 1]$—in that it does not imply a discontinuity of ϕ^i in guns at $G^1 = G^2 = 0$. That is to say, with $f(0) > 0$, $\phi^i_{G^i}(0, 0) = f'(0)/4f(0)$ is defined. One example is $f(G) = (\delta + G)^b$ with $b \in (0, 1]$ and $\delta > 0$. Assuming that $f(0)$ is positive but arbitrarily close to 0 is helpful in our proofs of existence of equilibrium in arming choices under the trade regimes studied.

[17] See Anbarci et al. (2002) for an analysis of how alternative bargaining solution concepts translate into rules of division that differ in their sensitivity to guns.

of these primary resource factors. Each country i combines its own residual resources, L_Z^i and K_Z^i, to produce an intermediate input, Z^i. Both factors are essential, and the technology is described by the unit cost function $c(w^i, r^i)$.[18] This function, like that for guns, is identical across countries, and increasing, concave and linearly homogeneous in factor prices. Furthermore, c_w^i and c_r^i give the conditional demands respectively for labor and capital in the production of a unit of Z^i, with c_r^i/c_w^i indicating the capital-labor ratio demanded in that sector of the economy.

The intermediate good Z^i, in turn, is used to produce goods for final consumption. Suppose, in particular, that to produce one unit of good j $(= 1, 2)$ producers in country i $(= 1, 2)$ need $a_j^i > 0$ units of Z^i. Then, with X_j^i denoting the supply of good j in country i, the following expression describes the production-possibility set of country i's tradables:

$$a_1^i X_1^i + a_2^i X_2^i = Z^i, \text{ where } X_j^i \geq 0. \tag{2}$$

Observe from (2) that, as in the canonical 2-country, 2-good, one-factor Ricardian trade model, comparative advantage is driven by international differences in technology (i.e., productivity) and not by differences in factor endowments.[19] To fix ideas, we assume that country i has a comparative advantage in good i. That is, $a_1^1/a_2^1 < a_1^2/a_2^2$, which states that the opportunity cost in country i to produce good i $(= 1, 2)$ is lower than the corresponding opportunity cost in country j $(\neq i)$. To economize on notation and emphasize the importance of other variables of interest, we normalize a_i^i $(= 1)$ for $i = 1, 2$ and let $\alpha^i = a_j^i > 1$ for $i \neq j = 1, 2$. Note, in any case, this model differs from the Ricardian trade model in a significant way. Specifically, each country i's production of Z^i (and thus each country's productive potential and income) depends on the security policies of both countries (G^i for $i = 1, 2$), which are endogenous.

Bringing the essential elements of the model together, the sequence of events is as follows:

Stage 1. The two countries $(i = 1, 2)$ simultaneously choose their guns G^i using their secure endowments of labor and capital.

Stage 2. The contested resource K_0 is divided according to the ϕ^i combination induced by these choices. Each country i then uses all its available inputs ($L_Z^i =$

[18] Below, we also consider briefly the possibility that the factor inputs are perfectly substitutable in the production of Z. As will become obvious, imperfect substitutability of inputs in the production of Z (along with a sharp uneven mix of labor and capital endowments across countries) is a necessary condition for trade to induce greater arming given non-binding resource constraints in the production of guns.

[19] We impose this production structure with an intermediate input for convenience only. An analytically equivalent approach would be to assume that the production functions of traded goods in each country differ by a Hicks neutral factor of proportionality, but not in factor intensities. Abstracting from differences in factor intensities allows us to rule out factor-endowment based rationales for trade patterns and the possible adjustments in arming decisions due to trade related factor-price effects (as in extensions of the Heckscher-Ohlin model) and focus instead on terms-of-trade effects.

$L_g^i - \psi_w^i G^i$ units of labor and $K_Z^i = K_g^i - \psi_r^i G^i$ units of capital) to produce the intermediate input, Z^i.

Stage 3. Each country i uses output of Z^i to produce X_i^i and X_j^i units of consumption goods i and j respectively.

Stage 4. These final goods are traded domestically and/or internationally depending on the trade regime in place.

In the subsections to follow, we complete the specification of the model and present some preliminary analysis that allows us to subsequently analyze the equilibrium of this model contingent on the prevailing trade regime.

2.1 Factor Markets and Production of the Intermediate Input

Whether the two countries trade or not, perfect competition within each one requires the price of the intermediate input (p_Z^i) to be equal to its corresponding marginal cost (c^i); as such, we use these concepts interchangeably. Additionally, the amount of each factor used to produce Z^i and G^i must equal its gross supply:

$$c_r^i Z^i + \psi_r^i G^i = K_g^i \ \ (\equiv K^i + \phi^i K_0) \tag{3a}$$

$$c_w^i Z^i + \psi_w^i G^i = L_g^i \ \ (\equiv L^i), \tag{3b}$$

for $i = 1, 2$. These conditions jointly determine the wage-rental ratio $\omega^i \equiv w^i / r^i$ and output of Z^i that ensure factor markets in each country i clear.

To proceed, we now derive an expression for the value of each country's production of the intermediate input, which we denote by R^i, given the two countries' gun choices. Specifically, multiply both sides of (3a) by r^i and the two sides of (3b) by w^i and sum them; rearranging and using the linear homogeneity of the unit cost functions in factor prices, then give the following:

$$R^i = c^i Z^i = w^i L_Z^i + r^i K_Z^i = w^i L_g^i + r^i K_g^i - \psi^i G^i. \tag{4}$$

Equation (4) implicitly defines Z^i as a function of ω^i, L_g^i, K_g^i, and G^i. In particular, solving (4) for Z^i, while again appealing to the linear homogeneity of the cost functions that implies $\psi(w^i, r^i)/r^i = \psi(\omega^i, 1)$ and $c(w^i, r^i)/r^i = c(\omega^i, 1)$, shows:

$$Z^i(\omega^i, K_g^i, L_g^i, G^i) = \frac{w^i L_g^i + r^i K_g^i - \psi(w^i, r^i) G^i}{c(w^i, r^i)} = \frac{\omega^i L_g^i + K_g^i - \psi(\omega^i, 1) G^i}{c(\omega^i, 1)}. \tag{5}$$

This function, as will become obvious below, plays a pivotal role in the definition of the countries' payoff functions and thus in our analysis of security policies.

For now, let $k_Z^i \equiv K_Z^i/L_Z^i$ denote country i' residual capital-labor ratio, and observe that combining (3a) and (3b) yields

$$\frac{c_r^i}{c_w^i} = k_Z^i \equiv \frac{K_g^i - \psi_r^i G^i}{L_g^i - \psi_w^i G^i}. \tag{6}$$

This equation implicitly defines the equilibrium wage-rental ratio $\omega^i \equiv w^i/r^i$ in country i as a function of K_g^i, L_g^i, and G^i: $\omega^{ie} \equiv \omega^{ie}(K_g^i, L_g^i, G^i)$. Then, using ω^{ie} in the right-hand side (RHS) of (5) gives $Z^{ie} \equiv Z^{ie}(K_g^i, L_g^i, G^i)$, which we refer to as the optimized production of the intermediate input.[20]

2.2 Product Markets and Demand

Let p_j^i and D_j^i respectively denote the price and quantity of consumption good j $(= 1, 2)$ demanded in country i $(= 1, 2)$. To keep the analysis as transparent as possible, we abstract from trade costs.[21] Competitive "domestic" (or "internal") pricing in output markets always requires $p_i^i = c^i$ for $i = 1, 2$, since country i produces the good in which it has a comparative advantage (good i) whether or not it trades with country j. Furthermore, absent trade costs, perfect competition requires that the price of country i's importable good satisfy $p_j^i = \min[\alpha^i c^i, c^j]$ for $i \neq j = 1, 2$. When trade is possible, at least one country will cease to produce its importable; in this case $p_j^i = p_j^j = c^j$ for $i \neq j = 1$ and/or 2. But, if trade is simply ruled out, then $p_j^i = \alpha^i c^i$.

Consumer preferences in each country i are given by $U^i = U(D_i^i, D_j^i)$ for $j \neq i = 1, 2$, which we assume to be increasing, quasi-concave, homogeneous of degree one and symmetric over the two goods, with a constant and finite elasticity of substitution, denoted in absolute terms by $\sigma \in (0, \infty)$.[22] Let Y^i denote country i's national income. Under both autarky and trade (absent tariffs), we have $Y^i = R^i = c^i Z^i$. Then, country i's indirect utility function can be written as $V^i = \mu(p_i^i, p_j^i) Y^i = \mu(p_i^i, p_j^i) c^i Z^i$, where $\mu^i \equiv \mu(p_i^i, p_j^i) = [(p_i^i)^{1-\sigma} + (p_j^i)^{1-\sigma}]^{1/(\sigma-1)}$ for $i \neq j = 1, 2$ represents the marginal utility of income, which is decreasing and homogeneous of degree -1 in prices.[23] By Roy's identity, the Marshallian

[20]Lemma A.1 presented in the Appendix shows how Z^{ie} and ω^{ie} depend on L_g^i, K_g^i and G^i.

[21]However, as discussed below in Section 5.1, our findings regarding the negative effect of trade openness on equilibrium arming and the resulting positive effect on welfare are robust to the presence of trade costs.

[22]Assuming CES preferences is not critical here, but helps simplify notation. Our assumption that the consumption goods are imperfect substitutes ($\sigma < \infty$) eases the exposition. As will become clear, when $\sigma = \infty$, the trade-dependency of arming incentives disappears.

[23]More precisely, μ^i is the inverse of the price index that is dual to the CES aggregator. Observe the linear homogeneity of utility implies risk neutrality; however, due to the concavity of the technology for Z^i in capital, this assumption does not imply the equivalence of the contest we are considering and a "winner-take-all contest," with ϕ^i equaling country i's probability of taking control of the entire quantity of the disputed resource. As can easily be confirmed, under autarky given their guns choices, the two countries strictly prefer (ex ante) to "share" the disputed resource according to (1).

demand function for good j in country i is

$$D_j^i = \gamma_j^i Y^i / p_j^i = \gamma_j^i c^i Z^i / p_j^i, \tag{7}$$

where $\gamma_j^i \equiv \gamma_j^i(p_i^i, p_j^i) = -(\partial\mu^i/\partial p_j^i)/(\mu^i/p_j^i) = (p_j^i)^{1-\sigma}/[(p_j^i)^{1-\sigma}+(p_i^i)^{1-\sigma}]$ is the expenditure share on good j $(= 1, 2)$ in country i $(= 1, 2)$. Naturally, the relative demand $RD^i = D_j^i/D_i^i$ for good j in country i depends solely on prices.

Define $p^i \equiv p_j^i/p_i^i$ as the price of good j in country i relative to the good in which country i enjoys a comparative advantage, and let a hat "ˆ" denote percentage change (e.g., $\widehat{x} \equiv dx/x$). Then, we have $\widehat{RD}^i = -\sigma\widehat{p}^i$, which implies the relative demand for the importable good j in country i depends negatively on p^i, and more so the greater is σ. Furthermore, we have $\partial\gamma_j^i/\partial p_j^i = -(1-\gamma_j^i)(\sigma-1)(\gamma_j^i/p_j^i)$, which shows that the expenditure share for the importable good is decreasing (increasing) in the price of that good p_j^i when σ is greater (less) than one.

3 Trade Regimes and Payoff Functions

To explore how the two countries' incentives to arm depend on the trade regime in place—namely, autarky and trade—we first analyze how equilibrium and thus payoffs are determined under each of these regimes for given security policies, G^i, and gross factor endowments, K_g^i and L_g^i.

3.1 Autarky

We start our analysis of a closed economy with two observations. First, self-sufficiency requires both consumption goods to be produced in each country i; therefore, $p_i^i = c^i$ and $p_j^i = \alpha^i c^i$, which give the equilibrium relative price under autarky: $p_A^i = p_j^i/p_i^i = \alpha^i$. Second, with the Marshallian demand functions (7), the just-described market exchange ratio between the two consumption goods determines the division of income between them. This division along with the product market-clearing requirement that $D_j^i = X_j^i$ for each good j determines (by (2)) equilibrium quantities.

Using the competitive pricing relations noted above in $\mu(p_i^i, p_j^i)$ and the fact that under autarky $Y^i = R^i = c^i Z^i$, the indirect utility function of the representative consumer in country i can be written as

$$V_A^i = \mu(c^i, \alpha^i c^i)Y^i = m_A^i Z^{ie}(K^i + \phi^i K_0, L^i, G^i), \;\; i = 1, 2, \tag{8}$$

where, since μ^i is homogeneous of degree -1 in prices, $m_A^i \equiv \mu(c^i, \alpha^i c^i)c^i = \mu(1, p_A^i)$, and where $Z^{ie} \equiv Z^i(\omega^{ie}, \cdot)$ satisfies (5) and (6). An important feature of country i's payoff function under autarky (8) is that, since $p_A^i = \alpha^i$ and α^i is a constant parameter, m_A^i is also a constant. Thus, each country i's arming decision under autarky depends solely on how G^i influences its optimized production of the intermediate input, Z^{ie}.

3.2 Free Trade

Before examining arming incentives under autarky, we characterize the equilibrium payoff functions given guns under trade. We focus on the case where each country specializes production completely in the good for which it has a comparative advantage: $X_i^i = Z^i$ for $i = 1, 2$. Thus, country i's demand for good j is fulfilled entirely by imports, and the equilibrium determination of world prices that clears the market for good j satisfies $D_j^i + D_j^j = X_j^j = Z^j$ for $j\ (\neq i)\ = 1, 2$.

To proceed, recall that competitive forces under free trade imply consumers face identical prices for each good: $p_j^i = p_j^j = c^j$ for $j = 1, 2$. Since consumers' preferences are identical across countries, the equality of the prices of good j across countries implies that their expenditure shares on that good are identical: $\gamma_j^i = \gamma_j^j = \gamma_j$ for $j \neq i = 1, 2$. Maintaining our focus on the market for good j, now let p_T^i $(\equiv p_j^j/p_i^j = c^j/c^i)$ denote the world (or domestic) price of country i's importable (good $j \neq i$) in terms of its exportable good. Substitution of the demand functions (7) with the competitive pricing relations into the relevant market-clearing condition shows

$$p_T^i = \gamma_j Z^i / \gamma_i Z^j, \qquad i \neq j, \tag{9}$$

where the expenditure share on good j can be written as $\gamma_j = (p_T^i)^{1-\sigma}/[1 + (p_T^i)^{1-\sigma}]$ and $\gamma_i = 1 - \gamma_j$.[24]

One can see the impact of the relative supply of the countries' intermediate inputs on p_T^i and thus on country i's terms of trade by logarithmically differentiating (9), while accounting for the dependence of the expenditure shares on p_T^i. After rearranging terms, we have

$$\widehat{p}_T^i = \frac{1}{\sigma}\left[\widehat{Z}^i - \widehat{Z}^j\right], \tag{10}$$

for $i \neq j = 1, 2$. An exogenous increase in the relative supply of country i's intermediate input Z^i/Z^j expands the relative supply of its exportable and thus increases the relative world price of country i's importable p_T^i—a deterioration of its terms of trade that is decreasing in the elasticity of substitution in consumption $\sigma > 0$.

Since $Y^i = R^i = c^i Z^i$, we write country i's payoff function under trade as:

$$V_T^i = \mu^i(p_i^i, p_j^i) Y^i = m_T^i Z^{ie}(K^i + \phi^i K_0, L^i, G^i), \quad i \neq j = 1, 2, \tag{11}$$

[24] Of course, depending on the countries' security policies, factor endowments, technology, and consumer preferences, p_T^i could be determined by one country's autarkic price. We revisit this possibility below.

where $m_T^i \equiv \mu(p_i^i, p_j^i)c^i = \mu(1, p_T^i)$ and Z^{ie} satisfies (5) and (6). Guns production influences the payoffs under trade, like the payoffs under autarky, through its effect on the maximized value of the country's intermediate output Z^{ie}. However, country i's production of guns influences V_T^i through an additional channel—namely, through its effect on the relative price of country i's importable, p_T^i. Specifically, from (10), an increase in G^i, given G^j, increases Z^i and decreases Z^j, and thereby increases the relative price of country i's importable good, p_T^i. Since $\frac{\partial m_T^i}{\partial p_T^i}\frac{p_T^i}{m_T^i} = -\gamma_j < 0$, equation (11) implies an increase in p_T^i alone reduces country i's payoff under trade V_T^i.

3.3 Incentives to Trade (for Given Guns)

While one of our primary goals is to explore the trade-regime dependency of arming incentives and the associated welfare implications, it is instructive to see how payoffs under autarky and trade compare for given guns. Based on a standard, gains-from-trade argument using (8) and (11), one can verify the following:

Lemma 1 *For any given feasible guns and gross factor endowments, payoffs under autarky and trade are ranked as follows: $V_A^i \leq V_T^i$, for each $i = 1, 2$.*

Intuitively, when country i's cost of importing good j (p_T^i) is greater than its opportunity cost $\alpha^i = p_A^i$, country i produces both goods locally, implying identical payoffs under the two trade regimes. Trade flows between the two countries will be strictly positive, given guns, only when $p_T^i \leq \alpha^i$ with strict inequality for at least one country to make the payoffs for both countries under trade at least as high as they are under autarky and strictly greater under trade for at least one of them.[25] Any added payoff generated by a shift to free trade (given arming) reflects the *familiar gains from trade* that follow from canonical trade models based on comparative advantage.

4 Endogenous Security Policies

We now turn to the determination of noncooperative equilibria in security policies and their dependence on trade regimes. Inspection of the objective functions under autarky (A) in (8) and under trade (T) in (11) reveals that the equilibrium production of the intermediate input, represented by the envelope function Z^{ie}, is of central importance here.[26] As established in Lemma A.1 (presented in the Appendix), Z^{ie} and the equilibrium wage-rental

[25] To be more precise, comparing $m_A^i = \mu(1, p_A^i)$ and $m_T^i = \mu(1, p_T^i)$ shows that, if $p_T^i < p_A^i = \alpha^i$ holds for both countries, then each country i specializes completely in the production of good i, and both countries are strictly better off under trade.

[26] While Z^{ie} might appear to be a regular production function, it is best thought of as an equilibrium quantity that is jointly determined with factor prices and with gross inputs K_g^i and L_g^i being employed (possibly) in different proportions than in the production of G^i.

ratio ω^{ie} depend, under each trade regime, on guns and gross labor and capital endowments. Given these quantities, however, Z^{ie} and ω^{ie} are independent of the prevailing trade regime. Thus, as noted earlier, our analysis rules out factor prices as a channel through which a change in trade regimes can influence arming incentives; instead, the trade-regime dependency of arming incentives operates solely through a terms-of-trade channel.[27]

To set the stage for this analysis, we make two additional observations that can be verified from (5).[28] First, the effect of a marginal increase in country i's gross endowment of capital $K_g^i = K^i + \phi^i K_0$ on Z^{ie} is given by $Z_K^{ie} = r^i/c^i$ and the effect of a marginal increase in its arming G^i (given K_g^i) on Z^{ie} is $Z_{G^i}^{ie} = -\psi^i/c^i$. Second, an increase in G^i also affects the rival's optimized production of the intermediate good Z^{je} through its influence the rival's gross capital endowment, $Z_K^{je} = r^j/c^j$. Bringing these observations together, while noting from (1) $\phi_{G^i}^j = -\phi_{G^i}^i$, shows:

$$\frac{dZ^{ie}}{dG^i} = K_0 \phi_{G^i}^i Z_K^{ie} + Z_{G^i}^{ie} = \frac{1}{c^i}\left[r^i K_0 \phi_{G^i}^i - \psi^i\right] \tag{12a}$$

$$\frac{dZ^{je}}{dG^i} = K_0 \phi_{G^i}^j Z_K^{je} = -\frac{1}{c^j}[r^j K_0 \phi_{G^i}^i], \tag{12b}$$

given G^j, for $i \neq j = 1, 2$.

4.1 Autarky

As revealed by the payoff functions under autarky (8) and noted earlier, country i's arming choice influences V_A^i only through its effect on the maximized value of country i's intermediate input, Z^{ie}. That is to say, since m_A^i is a constant, $\widehat{V}_A^i = \widehat{Z}^{ie}$. Thus, from (12a), each country i's first-order condition (FOC) for the choice of guns G^i, taking G^j as given, can be written as

$$\frac{1}{m_A^i}\frac{\partial V_A^i}{\partial G^i} = \frac{dZ^{ie}}{dG^i} = \frac{1}{c^i}\left[r^i K_0 \phi_{G^i}^i - \psi^i\right] \leq 0, \quad i = 1, 2. \tag{13}$$

The first term in the brackets of the RHS of (13) reflects the marginal benefit of producing an additional gun for country i. An additional gun increases the share of the disputed resource that country i captures in the contest (given the rival's choice G^j) and thereby increases the country's income and payoff. However, as shown in the second term, that additional gun comes at a cost. In particular, capital and labor resources are diverted away from the production of the intermediate input used in the production of consumption goods. Each country's optimal security policy balances this trade-off at the margin. Observe that

[27] By contrast, focusing on small countries, Garfinkel et al. (2015) abstract from the terms-of-trade channel, while featuring instead the factor-price channel.

[28] Also see the proof of Lemma A.1 presented in the Supplementary Appendix (B.1).

the negative influence of country i's security policy on country j's payoff through its effect on Z^{je} (shown in (12b)) does not directly enter this calculus.

Maintaining focus on the case in which the secure resource constraints on guns (i.e., $K^i - \psi_r^i G^i \geq 0$ and $L^i - \psi_w^i G^i \geq 0$) are not binding, equation (13) is an exact statement of country i's FOC under autarky. Furthermore, the conflict technology (1) implies interior solutions with (13) holding as an equality. Since Z^{ie} is concave in the country's own guns G^i and in the country's gross capital endowment K_g^i (see parts (c) and (e) of Lemma A.1) and K_g^i is concave in G^i through the conflict technology (1), we can show that V_A^i is strictly quasi-concave in G^i. This property ensures the existence of an interior (pure-strategy) equilibrium in security policies.

Let (G_A^{1*}, G_A^{2*}) be an equilibrium pair of guns. Some additional (but relatively mild) assumptions imply the equilibrium is unique:

Proposition 1 (*Equilibrium security policies under autarky.*) *An interior equilibrium in security policies exists under autarky: $G_A^{i*} > 0$, for $i = 1, 2$. Furthermore, if labor and capital are sufficiently substitutable in the production of arms and/or the intermediate good, this equilibrium is unique.*

Proof. See the Appendix.

Observe from (13) that the equilibrium in security policies under autarky is independent of the elasticity of substitution in consumption. This independence follows since each country's problem under autarky is effectively one of maximizing income or, equivalently, the quantity of the intermediate good used in the production of traded goods. Matters differ, however, in the case of trade.

4.2 Free Trade

Security policies under trade, like those under autarky, affect payoffs through their impact on the output levels of the intermediate good. However, when trade is possible between adversaries, these output changes also affect world prices as shown in (10). Then, using (10) and (11) and recalling that $\frac{\partial m_T^i / \partial p_T^i}{m_T^i / p_T^i} = -\gamma_j$, one can verify the following:

$$\widehat{V}_T^i = \widehat{Z}^{ie} + \left(\frac{\partial m_T^i / \partial p_T^i}{m_T^i / p_T^i} \right) \widehat{p}_T^i = \widehat{Z}^{ie} - \frac{\gamma_j}{\sigma} \left(\widehat{Z}^{ie} - \widehat{Z}^{je} \right), \quad i \neq j = 1, 2. \tag{14}$$

The second term in the RHS of this expression shows the terms-of-trade effect.

Combining (12) with (14), the FOC for country i's arming choice becomes:

$$\frac{1}{m_T^i} \frac{\partial V_T^i}{\partial G^i} = \frac{1}{c^i} \left\{ \left[1 - \frac{\gamma_j}{\sigma} - \left(\frac{r^j / c^j Z^{je}}{r^i / c^i Z^{ie}} \right) \frac{\gamma_j}{\sigma} \right] r^i K_0 \phi_{G^i}^i - \left[1 - \frac{\gamma_j}{\sigma} \right] \psi^i \right\} \leq 0, \tag{15}$$

for $i = 1, 2$ and $j \neq i$. Similar to the FOC under autarky (13), this FOC consists of two components: the marginal benefit and the marginal cost of an additional gun. However, the negative effect of an additional gun on country i's terms of trade modifies these two components as compared with autarky, thereby suggesting that whether trade is possible or not matters for the countries' incentives to arm.

Before exploring that influence, we characterize the equilibrium under trade. Henceforth, we assume that $\sigma > \gamma_j$ for $j = 1, 2$, such that the marginal cost of arming is strictly positive for both countries. This assumption, however, does not ensure that the marginal benefit, when evaluated at $G^i = 0$, is strictly positive. Digging a little deeper, let us define

$$\xi^j \equiv \frac{r^i/c^i Z^{ie}}{r^j/c^j Z^{je} + r^i/c^i Z^{ie}} = \frac{c^j Z^{je}/r^j}{c^i Z^{ie}/r^i + c^j Z^{je}/r^j}, \text{ for } i \neq j.$$

The parameter ξ^j reflects country j's relative size in terms of the countries' GDP, net of arming and measured in domestic units of the insecure resource. Then, the sign the marginal benefit for country i (i.e., the first term in (15)) is determined by the sign of $(\sigma - \gamma_j/\xi^j)$. Hence, there exists a critical value of σ, $\overline{\sigma}^i \equiv \gamma_j/\xi^j$ evaluated at $G^i = G^j = 0$ for each country $i \neq j = 1, 2$, such that country i's marginal benefit of arming when $G^i = 0$ is strictly positive (non-positive) if $\sigma > \overline{\sigma}^i$ ($\sigma \leq \overline{\sigma}^i$).

Since $\gamma_1 + \gamma_2 = \xi^1 + \xi^2 = 1$, there are two distinct possibilities to consider: (i) $\overline{\sigma}^1 = \overline{\sigma}^2 = 1$, which arises when the two countries have identical secure endowments;[29] and, (ii) $\overline{\sigma}^i > 1 > \overline{\sigma}^j$ for $i \neq j = 1, 2$, which arises in the presence of asymmetries. Thus, if $\sigma \leq 1$, the marginal benefit of arming must be non-positive for at least one country i (the relatively larger one) and possibly both; if $\sigma > 1$, the marginal benefit of arming must be positive for at least one country j (the relatively smaller one) and possibly both. Also observe that the maximum value of $\overline{\sigma}^i$ across i, $\overline{\sigma} \equiv \max[\overline{\sigma}^1, \overline{\sigma}^2]$, is greater than or equal to 1.

Building on these ideas and using the FOC under trade (15), it is possible to verify that an equilibrium always exists. However, multiple pure-strategy equilibria or mixed-strategy equilibria are possible. Nonetheless, provided that the strength of the two countries' comparative advantage, as determined by domestic opportunity costs and measured by α^i, is sufficiently strong, a pure-strategy equilibrium that differs from the one under autarky exists under free trade.[30] The next proposition focuses on this case, letting (G_T^{1*}, G_T^{2*})

[29] Specifically, as discussed in Section 5.1, a symmetric equilibrium emerges with $Z^{ie} = Z^{je}$, $\gamma_i = \gamma_j = \frac{1}{2}$ and $p_T^i = 1$ for $i = 1, 2$. Since equilibrium factor prices are identical across countries, we have additionally $\xi^j = \frac{1}{2}$ in this benchmark case. Thus, $\sigma > 1$ ($\sigma \leq 1$) implies that the marginal benefit of arming is strictly positive (non-positive) at $G^i = 0$ for both i.

[30] The potential problem, as analyzed in the Supplementary Appendix (B.1), is that p_T^i can vary only within the range $[1/\alpha^j, \alpha^i]$, giving rise to a possible discontinuity in country i's best-response function under trade such that it coincides with country i's best-response function under autarky for some G^j. Requiring that comparative advantage α^i be sufficiently large for both $i = 1, 2$ ensures the existence of a pure-strategy

denote that equilibrium:

Proposition 2 (*Equilibrium security policies under free trade.*) *Suppose the conditions that ensure a unique equilibrium under autarky are satisfied and each country's comparative advantage (α^i) is sufficiently large. Then, there exists a pure-strategy equilibrium in security policies under free trade that is distinct from the equilibrium under autarky: (i) $G_T^{i*} = 0$ for $i = 1, 2$ if $\sigma \leq \overline{\sigma} \equiv \max[\overline{\sigma}^1, \overline{\sigma}^2]$ and (ii) G_T^{i*} $(\neq G_A^{i*})$ > 0 for $i = 1, 2$ if $\sigma \in (\overline{\sigma}, \infty)$.*

Proof. See the Appendix.

Clearly, the elasticity of substitution in consumption (σ) plays an essential role under free trade, as it determines the magnitude of the terms-of-trade effect. When the two goods are not close substitutes (i.e., $\sigma \leq 1$), then $\sigma \leq \overline{\sigma}^i$ for one country $i = 1$, 2, or both. Country i's payoff under trade in this case is highly dependent on country j's production, through relative world prices. Even though an increase in guns by country i, where initially $G^i = 0$ and given $G^j \geq 0$, generally implies a positive net marginal benefit of arming for given world prices (i.e., $r^i K_0 \phi^i_{G^i} - \psi^i > 0$), that additional gun by country i implies at the same time a worsening of its terms of trade. More precisely, it induces not only an increase in the supply of its exportable good but also a decrease in the supply of its importable good. The terms-of-trade effect, then, tends to reduce the effective marginal benefit of an additional gun by more than it reduces the marginal cost. Indeed, when the two consumption goods are not sufficiently substitutable ($\sigma \leq 1$), the negative net terms-of-trade effect swamps the positive net marginal gain given world prices for at least one country, i. Then, given (1) with $\phi^i_{G^i} \in (0, \infty)$ at $G^i = 0$, country i chooses to produce no guns at all. If, in addition, $\sigma \leq \overline{\sigma}^j$, then $G_T^{i*} = G_T^{j*} = 0$. But, even if $\sigma > \overline{\sigma}^j$ while $\sigma \leq \overline{\sigma}^i$, our specification of the conflict technology (1) requiring that $f(0)$ be arbitrarily close to zero (even if positive) implies that country j's best response to $G^i = 0$ is to produce just an infinitesimal amount of guns. In this case, security costs are effectively equal to zero. Hence, we write in the case where $\sigma < 1 \leq \overline{\sigma} \equiv \max[\overline{\sigma}^1, \overline{\sigma}^2]$, $G_T^{1*} = G_T^{2*} = 0$.[31]

When the two consumption goods are sufficiently substitutable (i.e., $\sigma > \overline{\sigma} \geq 1$), the negative effect through the terms of trade channel is not large enough to wipe out the positive net marginal benefit of arming given world prices when evaluated at $G^i = 0$ for either country i. The conditions specified in the proposition along with (1) ensure, in this case, the existence of an interior equilibrium in security policies under trade that differs from the equilibrium under autarky. At this internal equilibrium, the favorable

equilibrium in security policies that is distinct from the equilibrium under autarky. Nevertheless, even when this requirement is satisfied, we cannot rule out the possibility that there exist multiple pure-strategy, trade equilibria.

[31] Observe $\sigma \leq 1$ is sufficient, but not necessary, for trade to effectively remove both countries' incentives to arm; the same result obtains when $\sigma > 1$, as long as $\sigma < \overline{\sigma}$.

effect of an increase in the rival country's arming G_T^{j*} on country i's terms of trade is not sufficiently large to swamp the negative effect on country i payoff for given world prices. Put differently, the externality of each country j's arming remains negative in equilibrium: $dV_T^{i*}/dG^{j*} < 0$.[32]

5 The Relative Appeal of Free Trade

We are now in a position to compare the equilibrium payoffs under the two trade regimes, denoted by $V_A^{*i} \equiv V_A^i(G_A^{i*}, G_A^{j*})$ in the case of autarky and $V_T^{*i} \equiv V_T^i(G_T^{i*}, G_T^{j*})$ in the case of trade, for $i \neq j = 1, 2$. Combining Lemma 1 with the equilibrium analysis underlying Propositions 1 and 2, the next lemma establishes that a sufficient condition for trade to dominate autarky is simply that trade induce lower arming by both countries:

Lemma 2 (*Equilibrium payoffs under autarky vs. free trade.*) *Suppose that trade induces both countries to arm less as compared with autarky. Then, each one is strictly better off under free trade than under autarky (i.e.,* $V_T^i(G_T^{i*}, G_T^{j*}) > V_A^i(G_A^{i*}, G_A^{j*})$ *for* $i \neq j = 1, 2$*), and the difference in payoffs for both is greater than the standard gains from trade.*

Proof. See the Appendix.

The benefits for each country in moving to free trade can be decomposed into three parts. First, given both countries' arming choices, each country enjoys the standard gains from trade. As established in Lemma 1, these are non-negative for both countries and possibly positive for at least one if not both. Second, each country enjoys, given its own arming choice, a positive strategic effect as the opponent reduces its arming. Finally, each country's payoff rises, as it optimally adjusts its own arming choice in response to the new trade regime. Of course, in the presence of asymmetries in secure resources, one country could become less powerful under trade relative to its position under autarky. However, provided that both countries reduce their arming, each realizes lower security costs on top of the standard gains from trade.

Using Propositions 1 and 2, we now turn to study the difference in equilibrium guns chosen by the two adversarial countries under the two trade regimes. Proposition 1 indicates the optimizing choice of guns for each country i under autarky is strictly positive regardless of the elasticity of substitution in consumption (σ). By contrast, Proposition 2 indicates that

[32] To confirm this claim, which also holds true when $\overline{\sigma}^i > \sigma > 1$ (such that country i chooses not to arm whereas country j chooses an infinitesimal quantity of arms), evaluate

$$\frac{dV_T^i/dG^j}{V^i} = \frac{dZ^{ie}/dG^j}{Z^{ie}} - \frac{\gamma_j}{\sigma}\left[\frac{dZ^{ie}/dG^j}{Z^{ie}} - \frac{dZ^{je}/dG^j}{Z^{je}}\right],$$

using the value of $\frac{dZ^{ie}/dG^j}{Z^{ie}}$ implied by country j's FOC under trade (15) for an interior solution, to find that a necessary and sufficient condition for $dV_T^i/dG^j < 0$ is that $\sigma > 1$.

the equilibrium choice of guns for each country i equals zero when $\sigma \leq \overline{\sigma} \equiv \max[\overline{\sigma}^1, \overline{\sigma}^2]$.[33] Thus, when the two goods are sufficiently complementary in consumption, equilibrium arming is lower under free trade and, by Lemma 2, welfare for each country is higher; each country enjoys not only the standard gains from trade, but also the elimination of security costs it induces.

When the two consumption goods are sufficiently substitutable (i.e., $\sigma > \overline{\sigma}$), each country's optimizing choice of guns is strictly positive under both autarky and free trade. But, a comparison of the FOCs under autarky (13) and trade (15) reveals the adverse terms-of-trade effect of a country's own arming reduces the marginal benefit of an additional gun relative to the analogous marginal benefit under autarky by more than it reduces the relative marginal cost. Thus, for any given G^j, country i's best response under free trade is strictly less than its best response under autarky: $B_T^i(G^j) < B_A^i(G^j)$ for any $G^j \geq 0$, $i \neq j = 1, 2$.[34] This finding gives us a sufficient but not necessary condition for trade to induce lower equilibrium arming, when arming is strictly positive under both trade regimes—that is, if neither country's best-response functions exhibits strategic substitutability in the neighborhood of the autarkic equilibrium, then there exists a pure-strategy equilibrium under free trade where both countries choose lower arms and are strictly better off than under autarky.

Whether a country's best-response function is positively or negatively sloped in the neighborhood of the autarkic equilibrium generally depends on how an increase in the opponent's guns G^j influences country i's marginal benefit and marginal cost of arming (see equation (A.5) in the Appendix). As one can easily verify, an increase in G^j (given G^i) decreases K_g^i and thus, given L_g^i, decreases country i's equilibrium wage-rental ratio ω^{ie}. Since country i's marginal cost of arming depends positively on ω^i (i.e., $\psi_\omega^i > 0$), the increase in G^j decreases country i's marginal cost and alone induces it to produce more guns. However, the effect of an increase in G^j on country i's marginal benefit of arming can be positive or negative. Specifically, equation (1) implies $\phi^i_{G^iG^j} \gtreqless 0$ as $B_A^i(G^j) \gtreqless G^j$. Thus, when $B_A^i(G^j) > G^j$ ($B_A^i(G^j) < G^j$), the implied positive (negative) effect of G^j on country i's marginal benefit of arming alone induces it to produce more (less) guns.

This analysis suggests that the sufficient condition for trade to lower equilibrium arming and enhance each country's payoff can be traced back to fundamentals that influence

[33] As noted above, arming by one country could be strictly positive, but would be infinitesimal.

[34] By rearranging (14) as

$$\frac{dV_T^i/dG^i}{V_T^i} = \left[1 - \frac{\gamma_2}{\sigma}\right] \frac{dZ^{ie}/dG^i}{Z^{ie}} + \frac{\gamma_2}{\sigma} \frac{dZ^{je}/dG^i}{Z^{je}}$$

and recalling from (12b) that $dZ^{je}/dG^i < 0$, one can confirm our assumption $1 - \gamma_2/\sigma > 0$ that ensures a positive marginal cost of arming is both sufficient and necessary to rule out the possibility that, given G^j, $dZ^{ie}/dG^i < 0$ at an interior optimum for country $i \neq j = 1, 2$ under trade. Thus, the finding that trade openness lowers each country i's arming incentive (given G^j) cannot be attributed to the presence of immizerising growth (Bhagwati, 1959).

relative arming by the two countries under autarky. To examine this issue more carefully, we proceed in two steps. First, we focus in the next subsection on the following benchmark case, which we refer to as *complete symmetry*: in addition to having identical preferences (defined symmetrically over the two consumption goods) and identical technologies in the production of G^i and Z^i, the two contending countries have identical endowments of secure capital and labor, $K^i = K$ and $L^i = L$ for $i = 1, 2$. Then, in the following subsection, we consider asymmetric distributions. The appeal of this approach over considering, for example, differences in the countries' technologies for producing guns stems from the fact that relative endowments are observable and contain information regarding differences in the degree of resource security across countries. Throughout, we assume sufficient substitutability between consumption goods (i.e., $\sigma > \overline{\sigma}$) such that arming is strictly positive under trade as well as under autarky.

5.1 Complete Symmetry

Suppose that the two countries arm identically: $G^1 = G^2$. Regardless of the trade regime in place, this assumption under the conditions of complete symmetry generally implies from (1) that $\phi^i = \frac{1}{2}$ for $i = 1, 2$, which in turn implies that each country holds, upon dividing the contested resource K_0, an identical amount of capital to employ in the production of the intermediate input: $K_Z^i = K_Z$ for $i = 1, 2$. Then, (5) and (6) imply identical relative factor prices and production of the intermediate good: $\omega^i = \omega$ and $Z^i = Z$ for $i = 1, 2$. The unit costs of producing Z and G, measured in units of capital (respectively c^i/r^i and ψ^i/r^i), are also identical across countries.

Under autarky, these simplifications mean that the marginal benefit and the marginal cost of arming are identical across countries when $G^1 = G^2$. As such, from the FOC under autarky (13), identical arming across countries is, in fact, consistent with optimizing behavior by both and thus represents a possible equilibrium. From Proposition 1, we know further that, provided labor and capital are sufficiently substitutable in the production of the intermediate good and guns, the equilibrium is unique, and arming is strictly positive regardless of the elasticity of substitution in consumption: $G_A^{i*} = G_A^* > 0$ for $i = 1, 2$ and all $\sigma > 0$.

Under free trade, the simplifications that follow from complete symmetry imply $p_T^i = 1$ for $i = 1, 2$, and thus $\gamma_1 = \gamma_2 = \frac{1}{2}$ when $G^1 = G^2$. Accordingly, from the relevant FOC (15) and our maintained assumptions that comparative advantage for each country is sufficiently strong and the two final goods are sufficiently complementary in consumption (i.e., $\sigma > \bar{\sigma}^i = 1$), the equilibrium in security policies under free trade is also unique and symmetric, with strictly positive arming: $G_T^{i*} = G_T^* > 0$ for $i = 1, 2$.[35] More importantly

[35] When $\sigma < 1$, we have $G_T^{i*} = G_T^* = 0$ for $i = 1, 2$. As indicated below, details on uniqueness of equilibrium

for our purposes here, since the two countries arm identically under autarky, both countries' best-response functions necessarily exhibit strategic complementarity in the neighborhood of the autarkic equilibrium, as illustrated in Fig. 1. Hence, starting at the autarkic equilibrium (point A in the figure where $G_A^{i*} = G_A^* > 0$ for $i = 1, 2$), a shift to free trade induces both countries' best-response functions to shift inward to intersect at a new equilibrium with less arming by both countries (point T in the figure where $0 < G_T^{i*} = G_T^* < G_A^*$ for $i = 1, 2$). The effect of trade to induce each country to produce fewer arms than under autarky, without influencing the distribution of the contested resource, implies that both countries enjoy equally lower security costs under trade. Thus, as implied by Lemma 2, a shift to free trade is welfare-improving for both countries, and the gains they realize are strictly greater than those predicted by the traditional paradigm that abstracts from conflict all together.

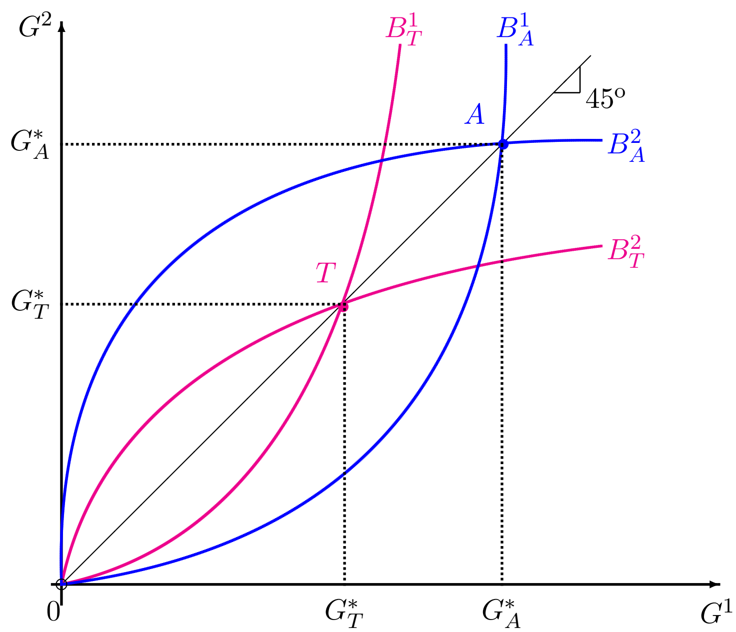

Figure 1: The Impact of Trade on Equilibrium Arming under Complete Symmetry

Given our focus above on free trade, it is natural to ask how trade costs, both import tariffs and non-tariff barriers, matter here. As one can show, under the conditions of complete symmetry, there exists a unique and symmetric, pure-strategy equilibrium under trade in the presence of (non-prohibitive) identical trade costs that is distinct from the equilibrium under autarky—namely, with less arming: $0 < G_T^{1*} = G_T^{2*} = G_T^* < G_A^*$ for $\sigma \in (1, \infty)$.[36] Furthermore, reciprocal reductions in physical trade barriers (globalization)

under trade for all $\sigma > 0$ in the case of complete symmetry are provided in Proposition A.1 presented in the Appendix.

[36]See Proposition A.1, which is presented in the Appendix, with details on existence and uniqueness of equilibrium. Also see Proposition A.2 that shows G_T^* is increasing in the elasticity of substitution in

and in tariffs (trade liberalization) reduce equilibrium arming. The welfare implications follow clearly from the following two observations. First, at constant (and equal) quantities of arms, reductions in trade costs expand the volume of trade between the two countries, while leaving their terms of trade intact. Second, such reductions lower the incentives for both countries to arm without affecting the distribution of the contested resource; with less arming, more resources are available for the production of goods for final consumption. Thus, a reciprocal reduction in trade costs enhances not only the familiar gains from trade, but also trade's conflict-reducing effect.[37]

5.2 Resource Asymmetries

While our results above illustrate how the pacifying effect of trade openness can follow intuitively from a formal game-theoretic analysis, the assumption of complete symmetry is not innocuous. In this subsection, we explore specifically the possible importance of asymmetries in the initial distribution of secure resources. We start by asking whether there exist asymmetric distributions of secure resources that similarly imply a symmetric equilibrium in security policies under autarky and thus satisfy the sufficient condition for trade to induce lower arming. The answer is yes, depending in part on the nature of the technologies in producing the intermediate good Z and guns G.

Suppose, for example, that labor L and capital K are perfect substitutes in producing both Z and G. In this special case, there is effectively only one factor input, bringing us squarely into the world of classical economics. The marginal cost of arming under autarky, measured in units of K, is a constant and thus identical across countries even when they have different initial resource endowments. Assuming neither country is resource constrained in the production of G, the slope of country i's best-response function depends only on how the rival's arming G^j influences the country i's marginal benefit. From the conflict technology specified in (1), that influence depends solely on the ranking $B_A^i(G^j) \gtreqless G^j$, with the slope being non-negative for $B_A^i(G^j) \geq G^j$ and strictly negative otherwise. But, given the symmetry of the conflict technology, the marginal benefit is similarly identical across nations when $G^1 = G^2$. Thus, for all distributions of initial secure resources where neither country's secure resource constraint binds in the production of G, the equilibrium

consumption for $\sigma > 1$ $(= \overline{\sigma})$, approaching G_A^* (which is independent of σ) as $\sigma \to \infty$. Thus, consistent with Hirshleifer's (1991) argument, the savings in security costs afforded by trade equal zero when the two goods are perfect substitutes and increase as σ falls approaching 1 (or equivalently as the two economies become increasingly integrated).

[37]See Proposition A.3 presented in the Appendix. Furthermore, it is possible to show that reciprocal reductions in trade costs can improve welfare in each country in a Pareto sense. Put differently, such reductions are not only efficient, but could also generate gains in payoffs to both factors of production—i.e., labor and owners of capital—and thus could be politically feasible. A sufficient condition for this result is that factor intensities in the guns and intermediate goods sectors are not too different from each other. (See Proposition B.1 presented in Supplementary Appendix B.1.)

in security policies under autarky is symmetric.[38] Since neither country's best-response function exhibits strategic substitutability in the neighborhood of this equilibrium, a shift from autarky to trade necessarily results in less arming and hence higher payoffs.[39]

Next, let us continue to assume that L and K are perfect substitutes in the production of Z, but not in the production of G. When neither country's arming decision is constrained by its secure endowments, the optimal allocation of resources to G (and residually to Z) for each country i requires that the marginal rates of technical substitution between L and K in the production of G and Z be equalized and that they be equal to the relative wage $\omega^i = w^i/r^i$. Supposing for simplicity that $Z^i = K^i_Z + L^i_Z$ for $i = 1, 2$, the marginal rate of technical substitution in the production of Z is equal to 1, and thus we have $\omega^i = 1$ for both i. Accordingly, each country's opportunity cost of producing an extra gun under autarky, measured in units of K (i.e., $\psi^i/r^i = \psi(\omega^i, 1)$), equals a constant that is independent of the distribution of secure resources (provided neither country's resource constraint in arming binds). This result, which implies further that each country's marginal cost of producing an additional gun is independent of its rival's arming choice, means that, as in the previous special case, the slope of each country i's best-response function depends only on the value of $B^i_A(G^j)$ relative to G^j. But, given the symmetry of the conflict technology (1), we have a symmetric equilibrium in security policies under autarky, implying once again that neither country's best-response functions exhibits strategic substitutability in the neighborhood of that equilibrium.[40] Thus, for initial distributions where neither country is resource constrained, trade dominates autarky due to the lower security costs it induces as well as the traditional gains from trade.[41]

Let us now return to the setting presented in Section 2 where L and K are imperfect substitutes in the production of both Z and G. We can establish, in this case, that there exists a set of initial distributions of secure resources (aside from the symmetric distribution) that similarly imply a symmetric equilibrium in security policies under autarky. To this end, recall that $k^i_Z \equiv K^i_Z/L^i_Z$ denotes country i' residual capital-labor ratio. Equation (6)

[38]Because $\partial B^i_A(G^j)/\partial G^j = 0$ for both $i \neq j = 1, 2$ in the autarkic equilibrium, equation (A.3) presented in the Appendix implies the equilibrium is unique.

[39]Even when the resource constraint on arming decisions binds for one or both countries, neither country's best-response function will be negatively sloped in the neighborhood of the autarkic equilibrium. Thus, arming under trade is never greater than arming under autarky, and trade weakly dominates autarky.

[40]Uniqueness of this equilibrium follows again from the fact that $\partial B^i_A(G^j)/\partial G^j = 0$ for both $i \neq j = 1, 2$ in the autarkic equilibrium and equation (A.3) in the Appendix.

[41]Binding resource constraints, however, can induce an asymmetric equilibrium in security policies under autarky. If, for example, country i absorbs its entire secure holding of labor L^i in the production of G^i, then $\omega^i > 1$ and an asymmetric equilibrium emerges under autarky with $G^{i*}_A < G^{j*}_A$. In this case, country i's best-response function will exhibit strategic substitutability in the neighborhood of the autarkic equilibrium. Thus, even with perfect substitutability of inputs in the production of Z, a binding resource constraint in the production of G where inputs are not perfect substitutes could undermine the dominance of free trade over autarky.

shows that $k_Z^i = c_r^i / c_w^i$ determines the equilibrium wage-rental ratio ω_A^i, and these ratios are identical across the two countries under the conditions of complete symmetry in the autarkic equilibrium: $k_{ZA}^i = k_{ZA}^*$ and $\omega_A^i = \omega_A^*$ for $i = 1, 2$. Now, fix each country's guns production at G_A^* and reallocate both labor and secure capital from country 2 to country 1 such that $dK^i = k_{ZA}^* dL^i$ for $i = 1, 2$, so as to leave their residual capital-labor ratios unchanged at k_{ZA}^*. (For future reference, denote the set of the resulting distributions of secure resources by $\mathcal{S}_0$.) But, since k_{ZA} does not change by assumption, the equilibrium value ω_A also remains unchanged. Thus, by the FOC under autarky (13), neither country has an incentive to change its arming with the redistribution of labor and capital. As such, for asymmetric distributions of secure resources in $\mathcal{S}_0$, equilibrium arming under autarky remains the same as when secure resources are evenly distributed across countries. Since arming also remains identical across countries, their best-response functions continue to exhibit strategic complementarity in the neighborhood of the autarkic equilibrium. Therefore, for distributions of secure endowments within $\mathcal{S}_0$ (including but not limited to the case of complete symmetry), there exists a pure-strategy equilibrium under free trade with less arming by both countries than that chosen under autarky (i.e., $G_T^{i*} < G_A^*$ for $i = 1, 2$).[42]

Now, consider a slightly different experiment. Starting from any distribution of secure resources in $\mathcal{S}_0$, transfer one unit of labor from country 2 to country 1. One can confirm this sort of transfer for given arming choices increases the relative wage and thus the marginal cost of arming in the donor country (2) and has the opposite effects in the recipient country (1).[43] Thus, by the FOC under autarky (13) along with the strict quasi-concavity of payoffs under autarky demonstrated in the proof to Proposition 1, this transfer of labor induces both countries' best-response functions under autarky to rotate clockwise, with country 1 producing more guns and country 2 producing less: $G_A^{1*} > G_A^{2*}$. The logic spelled out above, in turn, implies that country 1's best-response function continues to be positively sloped in the neighborhood of the autarkic equilibrium. Furthermore, for small transfers of labor from country 2 to country 1, country 2's best-response function also remains positively sloped. By continuity, then, both countries' best-response functions exhibit strategic complementarity for distributions adjacent to $\mathcal{S}_0$, and the result that $G_A^{i*} > G_T^{i*}$ for $i = 1, 2$ remains intact. As one can verify, transfers of secure capital this time from country 1 to country 2, again starting from any distribution of secure resources in $\mathcal{S}_0$, generate similar effects.[44] Hence, provided that the distribution of initial resource endowments across the two countries imply

[42] Note that, successive transfers of secure labor and capital resources from country 2 to country 1 within $\mathcal{S}_0$ eventually imply $\sigma - \gamma_2/\xi^2 < 0$ and thus drive arming by both countries under free trade (effectively) to zero. More generally, uneven distributions of secure resources in $\mathcal{S}_0$ imply that the two countries produce different quantities of Z^i, and such differences cause their FOC's under trade (15) to differ in equilibrium, such that $G_T^{1*} \neq G_T^{2*}$ even though $G_A^{1*} = G_A^{2*}$. For more details, see the Supplementary Appendix (B.2).

[43] See Lemma A.1(b) presented in the Appendix.

[44] See the Supplementary Appendix (B.2) for more details.

sufficiently similar residual capital-labor ratios, a shift from autarky to trade will lower equilibrium arming and thus, by Lemma 2, will bring positive gains to both countries that exceed the gains predicted by traditional models that ignore conflict.

5.3 Discussion

Our results above indicate that the validity of the notion that a shift to trade from autarky can induce lower equilibrium arming, at least in the context of the Ricardian model where comparative advantage is driven by differences in technology, depends jointly on the nature of technology and the initial distribution of secure resources. Specifically, perfect substitutability of factor inputs (i.e., L and K) in the production of the intermediate input implies that, whether the initial distribution is symmetric or asymmetric, as long as the distribution is such that neither country is resource constrained in the production of guns, a shift from autarky to trade reduces equilibrium arming by both countries and thus augments their equilibrium payoffs.[45]

However, when factor inputs are imperfect substitutes in the production of the intermediate input, a shift from autarky to free trade need not imply reduced equilibrium arming. A necessary condition for trade to induce greater arming by one country is that the mix of secure labor and capital resources held initially by the two countries be sufficiently uneven to generate large differences in the countries' arming choices under autarky that make one country's best-response function negatively related to the opponent's arming in the autarkic equilibrium.[46] But, even in this case, it is possible that a move to trade induces lower equilibrium arming by both countries. A more extreme asymmetry in secure resource endowments across countries is required for trade to induce one country to become more aggressive.

Such a possibility is illustrated in Fig. 2, which assumes that country 2 is the country having the higher capital-to-labor ratio and thus arms less under autarky than its rival.[47] Starting from the autarkic equilibrium depicted by point A in the figure, a shift to free trade causes each country's best-response function to shift inward towards the 45^{o} line, resulting in a new intersection at point T, with decreased arming by country 1 but increased arming by country 2. In Supplementary Appendix B.2.1, we demonstrate this possibility (as well

[45] As mentioned earlier, if labor and capital are also perfect substitutes in the production of guns, then arming will be no greater under trade for either country even when the resource constraints bind in the production of guns. Binding resource constraints in the production of guns when labor and capital are not perfect substitutes in such production, though, could imply greater arming under trade by one country.

[46] Observe from equation (A.5) in the Appendix, the conflict technology (1) implies it is not possible for both countries' best-response functions to be negatively sloped in the neighborhood of the autarkic equilibrium. Still, there exist extremely large transfers of labor that could induce both countries to produce less guns relative to the benchmark (symmetric) equilibrium.

[47] Observe in the figure the scale for G^1 is more concentrated such that the slope of the "45^{o}" line drawn is greater than 45^{o}.

as the converse) numerically, depending on the distribution of secure labor and capital resources across the two countries, under a specific parameterization of the model.[48]

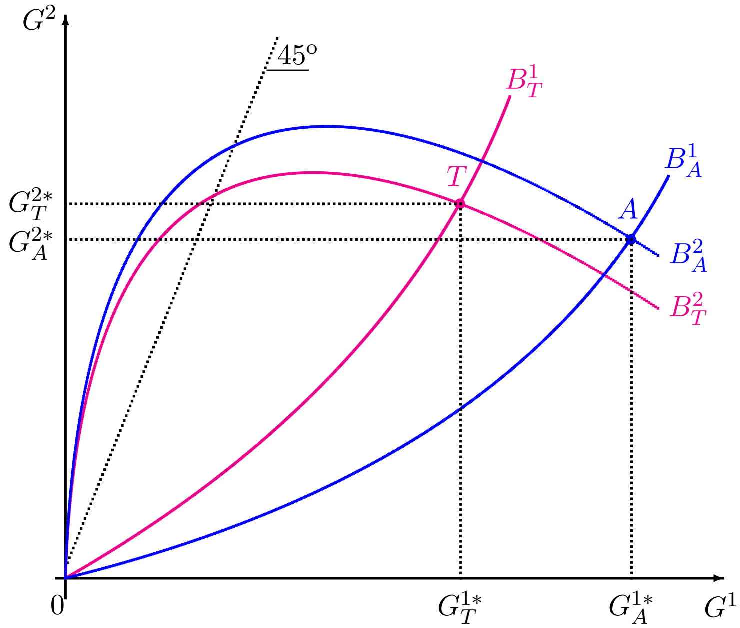

Figure 2: The Impact of Trade on Equilibrium Arming in the Presence of Sharply Asymmetric Factor Endowments

What are the welfare implications given that trade does induce one country to arm more aggressively? Although country 1 arms by more than country 2 under both autarky and trade, it arms less heavily under trade than under autarky, implying that country 2 enjoys a positive strategic welfare effect as well as the standard gains with a move from autarky to trade. The more powerful country (1) might also prefer trade over autarky despite the adverse strategic welfare effect it experiences in a move to trade. But, as demonstrated with our numerical analysis in Supplementary Appendix B.2.1, this preference ranking by the more powerful country is less likely to hold when the strength of comparative advantage and thus the standard gains from trade are relatively small.[49] Either way, the possibility that trade could induce greater arming by one country under some circumstances illustrates one potential (although perhaps remote) limit to the classical liberal view.

[48]Our numerical analysis, which assumes $\sigma > 1$, produces a unique equilibrium under free trade.

[49]Supplementary Appendix B.2.2 also analyzes the consequences of trade for power when secure resources are asymmetrically distributed. The numerical analysis shows that a shift to trade can tilt the balance of power towards the country that is less powerful under autarky, and in more extreme cases can reverse the balance of power.

6 Trading with Friends

While our analysis above provides analytical support to the notion that trade can be pacifying in international relations, we have just seen that the results cannot stand without some qualification, particularly when secure labor and capital resources are unevenly distributed across the two countries and labor and capital are imperfect substitutes in the production of the intermediate good. In this section, we explore how the nature of the trading relationship between the contending nations can matter, implying another possible limit to the optimism of the classical liberal view. In particular, in contrast to our setting above where differences in technology to produce consumption goods render trade between contending countries mutually advantageous, we now consider a setting where the structure of technology is such that the two contending countries do not trade with each other, but instead trade with a third, non-adversarial country. Extending the two-country, two-good model analyzed above to a three-country, two-good model, we show that a shift to trade can induce greater arming by both contending countries, even under the conditions of complete symmetry.

Suppose the two adversarial countries ($i = 1, 2$) are identical in all respects, including the comparative advantage they enjoy in producing good 1. In terms of the notation from (2), we assume $a_1^1 = a_1^2 = 1$ and $a_2^1 = a_2^2 \equiv a_2 > 1$, implying the relative price of good 2 under autarky in both countries ($p_A^i = p_A$ for $i = 1, 2$) satisfies $p_A = a_2$. Although only two goods are produced as before ($j = 1, 2$), there is now a third country ($i = 3$). Consumer preferences are identical worldwide, but country 3 is not involved in the contest over the capital resource. Moreover, country 3 has a comparative advantage in producing good 2: $a_1^3 > a_2^3 = 1$, which implies that its relative price of good 1 under autarky, $p_A^3 = a_1^3$, satisfies the inequality $1/p_A^3 < p_A$. Not surprisingly, the introduction of a third, friendly country has no relevance for equilibrium choices and outcomes under autarky. Most importantly, the FOC that must be satisfied for each contending country's optimal security policy under autarky is unchanged from that in the setting of the previous sections with just two countries, as shown in (13).

When trade is possible, the assumed production structure implies countries 1 and 2 export good 1 in return for imports of good 2 from country 3. To keep matters simple, we continue to assume no trade costs. In a trade equilibrium, then, all three countries face the same prices for goods 1 and 2, respectively denoted by p_1 and p_2, and thus the same relative price $p_T \equiv p_2/p_1$ for good 2, and this price balances world trade.[50] With each country specializing in the good in which it has a comparative advantage, balanced trade requires

[50] As before, we focus on the case where the world price of country 1 and 2's importable in terms of their exportable good satisfies $p_T \in (1/p_A^3, p_A)$ to abstract from the potential complications associated with discontinuities in the best-response functions. However, we briefly discuss these issues in the the proof of Proposition B.2 presented in Supplementary Appendix (B.1).

$p_2(D_2^1+D_2^2) = p_1 D_1^3$. Since the countries have identical and homothetic preferences and face identical world prices, their expenditures shares are identical. Furthermore, specialization in production implies $Y^i = p_1 X_1^i = p_1 Z^i$ for $i = 1, 2$, and $Y^3 = p_2 X_2^3 = p_2 Z^3$. It then follows that $D_2^i = \gamma_2 \frac{p_1 Z^i}{p_2}$ $(i = 1, 2)$ and $D_1^3 = \gamma_1 \frac{p_2 Z^3}{p_1}$. The world market-clearing condition, then, implies $p_T(\equiv \frac{p_2}{p_1}) = \gamma_2(Z^1 + Z^2)/\gamma_1 Z^3$, where $\gamma_2 = (p_T)^{1-\sigma}/[1 + (p_T)^{1-\sigma}]$ and $\gamma_1 = 1 - \gamma_2$. Now differentiate this expression logarithmically, while keeping the intermediate output of country 3 (Z^3) fixed in the background, and rearrange to find: $\widehat{p}_T = \frac{1}{\sigma}(\nu^1 \widehat{Z}^1 + \nu^2 \widehat{Z}^2)$, where $\nu^i \equiv Z^i/(Z^1 + Z^2)$ for $i = 1, 2$ and $\sigma > 0$.

As in the two-country case, a country's security policy under trade affects the relative price of its importable through its impact on the optimizing value of the intermediate good produced by the two adversaries, Z^{ie} for $i = 1, 2$, that satisfies (5) and (6). This effect, in turn, makes the incentive to arm trade-regime dependent.[51] Furthermore, as before, an increase in arming by country i reduces the opponent's production of the intermediate good given G^j: $dZ^{je}/dG^i < 0$ for $i \neq j = 1, 2$, as shown in (12b). The key difference here, shown in the expression for $\widehat{p}_T$, is that the decrease in Z^j improves country i's terms of trade.

To see how this difference matters in determining arming incentives, note first that each adversary's payoff function under free trade V_T^i can be written as before and as shown in (11), but now where $m_T^i = \mu(1, p_T)$ with $\frac{\partial m_T^i/\partial p_T}{m_T^i/p_T} = -\gamma_2$ for $i = 1, 2$. Then differentiate V_T^i to obtain

$$\widehat{V}_T^i = \widehat{Z}^{ie} + \left(\frac{\partial m_T^i/\partial p_T}{m_T^i/p_T}\right)\widehat{p}_T = \widehat{Z}^{ie} - \frac{\gamma_2}{\sigma}\left(\nu^i \widehat{Z}^{ie} + \nu^j \widehat{Z}^{je}\right). \tag{16}$$

Using (12) in (16), country i's FOC under free trade can be written as:

$$\frac{1}{m_T^i}\frac{\partial V_T^i}{\partial G^i} = \frac{1}{c^i}\left[\left(1 - \frac{\nu^i\gamma_2}{\sigma} + \frac{\nu^j\gamma_2}{\sigma}\left[\frac{r^j/c^j Z^{je}}{r^i/c^i Z^{ie}}\right]\right) r^i K_0 \phi_{G^i}^i - \left(1 - \frac{\nu^i\gamma_2}{\sigma}\right)\psi^i\right] \leq 0, \tag{17}$$

for $i \neq j = 1, 2$. The first term inside the outer square brackets reflects the marginal benefit of arming, whereas the second term reflects the marginal cost. Assuming $\sigma > \nu^i\gamma_2$ for $i = 1, 2$ ensures that both terms are positive for each adversary. Then, using (17) for $i = 1, 2$ and the FOC for each country i under autarky $(dZ^{ie}/dG^i = [r^i K_0 \phi_{G^i}^i - \psi^i]/c^i = 0)$, one can show the following:[52]

Proposition 3 (*Equilibrium arming under free trade with a third, friendly country.*) *Sup-*

[51] Note, if the two adversarial countries were so small that their production of the intermediate input had no influence on p_T, their security policies would not generate a terms-of-trade effect and therefore would not be trade-regime dependent. The sharp contrast of this result with that of Garfinkel et al. (2015), who focus on small adversarial countries, stems from the presence of a factor-price channel that is absent in the present analysis.

[52] As noted earlier, we assume here that $p_T \equiv p_2/p_1 \in (1/p_A^3, p_A)$.

pose the two identical adversarial countries potentially compete in the same market for exports to a third, non-adversarial country. Then, a shift from autarky to free trade induces both adversaries to arm more heavily.

Proof. See the Appendix.

The intuition is that, with trade, in addition to using its security policy to appropriate more K_0 and thus produce more Z, each country also has an interest in producing more guns at the margin to reduce its rival's output, because that improves its terms of trade.[53] Furthermore, as one can verify, an increase in the substitutability of consumption goods (σ) reduces the magnitude of this terms-of-trade effect.[54] But, with intensified conflict between the two contending countries, free trade brings higher security costs, and these higher security costs can swamp the gains from trade.[55]

7 Commitments in Trade-Regime Choices

The analysis of the previous two sections shows how the structure of comparative advantage matters not only for equilibrium arming by both contending countries, but also for the potential welfare gains under free trade relative to autarky. In this section, we briefly examine the implications for the countries' choice of trade regimes in both the two- and three-country settings. We assume that the two contestants choose their respective trade regimes simultaneously and non-cooperatively, but before arming decisions are made. Their trade-regime choices, then, naturally factor in the implications of that choice for subsequent arming.

7.1 Choosing between Autarky and Trade with the Enemy

In the two-country case, if either country chooses autarky, autarky necessarily prevails regardless of the other country's trade-regime choice. Since this is true for both countries, autarky is always a possible subgame perfect equilibrium, implying a payoff of $V_A^i(G_A^{i*}, G_A^{j*})$ for each $i \neq j = 1, 2$ as studied earlier. Now suppose one country chooses free trade. If

[53] In its quest for raw resources with an aim to match Great Britain's access and thus be better able to compete with Great Britain in the export of manufactures to third-countries, Germany invaded parts of Eastern Europe at the outset of WWII. Eventually shifting its efforts westward, Germany had hoped that it could negotiate some sort of peaceful settlement with Great Britain. But, of course, no such settlement was reached. Our analysis suggests that, insofar as Germany and Great Britain did not trade with each other, each side had an interest to fight, even if costly, for terms-of-trade (among other) reasons.

[54] In the limiting case where $\sigma = \infty$, this effect vanishes and arming incentives for both adversaries are identical across the two trade regimes.

[55] See Proposition B.2 (presented in Supplementary Appendix B.1), which establishes this possibility based on sufficiently weak comparative advantage that makes the gains from trade sufficiently small. Furthermore, we expect that admitting the possibility of trade in arms between the third friendly country and each of the two adversaries would not change our results qualitatively and, in fact, could amplify the positive effect of trade on arming incentives, implying even greater security costs and a larger likelihood of negative welfare consequences.

the sufficient condition of Lemma 2 is satisfied such that $V_T^i(G_T^{i*}, G_T^{j*}) > V_A^i(G_A^{i*}, G_A^{j*})$, the other country's best response is to choose free trade too. Thus, both autarky and free trade are possible subgame perfect equilibria of the extended game with just two contending countries. Of course, since $V_T^i(G_T^{i*}, G_T^{j*}) > V_A^i(G_A^{i*}, G_A^{j*})$ for $i \neq j = 1, 2$, the two countries would, if possible, coordinate their trade-regime choices and choose free trade.

But, if the two countries were sufficiently asymmetric to imply that one country arms more heavily under free trade and so much so that the added security costs for the other country swamp the familiar gains from trade, then the subgame perfect equilibrium of the extended game would be unique, involving no trade between the two countries. While this finding has a similar flavor to Gowa and Mansfield's (1993) argument (in a different setting) that trade between foes is disadvantageous, the logic of our argument contrasts sharply with theirs as it also implies that trade between (sufficiently symmetric) foes can be advantageous. Thus, the validity of their argument would seem to depend on the extent to which adversaries differ from one another.[56]

7.2 Choosing between Autarky and Trade with a Friend

In the three-country case where the two contending countries do not trade with each other, the choice of autarky by one of them need not force the rival to foreclose on trade. Of course, if country j chooses autarky, then country i can also choose autarky for a payoff of $V_A^i(G_A^*, G_A^*)$, as analyzed above. If country i chooses instead free trade, its arming chosen subsequently will influence its terms of trade with the friendly country. However, since this influence operates only through the amount of the contested resource it captures—not through the amount of that resource that the adversary (j) captures—the terms-of-trade effect influences country i's marginal benefit and marginal cost of arming proportionately. Thus, arming by the two countries will be identical and equal to that when they both choose autarky: G_A^*, as implicitly defined by (13) with complete symmetry imposed.[57] But, while country i's choice of trade regimes has no consequence for its security policy and thus for the security policy of its rival that remains in autarky, it does matter for its own payoff. In particular, since free trade brings the familiar gains that come with comparative advantage, country i will necessarily choose that regime given that country j chooses autarky, for a payoff of $V_T^i(G_A^*, G_A^*) > V_A^i(G_A^*, G_A^*)$.

To see when this asymmetric outcome is a stable equilibrium, immune to unilateral deviations by either country, suppose that country i chooses free trade. The relevant payoffs

[56]Our result is also related to but distinct from the important finding of Anderson and Marcouiller (2005) and Anderton et al. (1999), who analyze Ricardian models in which traded goods are insecure, that the anarchic nature of international relations can hinder world trade.

[57]To verify this claim, note that, when country i trades while country j does not, $\nu^i = 1$ and $\nu^j = 0$, and the relevant FOC (17) in this case simplifies to the FOC under autarky (13).

that inform country j's best response are $V_A^j(G_A^*, G_A^*)$ when country j alone chooses autarky (and thus $G^i = G^j = G_A^*$) and $V_T^j(G_T^*, G_T^*)$ when country j joins country i in choosing free trade. The difference in these payoffs depends on how the added costs of security under trade ($G_T^* > G_A^*$) as emphasized in Proposition 3 compare with the offsetting gains from trade given guns. If those gains outweigh the higher security costs, then $V_T^j(G_T^*, G_T^*) > V_A^j(G_A^*, G_A^*)$, such that the subgame perfect equilibrium of the extended game is unique and symmetric, involving free trade for each contending country with the third, friendly country. However, the added security costs can outweigh those gains to reverse the relative ranking of payoffs, such that $V_T^j(G_T^*, G_T^*) < V_A^j(G_A^*, G_A^*)$.[58] In this case, the subgame perfect equilibrium is necessarily asymmetric: country i trades freely with the third country and obtains a higher payoff than its rival (country j) that forecloses on trade.

Whether the subgame perfect equilibrium of the extended game is symmetric or asymmetric, each adversary would have an incentive to block the rival country's trade with the friendly country, not only to reap the full gains from trade, but also to reduce equilibrium arming. Meanwhile, the third, friendly country might prefer to trade with just one of the two contending countries. To be sure, trading with them both, for fixed arming, necessarily implies a more favorable terms of trade for the third country. However, the added arming by the contending countries implied when the third country trades with them both would have an adverse effect on its own terms of trade, and this latter effect could dominate. Although beyond the scope of the present paper, one could analyze in this three-country setting the incentives of the third, friendly country's to impose an embargo on one of the two contending countries, particularly when the two contending countries both choose free trade in the equilibrium of the extended game.

The possibility of an asymmetric outcome emerging as the equilibrium in a setting populated by otherwise identical countries is interesting in its own right, as it suggests like Matsuyama (2002) that the opportunity for trade with a third, friendly country could be "symmetry-breaking" in a global setting and thus helps us understand, at least partly, the diversity of economic performance across countries. But, whereas agglomeration effects are a key element in Matsuyama (2002), the presence of insecure resources is key in the present analysis.

8 Concluding Remarks

This paper develops a Ricardian-type model of trade where comparative advantage is driven by differences in technology. The model, augmented by conflict between two large countries over a productive resource, features a terms-of-trade channel that makes arming choices trade-regime dependent. Specifically, a country's arming under trade has an additional

[58]See Proposition B.2, presented in the Supplemetary Appendix (B.1).

effect on its own payoff, by influencing the adversary's production and thus international prices. How trade influences arming incentives depends on the structure of comparative advantage.

When the two countries in conflict also trade with each other, the terms-of-trade effect of arming is negative. Provided the two countries are sufficiently symmetric, not only in terms of technologies and preferences, but also in terms of the mix of their secure resource endowments, equilibrium arming by both is lower and their payoffs are higher under trade than under autarky. These results provide theoretical support to the long-standing classical liberal hypothesis that increased trade openness can ameliorate conflict and thus amplify the gains from trade. By contrast, with sufficiently extreme differences in the distribution of the primary resources, a shift to trade induces one country to arm more heavily, implying that autarky might be preferable to the other country.[59]

When the structure of comparative advantage is such that the two adversaries do not trade with each other, but instead trade with a third, friendly country and they compete in the same export market, the terms-of-trade effect of security policies is positive. As such, a shift from autarky to trade unambiguously intensifies international conflict, possibly with negative net welfare consequences. This finding is similar to Martin et al. (2008) who show that increasing opportunities for multilateral trade can aggravate bilateral conflict. Our analysis, however, captures not only the possible gains from trade, but also the endogeneity of arming and thus security costs.[60]

The analysis' welfare implications naturally raise the question of how security considerations can influence the endogenous choice of trade policy.[61] We have already seen how the non-cooperative choice of trade regimes can interact with subsequently chosen non-cooperative security policies. One interesting and important extension left for future research involves an exploration of the countries' non-cooperative choice of (continuous) revenue-generating tariff policies, when chosen simultaneously with security policies. In our setting given arming decisions, as in typical trade policy games absent security concerns, each country's optimal tariff would balance a favorable terms-of-trade effect against an un-

[59]The analysis could be extended to capture the presence of a non-tradable goods sector—a salient feature of the world economy. In ongoing research we have shown that, even in the case of complete symmetry, different factor intensities across the tradable and non-tradable sectors can cause the classical liberal view to fail. One can also show that, in Heckscher-Ohlin settings with two large countries (one relatively more endowed with capital and the other having relatively more labor), the terms-of-trade channel would vanish under circumstances where factor-price equalization holds. Nonetheless, the factor-price channel preserves the trade-regime dependency of security policies, and in fact trade tends to induce greater arming by one country and less by the other, and therefore could make the latter country worse off. It is worthwhile noting, however, that a $2 \times 2 \times 2$ Heckscher-Ohlin model with complete specialization would imply terms-of-trade effects of arming similar to those implied by our modified Ricardian model.

[60]One could move our three-country analysis closer to the multi-country world of Martin et al. (2008) by extending the model along the lines of Eaton and Kortum (2002).

[61]Hirschman (1945) wrote long ago that trade policy can serve as an extension of security policy.

favorable reduction in the volume of trade and, thus, would depend on the price elasticities of import demand that provide a measure of the exporting country's market power in world trade. One would expect that the presence of security concerns matters in this context, depending specifically on how security policies (given tariffs) influence those price elasticities. Although a formal analysis is beyond the scope of the present paper, our analysis above indicates that equilibrium world prices depend on arming choices whether or not tariffs are present. Thus, provided non-cooperative tariff policies do not shut down trade between two contending countries entirely, the terms-of-trade effect of security policies will not vanish.[62] We also expect to find, in the three-country case, the possibility of an asymmetric equilibrium in the choice of trade policies. That is to say, once one country trades with the third, friendly one with or without tariffs, the adversary might have no incentive to trade at all, particularly if doing so creates sufficiently greater security costs that swamp the gains from trade.

Another potentially fruitful avenue for future research concerns the political economy implications of the model. In our two-country model, security policies can have different welfare implications for different factor owners when intermediate-goods production uses labor and capital in different proportions than guns production. Thus, one might consider the possibility that factor owners express their differing interests, say through lobbying, to influence the formulation of security policies. Such an extension will likely offer new insights on how differing domestic interests could affect the domestic distribution of income as well as external conflict.

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[62]This implication holds even when countries can coordinate their tariff policies but not their security policies.

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A Appendix

Lemma A.1 *For any given feasible quantities of guns and gross factor endowments, the equilibrium wage-rental ratio ω^{ie} is independent of the prevailing trade regime (autarky or trade). Furthermore,*

(a) $\partial Z^i(\omega^{ie},\cdot)/\partial\omega^i = 0$ *and* $\partial^2 Z^i(\omega^{ie},\cdot)/\partial(\omega^i)^2 > 0$*, s.t.* $\omega^{ie} = \arg\min_{\omega^i} Z^i(\omega^i,\cdot)$;

(b) $\partial\omega^{ie}/\partial K_g^i > 0$*,* $\partial\omega^{ie}/\partial L_g^i < 0$*, and* $\partial\omega^{ie}/\partial G^i \gtreqless 0$ *if* $c_r^i/c_w^i \gtreqless \psi_r^i/\psi_w^i$;

(c) $\partial Z^{ie}/\partial K_g^i > 0$ *and* $\partial^2 Z^{ie}/(\partial K_g^i)^2 < 0$;

(d) $\partial Z^{ie}/\partial L_g^i > 0$ *and* $\partial^2 Z^{ie}/(\partial L_g^i)^2 < 0$;

(e) $\partial Z^{ie}/\partial G^i < 0$ *and* $\partial^2 Z^{ie}/(\partial G^i)^2 < 0$ *for fixed* K_g^i.

Proof: See the Supplementary Appendix B.1.

Proof of Proposition 1. Let $\overline{G}^i$ be the quantity of guns country i would produce if all of L^i and K^i were employed in that sector. Assuming $f(0)$ is arbitrarily close to 0 implies that $\lim_{G^i\to 0} f'(G^i)$ and thus $\lim_{G^i\to 0}\phi_{G^i}^i$ in (13) are arbitrarily large for any $G^j \geq 0$. Therefore, $\partial V_A^i/\partial G^i > 0$ as $G^i \to 0$. Furthermore, by the definition of $\overline{G}^i$ and our assumption that both factors are essential in the production of Z^i, $V_A^i(\overline{G}^i, G^j) < V_A^i(G^i, G^j)$ for all $G^i < \overline{G}^i$ that imply $Z^{ie} > 0$; therefore, $\partial V_A^i/\partial G^i < 0$ for a sufficiently large $G^i \in [0, \overline{G}^i]$. The continuity of V_A^i in G^i, then, implies that there exists a best-response function for each country i, $G^i = B_A^i(G^j) \in (0, \overline{G}^i)$, such that (13) holds as an equality.

Existence: As in Garfinkel et al. (2015), it suffices to show $\partial^2 V_A^i/\left(\partial G^i\right)^2 < 0$ at $G^i = B_A^i(G^j)$ to establish existence. In the proof of Lemma A.1 (presented in the Supplementary Appendix (B.1)), we establish the following effects of marginal changes in w^i, K_g^i and G^i on Z_w^{ie}:

$$\begin{aligned}
Z_{ww}^{ie} &= -(c_{ww}^i Z^{ie} + \psi_{ww}^i G^i)/c^i > 0\\
Z_{wK}^{ie} &= -r^i c_w^i/(c^i)^2 < 0\\
Z_{wG^i}^{ie} &= -(\psi_w^i c^i - c_w^i \psi^i)/(c^i)^2.
\end{aligned}$$

Then, while keeping r^i fixed in the background (thus attributing any implied changes in ω^i to changes in w^i) and using the FOC associated with B_A^i in (13), an application of the implicit function theorem to the envelope condition $Z_\omega^{ie}(\omega^{ie},\cdot) = 0$ shows:

$$w_{G^i}^i\big|_{G^i=B_A^i} = -\frac{K_0 Z_{wK}^{ie}\phi_{G^i}^i + Z_{wG^i}^{ie}}{Z_{ww}^{ie}} = -\frac{\psi_w^i}{c_{ww}^i Z^{ie} + \psi_{ww}^i G^i} > 0. \tag{A.1}$$

Since $\psi_w^i > 0$, the concavity of the unit cost functions in factor prices (i.e., $c_{ww}^i, \psi_{ww}^i < 0$) implies, in turn, that an increase in G^i in the neighborhood of B_A^i increases the country's market-clearing (relative) wage regardless of factor intensities.

Differentiating country i's FOC in (13) with respect to G^i, after simplifying and using (A.1), shows

$$\left.\frac{\partial^2 V_A^i}{(\partial G^i)^2}\right|_{G^i=B_A^i} = \frac{m_A^i}{c^i}\left[r^i K_0 \phi_{G^i G^i}^i + \frac{\left(\psi_w^i\right)^2}{c_{ww}^i Z^{ie} + \psi_{ww}^i G^i}\right] < 0. \tag{A.2}$$

The negative sign of (A.2) follows from the concavity of the conflict technology in G^i (i.e., $\phi_{G^i G^i}^i < 0$) and equation (A.1). The strict quasi-concavity of V_A^i in G^i, in turn, implies the existence of an interior equilibrium.

Uniqueness: To prove uniqueness of equilibrium, it suffices to show that

$$|J| \equiv \frac{\partial^2 V_A^i}{(\partial G^i)^2}\frac{\partial^2 V_A^j}{(\partial G^j)^2} - \frac{\partial^2 V_A^i}{\partial G^i \partial G^j}\frac{\partial^2 V_A^j}{\partial G^j \partial G^i} > 0, \tag{A.3}$$

at any equilibrium point or, equivalently,

$$\frac{\partial B_A^i}{\partial G^j}\frac{\partial B_A^j}{\partial G^i} < 1, \text{ where } \quad \frac{\partial B_A^i}{\partial G^j} = -\frac{\partial^2 V_A^i}{\partial G^i \partial G^j}\Bigg/\frac{\partial^2 V_A^i}{\left(\partial G^i\right)^2} \quad \text{for } i \neq j = 1, 2.$$

From the existence part of this proof, the sign of $\partial B_A^i/\partial G^j$ is determined by the sign of $\partial^2 V_A^i/\partial G^i \partial G^j$. To proceed, apply the implicit function theorem to $Z_\omega^{ie}(\omega^{ie}, \cdot) = 0$ to find the following:

$$\left.w_{G^j}^i\right|_{G^i=B_A^i} = -\frac{K_0 Z_{wK}^{ie}\phi_{G^j}^i}{Z_{ww}^{ie}} = -\frac{r^i K_0 \phi_{G^j}^i c_w^i}{c^i(c_{ww}^i Z^{ie} + \psi_{ww}^i G^i)} < 0, \tag{A.4}$$

where the negative sign follows from the facts that $c_w^i > 0$, $c_{ww}^i, \psi_{ww}^i < 0$, and $\phi_{G^j}^i < 0$. Differentiating (13) with respect to G^j, while using (A.4), gives

$$\left.\frac{\partial^2 V_A^i}{\partial G^i \partial G^j}\right|_{G^i=B_A^i} = \frac{m_A^i}{c^i}\left[r^i K_0 \phi_{G^i G^j}^i + \frac{r^i K_0 \phi_{G^j}^i c_w^i \psi_w^i}{c^i(c_{ww}^i Z^{ie} + \psi_{ww}^i G^i)}\right]. \tag{A.5}$$

The second term inside the square brackets is positive by (A.4). From (1), the first term is non-negative for all $B_A^i(G^j) \geq G^j$, implying that the expression above is positive, and thus G^i depends positively on G^j. However, G^i will depend negatively on G^j if $B_A^i(G^j)$ is sufficiently smaller than G^j. Thus, depending on fundamentals, two possibilities arise: (i) $\partial B_A^i/\partial G^j < 0$ and $\partial B_A^j/\partial G^i \geq 0$ for $i \neq j = 1, 2$; or (ii) $\partial B_A^i/\partial G^j > 0$ for both $i \neq j = 1, 2$. However, $|J| > 0$ in case (i), implying we need to consider only case (ii).

In the spirit of Jones (1965), let $\theta_{KZ}^i \equiv r^i c_r^i/c^i$ and $\theta_{LZ}^i \equiv w^i c_w^i/c^i$ denote respectively the cost shares of K^i and L^i in producing Z^i. Similarly let $\theta_{KG}^i \equiv r^i \psi_r^i/\psi^i$ and $\theta_{LG}^i \equiv$

$w^i\psi^i_w/\psi^i$ denote the corresponding cost shares in the production of G^i. In addition, let $\sigma^i_Z \equiv c^i c^i_{wr}/c^i_w c^i_r$ and $\sigma^i_G \equiv \psi^i\psi^i_{wr}/\psi^i_w\psi^i_r$ denote the elasticities of substitution between factor inputs in sectors Z^i and G^i respectively. Lastly, define

$$\lambda^i \equiv \frac{\theta^i_{LG}\psi^i G^i}{\theta^i_{LG}\psi^i G^i\theta^i_{KG}\sigma^i_G + \theta^i_{LZ}c^i Z^{ie}\theta^i_{KZ}\sigma^i_Z} > 0. \tag{A.6}$$

Applying the above definitions and country i's FOC (13), while using the linear homogeneity of c^i and ψ^i, allows us to rewrite (A.2) and (A.5) as

$$\left.\frac{\partial^2 V^i_A}{(\partial G^i)^2}\right|_{G^i=B^i_A} = -\frac{m^i_A\psi^i\phi^i_{G^i}}{c^i\phi^i}\left[-\frac{\phi^i\phi^i_{G^iG^i}}{\phi^i_{G^i}\phi^i_{G^i}} + \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LG}\right] \tag{A.7a}$$

$$\left.\frac{\partial^2 V^i_A}{\partial G^i\partial G^j}\right|_{G^i=B^i_A} = -\frac{m^i_A\psi^i\phi^i_{G^j}}{c^i\phi^i}\left[-\frac{\phi^i\phi^i_{G^iG^j}}{\phi^i_{G^i}\phi^i_{G^j}} + \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LZ}\right]. \tag{A.7b}$$

Combining these equations gives $\partial B^i_A/\partial G^j = (-\phi^i_{G^j}/\phi^i_{G^i})H^i$, where

$$H^i \equiv \frac{H^i_1}{H^i_2} \equiv \left[-\frac{\phi^i\phi^i_{G^iG^j}}{\phi^i_{G^i}\phi^i_{G^j}} + \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LZ}\right]\Bigg/\left[-\frac{\phi^i\phi^i_{G^iG^i}}{\phi^i_{G^i}\phi^i_{G^i}} + \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LG}\right]. \tag{A.8}$$

Since $(-\phi^i_{G^j}/\phi^i_{G^i})(-\phi^j_{G^i}/\phi^j_{G^j}) = 1$, we have $(\partial B^i_A/\partial G^j)(\partial B^j_A/\partial G^i) = H^i H^j$. Given our focus on the case of strategic complements (which implies $H^i > 0$ for $i = 1, 2$), establishing $|J| > 0$ requires only that $H^i < 1$. In the Supplementary Appendix (B.1), we establish that a sufficient (but hardly necessary) condition is that σ^i_G and/or σ^i_Z are not too much smaller than one. $||$

Proof of Proposition 2. Assuming zero trade costs, the world relative price of country i's importable good is identical to the domestic relative price: p^i_T. While p^i_T is endogenously determined as a function of G^i and G^j, it must satisfy $p^i_T = p^i_T \in \left[1/\alpha^j, \alpha^i\right]$. Our proof to follow abstracts from the boundary conditions on p^i_T. As such, it demonstrates the existence only of a local optimum for each country i, given G^j.[63] In the Supplementary Appendix (B.1), we return to this issue, showing how weak comparative advantage (i.e., low values of α^i) matters and identifying the existence of a critical value of α^i for each country i, denoted by α^i_{0T} (> 1), such that $\alpha^i > \alpha^i_{0T}$ for both i ensures that the boundary constraints on p^i_T do not bind in the equilibrium identified here.

Let $\widetilde{V}^i_T(G^i, G^j)$ denote the unconstrained value function for country i under free trade

[63] Our abstraction can be thought of as assuming that $\alpha^i \to \infty$ for $i = 1, 2$, which effectively reduces the model to one in which each country produces a nationally differentiated good, as is the case of the Armington (1969) model. Although convenient for ruling out possible discontinuities in the best-response functions, this simplification fails to capture the rich welfare implications we identify in our modified Ricardian model.

(i.e., abstracting from the limits on p_T^i), and recall our definition of $\overline{G}^i$, as the maximum quantity of arms produced by country i using all of K^i and L^i. Assuming both labor and capital are essential in producing Z^i, $\widetilde{V}_T^i(\overline{G}^i, G^j) < \widetilde{V}_T^i(G^i, G^j)$ for all $G^i < \overline{G}^i$ that imply $Z^{ie} > 0$. Therefore, $\partial\widetilde{V}_T^i/\partial G^i < 0$ for sufficiently large $G^i \in [0, \overline{G}^i]$. If $\sigma \leq \overline{\sigma}^i \equiv \gamma_j/\xi^j$ evaluated at $G^i = G^j = 0$ so that the marginal benefit of arming is non-positive at $G^i = 0$, the conflict technology (1) implies $\widetilde{V}_T^i(G^i, G^j)$ reaches a peak in the domain $[0, \overline{G}^i]$ at $G^i = 0$ for $G^j \geq 0$.[64] Alternatively, if $\sigma > \overline{\sigma}^i$ so that the marginal benefit of arming is strictly positive at $G^i = 0$, then $\widetilde{V}_T^i(G^i, G^j)$ reaches a peak in the interior of the domain. In this case, equation (1) and the continuity of $\widetilde{V}_T^i(G^i, G^j)$ in G^i imply the existence of an unconstrained best-response function for country i (i.e., ignoring the constraints on p_T^i), denoted by $\widetilde{B}_T^i(G^j)$, such that the FOC (15) holds with equality.

Without loss of generality, assume that $\overline{\sigma}^1 < \overline{\sigma}^2$. Then, since the above holds true for both countries $i = 1, 2$, we have three possibilities to consider:

(i) $\sigma \leq \overline{\sigma}^1$, which implies $\partial\widetilde{V}_T^i/\partial G^i|_{G^i=0} < 0$ for $i = 1, 2$;
(ii) $\sigma \in (\overline{\sigma}^1, \overline{\sigma}^2]$ which implies $\partial\widetilde{V}_T^1/\partial G^1|_{G^1=0} < 0$, while $\partial\widetilde{V}_T^2/\partial G^2|_{G^2=0} > 0$;
(iii) $\sigma > \overline{\sigma}^2$, which implies $\partial\widetilde{V}_T^i/\partial G^i|_{G^i=0} > 0$ for $i = 1, 2$.

In case (i), there exists an equilibrium where each country i chooses $G_T^i = 0$. Similarly, $G_T^i = 0$ in case (ii) for country i that has $\partial\widetilde{V}_T^i/\partial G^i < 0$ at $G^i = 0$ for $G^j = 0$. By contrast, in case (ii) for the other country $j \neq i$ and in case (iii) for both countries i, the marginal net benefit from producing arms, when evaluated at zero arming, is strictly positive. Hence, to establish the existence of a pure-strategy equilibrium in security policies under trade $(G_T^1, G_T^2) \neq (G_A^1, G_A^2)$, assuming that α^i is arbitrarily large for both i, we need to prove only that $\widetilde{V}_T^i(G^i, G^j)$ is strictly quasi-concave in G^i for country i when $\partial\widetilde{V}_T^i/\partial G^i = 0$ at $G^i = \widetilde{B}_T^i(G^j) > 0$ given $G^j \geq 0$, in cases (ii) and (iii).[65] We present this part of the proof in the Supplementary Appendix (B.1). ||

Proof of Lemma 2. To confirm that a shift from autarky to free trade induces a positive welfare effect that exceeds the traditions gains from trade (i.e., given guns) provided trade induces lower arming, we decompose the changes in payoffs into three parts. First, starting from the equilibrium under autarky, allow country j to reduce its guns from G_A^{j*} to $G_T^{j*} \geq 0$.

[64] As suggested in the main text, we cannot rule out the possibility that there exists another local maximum for $G^i \in (0, \overline{G})$.

[65] Note that, by the continuity of our specification of $\phi^i = \phi(G^i, G^j)$ in (1) at $G^1 = G^2 = 0$, with an appropriate parameterization of this conflict technology (e.g., in terms of the parameters δ and b for the example shown in footnote 16), we can ensure that $\sigma - \gamma_2/\xi^2$ remains negative when evaluated at country 2's best response to $G^1 = 0$ for case (ii).

For such moves along $B_A^i(G^j)$, only the strategic effect on country i's payoff matters:

$$\frac{1}{V_A^i}\frac{\partial V_A^i}{\partial(-G^j)}\bigg|_{G^i=B_A^i(G^j)} = -\frac{dZ^{ie}/dG^j}{Z^{ie}} > 0,$$

which implies $V_A^i(B_A^i(G_T^{j*}), G_T^{j*}) > V_A^i(G_A^{i*}, G_A^{j*})$. Second, consider the welfare effect of a shift to trade, given $G^i = B_A^i(G_T^{j*})$ and $G^j = G_T^{j*}$. From Lemma 1, such a shift implies a non-negative welfare effect: $V_T^i(B_A^i(G_T^{j*}), G_T^{j*}) \geq V_A^i(B_A^i(G_T^{j*}), G_T^{j*})$. Third, country i's shift to trade induces it to adjust its arming, from $B_A^i(G^j)$ to $B_T^i(G^j)$ given $G^j = G_T^{j*} \geq 0$. This adjustment also produces a non-negative welfare effect, since $V_T^i(B_T^i(G_T^{j*}), G_T^{j*}) \geq V_T^i(B_A^i(G_T^{j*}), G_T^{j*})$. Bringing these results together implies $V_T^i(G_T^{i*}, G_T^{j*}) > V_A^i(G_A^{i*}, G_A^{j*})$ for $i \neq j = 1, 2$. Since the first part of our decomposition is strictly positive (under the assumption that arming by the opponent falls under trade relative to autarky) and the third part is non-negative, each country's gain will exceed those predicted by models that abstract from conflict as captured by the second part.

Finally, observe that the proof of Proposition 2 indicates that country i's payoff under free trade is comparable to its payoff under autarky for any given $G^j \geq 0$, provided that α^i is sufficiently large. Thus, we need to consider only circumstances where the lower price constraint $p_T^i \geq 1/\alpha^j$ binds for country i. Fix G^j at some level, and suppose that $p_T^i = 1/\alpha^j$ for some value of G^i—call it G_c^i. Because the unconstrained optimal value of G^i level is less than G_c^i, we know that $\widetilde{V}_T^i$ and thus V_T^i rise as G^i approaches G_c^i from above. But, V_T^i also rises as G^i approaches G_c^i from below, since the optimal value of G^i at constant prices equals $B_A^i(G^j)$ which exceeds G_c^i, given G^j. Thus, V_T^i reaches a (kinked) peak at G_c^i. ||

The next three propositions deal with the special case of complete symmetry, but also allow for trade costs. Their proofs are presented in Supplementary Appendix B.1.

Proposition A.1 (*Equilibrium security policies under trade with trade costs and complete symmetry.*) *Suppose the conditions of complete symmetry are satisfied and that labor and capital are sufficiently substitutable in the production of arms and/or the intermediate good. Furthermore, assume trade costs are identical across countries and sufficiently low to allow for trade flows, and that each country's comparative advantage (α^i) is sufficiently strong. Then, there exists a unique, symmetric equilibrium in security policies under trade involving less arming than in the symmetric equilibrium under autarky: $G_T^{i*} = G_T^*$ for $i = 1, 2$, such that $G_T^* = 0$ if $\sigma \leq 1$ and $G_T^* \in (0, G_A^*)$ if $\sigma \in (1, \infty)$.*

Proposition A.2 (*Equilibrium arming and the elasticity of substitution.*) *Under the conditions of Proposition A.1 with $\sigma > 1$, an increase in the elasticity of substitution in consumption (σ) induces greater arming under trade (i.e., $dG_T^*/d\sigma > 0$). As σ approaches*

∞, *equilibrium arming under trade approaches equilibrium arming under autarky* (*i.e.*, $\lim_{\sigma\to\infty} G_T^* = G_A^*$).

Proposition A.3 (*Trade costs under complete symmetry.*) *Under the conditions of Proposition A.2, reciprocal reductions in physical trade barriers* (*globalization*) *and in tariffs* (*trade liberalization*) *reduce arming and amplify the total gains from trade.*

Proof of Proposition 3. Assume a unique, interior equilibrium in security policies exists under free trade. Given our assumption that the two contending countries are identical in all respects means that, when $G^i = G$, $Z^{1e} = Z^{2e}$, so that $\nu^i = \frac{1}{2}$, $w^i = w$, $r^i = r$, and $c^i = c$ for $i = 1, 2$. Provided the two consumption goods are sufficiently substitutable to ensure positive marginal costs of arming ($\sigma - \frac{1}{2}\gamma_2 > 0$), the FOCs under trade (17) can be written as

$$\operatorname{sign}\left\{ \left.\frac{\partial V_T^i}{\partial G^i}\right|_{G^i=G} \right\} = \operatorname{sign}\left\{ ArK_0\phi^i_{G^i}|_{G^i=G} - \psi \right\},$$

for $i = 1, 2$, where $A = 1 + \frac{\gamma_2/2\sigma}{1-\gamma_2/2\sigma} > 1$, for $\sigma < \infty$. Recall that the FOC for country i's arming choice under autarky (13) requires $dZ^i/dG^i = 0$ or equivalently $rK_0\phi^i_{G^i}|_{G^i=G_A^*} - \psi = 0$. Since $A > 1$, the sign of the FOC under trade when evaluated at $G^i = G_A^*$ for $i = 1, 2$ is strictly positive. The result then follows from the implicit function theorem. ||

B Supplementary Appendix (For Online Publication Only)

B.1 Additional Propositions and Proofs

This supplementary appendix provides additional propositions and proofs. It also contains some of the more technical parts of the proofs presented in Appendix A.

Proof of Lemma A.1. That the equilibrium wage-rental ratio ω^{ie} is independent of the trade regime in place, for given gross endowments and guns, follows directly from the equilibrium condition in (6).

To establish parts (a)–(e) of the lemma, let us temporarily omit country superscripts and recall that

$$Z(\omega, K_g, L_g, G) = \frac{wL_g + rK_g - \psi G}{c(w,r)} = \frac{\omega L_g + K_g - \psi(\omega,1)G}{c(\omega,1)}. \tag{B.1}$$

Now differentiate Z with respect to w, K_g, L_g, and G to obtain the following (after some algebra):[1]

$$\begin{aligned}
\frac{\partial Z}{\partial w} &\equiv Z_w = \left[\frac{rc_w}{c^2}\right](L_g - \psi_w G)\left[\frac{c_r}{c_w} - \frac{K_g - \psi_r G}{L_g - \psi_w G}\right] && \text{(B.2a)}\\
\frac{\partial Z}{\partial K_g} &\equiv Z_K = r/c && \text{(B.2b)}\\
\frac{\partial Z}{\partial L_g} &\equiv Z_L = w/c && \text{(B.2c)}\\
\frac{\partial Z}{\partial G} &\equiv Z_G = -\psi/c. && \text{(B.2d)}
\end{aligned}$$

Taking derivatives of the above expressions with respect to w shows

$$\begin{aligned}
Z_{ww} &= -(c_{ww}Z + \psi_{ww}G)/c > 0 && \text{(B.3a)}\\
Z_{Kw} &= -rc_w/c^2 < 0 && \text{(B.3b)}\\
Z_{Lw} &= rc_r/c^2 > 0 && \text{(B.3c)}\\
Z_{Gw} &= -\frac{\psi_w c - c_w\psi}{c^2} = \frac{r(\psi_r c_w - \psi_w c_r)}{c^2} = \frac{r\psi_w c_w}{c^2}\left(\frac{\psi_r}{\psi_w} - \frac{c_r}{c_w}\right). && \text{(B.3d)}
\end{aligned}$$

Part (a): As can be seen from (B.1), to trace the role of ω it is sufficient to consider the impact of variations in w keeping r fixed in the background. The expression inside the last set of square brackets in (B.2a) coincides with the domestic market-clearing condition (6) that produces the solution ω^e; therefore, $Z_w = 0$ at $\omega = \omega^e$ or, equivalently, $Z_\omega(\omega^e, \cdot) = 0$. Thus, the first component of this part of the lemma follows from (B.2a). The second

[1] We abstract here from the dependence of K_g on guns by treating gross factor endowments as exogenous; however, we explicitly consider this dependence in our analysis of endogenous security policies.

component follows from the sign of the expression for Z_{ww} in (B.3a), as implied by (5), (6), the linear homogeneity of the unit costs functions ψ and c, as well as their concavity in factor prices. That Z^e is strictly quasi-convex in ω and minimized at ω^e implies that Z^e is the envelope function of $Z(\omega, \cdot)$, which is useful for establishing the remaining parts of the lemma and for our subsequent characterization of equilibria in security policies.

Part (b): This part follows from the implicit function theorem applied to $Z^e_w = 0$, while using (B.3). The first two components conform to intuition that an increase in the relative supply of one factor (capital or labor) decreases its relative price in factor markets. The last component points out that, if the technology for guns is labor intensive (i.e., $c_r/c_w > \psi_r/\psi_w$), then, at constant factor prices, production of an additional gun will increase the demand for labor relative to capital, thus forcing the market-clearing wage-rental ratio to rise.

Parts (c)–(e): The proof of the first component of parts (c), (d) and (e) follows readily from (B.2b)–(B.2d) with $Z^e_\omega = 0$ and the fact that $dZ^e = Z^e_\omega d\omega^e + Z^e_K dK_g + Z^e_L dL_g + Z^e_G dG$. The strict concavity of Z^e in, say, K_g can be proven as follows. First, observe that $dZ^e/dK_g = Z^e_\omega(d\omega^e/dK_g) + Z^e_K$. Now differentiate this expression with respect to K_g to find

$$\frac{d^2 Z^e}{dK_g^2} = Z^e_{\omega\omega}\left(\frac{d\omega^e}{dK_g}\right)^2 + 2Z^e_{\omega K}\frac{d\omega^e}{dK_g} + Z^e_\omega \frac{d^2\omega^e}{dK_g^2} + Z^e_{KK}.$$

Since $Z^e_\omega = 0$, $Z^e_{KK} = 0$, and $d\omega^e/dK_g = -Z^e_{\omega K}/Z^e_{\omega\omega}$, the expression above simplifies as

$$\frac{d^2 Z^e}{dK_g^2} = -\frac{(Z^e_{\omega K})^2}{Z^e_{\omega\omega}} < 0,$$

giving us the desired result. The strict concavity of Z^e in L_g and in G in parts (d) and (e) respectively can be shown along the same lines. Parts (c) and (d) state that Z^e is increasing and strictly concave in the country's gross factor endowments. Part (e) establishes that, given K_g, Z^e is decreasing and strictly concave in G. ||

Proof of Proposition 1 continued. To derive the sufficient conditions for uniqueness of the equilibrium in security policies under autarky as stated in the proposition, we demonstrate that the expression for $H^i \equiv H^i_1/H^i_2$ shown in equation (A.8) of Appendix A is less than one. Because the numerator (H^i_1) and the denominator (H^i_2) are both positive, we can subtract the former from the latter to obtain

$$C^i = H^i_2 - H^i_1 = -\frac{\phi^i\phi^i_{G^iG^i}}{\phi^i_{G^i}\phi^i_{G^i}} + \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LG} - \left(-\frac{\phi^i\phi^i_{G^iG^j}}{\phi^i_{G^i}\phi^i_{G^j}} + \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LZ}\right).$$

If $C^i > 0$, then $H^i < 1$ holds. The properties of $\phi(G^i, G^j)$ imply

$$
\begin{aligned}
-\frac{\phi^i \phi^i_{G^iG^i}}{\phi^i_{G^i}\phi^i_{G^i}} &= \frac{1}{\phi^j}\left[2\phi^i - \frac{f(G^i)f''(G^i)}{f'(G^i)f'(G^i)}\right] > 0 \quad (\text{since } f'' \leq 0) \\
-\frac{\phi^i \phi^i_{G^iG^j}}{\phi^i_{G^i}\phi^i_{G^j}} &= \frac{\phi^i}{\phi^j} - 1.
\end{aligned}
$$

Using the above observations in C^i after rearranging terms gives

$$
C^i = \frac{1}{\phi^j}\left[\phi^i - \frac{f(G^i)f''(G^i)}{f'(G^i)f'(G^i)}\right] + \left[1 - \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LZ}\right] + \frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LG}.
$$

A sufficient (but hardly necessary) condition for $C^i > 0$ is that the expression inside the second set of square brackets is non-negative. The definition of λ^i in (A.6) and the FOC under autarky (13) together imply after tedious algebra

$$
\begin{aligned}
\frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LZ} &= \frac{r^i\phi^i K_0\lambda^i}{\psi^i G^i}\theta^i_{LZ} = \frac{\theta^i_{LG}\theta^i_{LZ}\left(r^i\phi^i K_0\right)}{\sigma^i_G\theta^i_{LG}\theta^i_{KG}\psi^i G^i + \sigma^i_Z\theta^i_{LZ}\theta^i_{KZ}c^i Z^{ie}} \\
&= \frac{\theta^i_{LG}\theta^i_{LZ}\left(r^i\phi^i K_0\right)}{M^i + \theta^i_{LG}\theta^i_{KG}\psi^i G^i + \theta^i_{LZ}\theta^i_{KZ}c^i Z^{ie}}. \qquad \text{(B.4)}
\end{aligned}
$$

where $M^i \equiv (\sigma^i_G - 1)\theta^i_{LG}\theta^i_{KG}\psi^i G^i + (\sigma^i_Z - 1)\theta^i_{LZ}\theta^i_{KZ}c^i Z^{ie}$. Solving for G^i and Z^{ie} from (3a) and (3b) and using these expressions in gives:

$$
\begin{aligned}
\psi^i G^i &= \frac{\theta^i_{KZ}w^i L^i - \theta^i_{LZ}r^i\left(K^i + \phi^i K_0\right)}{\theta^i_{KZ} - \theta^i_{KG}} \\
c^i Z^{ie} &= \frac{-\theta^i_{KG}w^i L^i + \theta^i_{LG}r^i\left(K^i + \phi^i K_0\right)}{\theta^i_{KZ} - \theta^i_{KG}},
\end{aligned}
$$

which together imply

$$
\theta^i_{LG}\theta^i_{KG}\psi^i G^i + \theta^i_{LZ}\theta^i_{KZ}c^i Z^{ie} = \theta^i_{KG}\theta^i_{KZ}w^i L^i + \theta^i_{LG}\theta^i_{LZ}r^i\left(K^i + \phi^i K_0\right).
$$

Substitution of the above in (B.4) shows

$$
\frac{\phi^i\lambda^i}{\phi^i_{G^i}G^i}\theta^i_{LZ} = \left[\frac{\theta^i_{LG}\theta^i_{LZ}r^i\left(K^i + \phi^i K_0\right)}{M^i + \theta^i_{KG}\theta^i_{KZ}w^i L^i + \theta^i_{LG}\theta^i_{LZ}r^i\left(K^i + \phi^i K_0\right)}\right]\left(\frac{\phi^i K_0}{K^i + \phi^i K_0}\right),
$$

which is less than one, implying $C^i > 0$, $H^i < 1$ and thus uniqueness of equilibrium under autarky, if M^i is not very negative. A sufficient condition is that σ^i_G and/or σ^i_G are not too much smaller than one. $\quad ||$

Proof of Proposition 2 continued.

Strict quasi-concavity of $\widetilde{V}_T^i$ in G^i. To present this proof, we introduce more compact notation. In particular, define $F^i \equiv Z^{ie}(K_Z^i, L_Z^i)$ and let $F_{G^i}^i \equiv dZ^{ie}/dG^i$ shown in (12a) and $F_{G^i}^j \equiv dZ^{je}/dG^i$ shown in (12b), for $j \neq i = 1, 2$, indicate the effects of a change in G^i on those optimized values. Then, with equation (14), we can rewrite country i's FOC under trade, focusing on interior solutions, as follows:

$$\frac{\partial \widetilde{V}_T^i}{\partial G^i} = \widetilde{V}_T^i \left[\frac{F_{G^i}^i}{F^i} - \frac{\gamma_j}{\sigma} \left(\frac{F_{G^i}^i}{F^i} - \frac{F_{G^i}^j}{F^j} \right) \right] = 0, \tag{B.6}$$

where $F_{G^i}^i/F^i - F_{G^i}^j/F^j > 0$ which implies $F_{G^i}^i/F^i > 0$. Note from (10) that $\partial p_T^i/\partial G^i = (p_T^i/\sigma)(F_{G^i}^i/F^i - F_{G^i}^j/F^j)$ and recall $\partial \gamma_j^i/\partial p_j^i = -(1-\gamma_j^i)(\sigma-1)(\gamma_j^i/p_j^i)$. Then, differentiation of the expression above evaluated at an interior solution yields:

$$\begin{aligned} \left.\frac{\partial^2 \widetilde{V}_T^i}{(\partial G^i)^2}\right|_{G^i=\widetilde{B}_T^i} &= \widetilde{V}_T^i \left\{ \frac{(\sigma-1)(1-\gamma_j)}{\gamma_j} \left(F_{G^i}^i/F^i\right)^2 \right. \\ &\quad + \left(1 - \frac{\gamma_j}{\sigma}\right) \left[\left(F_{G^iG^i}^i/F^i\right) - \left(F_{G^i}^i/F^i\right)^2 \right] \\ &\quad \left. + \left(\frac{\gamma_j}{\sigma}\right) \left[\left(F_{G^iG^i}^j/F^j\right) - \left(F_{G^i}^j/F^j\right)^2 \right] \right\}. \end{aligned} \tag{B.7}$$

To evaluate the sign of this expression, we apply the implicit function theorem to $Z_\omega^{ie}(\omega^{ie}, \cdot) = 0$, using equations (B.3) and (12) with $F^i \equiv Z^{ie}$ and (as before) attributing any implied changes in ω^i to changes in w^i alone:

$$\begin{aligned} \left. w_{G^i}^i \right|_{G^i=\widetilde{B}_T^i} &= -\frac{K_0 Z_{wK}^{ie} \phi_{G^i}^i + Z_{wG^i}^{ie}}{Z_{ww}^{ie}} = -\frac{c_w^i F_{G^i}^i + \psi_w^i}{c_{ww}^i F^i + \psi_{ww}^i G^i} > 0 \\ \left. w_{G^i}^j \right|_{G^i=\widetilde{B}_T^i} &= -\frac{K_0 Z_{wK}^{je} \phi_{G^i}^i}{Z_{ww}^{je}} = -\frac{c_w^j F_{G^i}^j}{c_{ww}^j F^j + \psi_{ww}^j G^j} < 0. \end{aligned}$$

With these expressions, one can verify the following:

$$\begin{aligned} \left. F_{G^iG^i}^i \right|_{G^i=\widetilde{B}_T^i} &= \frac{1}{c^i} \left[r^i K_0 \phi_{G^iG^i}^i + \frac{\left(c_w^i F_{G^i}^i + \psi_w^i\right)^2}{c_{ww}^i F^i + \psi_{ww}^i G^i} \right] < 0 \\ \left. F_{G^iG^i}^j \right|_{G^i=\widetilde{B}_T^i} &= \frac{1}{c^j} \left[r^j K_0 \phi_{G^iG^i}^j + \frac{\left(c_w^j F_{G^i}^j\right)^2}{c_{ww}^j F^j + \psi_{ww}^j G^j} \right]. \end{aligned}$$

Then, substitute the above into (B.7) and invoke the FOC under free trade (B.6), using the fact that $\phi^i_{G^i} = -\phi^j_{G^i}$. After rearranging, we have

$$\left.\frac{\partial^2 \widetilde{V}^i_T}{(\partial G^i)^2}\right|_{G^i=\widetilde{B}^i_T} = \widetilde{V}^i_T \left\{ - \left[\frac{1-\gamma_j + (\sigma-1)\gamma_j}{\gamma_j} \right] (F^i_{G^i}/F^i)^2 \right. + \left(\frac{1-\gamma_j/\sigma}{c^i F^i} \right) \frac{\phi^i_{G^iG^i}\psi^i}{\phi^i_{G^i}} + \left(\frac{1-\gamma_j/\sigma}{c^i F^i} \right) \frac{(c^i_w F^i_{G^i} + \psi^i_w)^2}{c^i_{ww}F^i + \psi^i_{ww}G^i} \left. + \left(\frac{\gamma_j/\sigma}{c^j F^j} \right) \frac{(c^j_w F^j_{G^i})^2}{c^j_{ww}F^j + \psi^j_{ww}G^j} \right\} < 0. \tag{B.8}$$

The negative sign follows from our assumptions that $\sigma > \gamma_j$ (to ensure that the marginal cost of arming is strictly positive) and $\phi^i_{G^iG^i}, c^i_{ww}, \psi^i_{ww} < 0$.

Sufficiently strong comparative advantage. We now turn to the condition in the proposition that α^i is sufficiently large. As noted earlier, for any given G^j, country i's choice of guns (G^i) influences its terms of trade and thus the domestic price of country i's importable, p^i_T, which equals p^i_T in the absence of trade costs. However, this price cannot rise above the analogous price in country i under autarky, $p^i_A = \alpha^i$; nor can it fall below the relative domestic price of that good that prevails in country j, $1/p^j_A = 1/\alpha^j$. Thus, we have $p^i_T \in \left[1/\alpha^j, \alpha^i\right]$. In the main text we refer to α^i as the strength of country i's comparative advantage in producing good i. The smaller are α^i and α^j, the smaller is the range of prices within which country i's domestic price for its importable good can lie. This limited range, in turn, can lead to the emergence of multiple peaks in each country's payoff function given the opponent's guns choice that might generate discontinuities in best-response functions and, thus, can imply the non-existence of a pure-strategy equilibrium.

Above, we analyzed the outcome in security policies (G^1_T, G^2_T) based on the countries' unconstrained best-response functions, $\widetilde{B}^i_T(G^j)$ $i = 1, 2$ that ignore these boundary conditions. The requirement for (G^1_T, G^2_T) to be an equilibrium point is that it belongs to both countries' constrained best-response functions, $B^i_T(G^j)$ for $i = 1, 2$ that take into account the boundaries on the feasible range for p^i_T. The potential problem is that depending on the value of α^i, $B^i_T(G^j)$ could consist of two segments in the neighborhood of G^j_T, one that lies on $B^i_A(G^j)$ and another that lies on $\widetilde{B}^i_T(G^j)$, as illustrated in Fig. B.1. Thus, whether (G^1_T, G^2_T) qualifies as a pure-strategy equilibrium or not hinges on the location of the discontinuity relative to G^j_T. We now show that location depends on the strength of the two countries' comparative advantage, α^i for $i = 1, 2$.

To proceed, observe that, from (8) and the fact that $p^i_A = \alpha^i$, we can write $V^i_A(G^i, G^j) = V^i_A(G^i, G^j; \alpha^i)$, where $\partial V^i_A/\partial\alpha^i < 0$; and, similarly from (11), we can write $\widetilde{V}^i_T\left(G^i, G^j\right) = V^i_T\left(p^i_T\left(G^i, G^j\right), G^i, G^j\right)$, which gives us country i's payoff under trade when it accounts for the terms-of-trade effect, without imposing the constraint that $p^i_T \leq \alpha^i$. Note especially that the strength of comparative advantage α^i has no direct effect on unconstrained payoffs under trade, $\partial\widetilde{V}_T^i/\partial\alpha^i = 0$.

Now consider a feasible value of G^j, and the following two values of α^i for country i associated with that level of arming by the rival:

(i) $\underline{\alpha}^i \equiv p_T^i(\widetilde{B}_T^i\left(G^j\right), G^j)$ denotes the value of α^i for which, given G^j, the terms-of-trade effect of country i's arming just vanishes, such that

$$V_A^i(\widetilde{B}_T^i(G^j), G^j; \underline{\alpha}^i) = \widetilde{V}_T^i(\widetilde{B}_T^i\left(G^j\right), G^j\ );$$

(ii) $\overline{\alpha}^i \equiv p_T^i(B_A^i(G^j), G^j)$ denotes the value of α^i that, given G^j, makes country i's payoff under autarky just equal to its payoff under trade if it were to operate along B_A^i, ignoring the terms-of-trade effect of its arming decision:

$$\widetilde{V}_T^i\left(B_A^i\left(G^j\right), G^j\right) = V_A^i\left(B_A^i\left(G^j\right), G^j; \overline{\alpha}^i\right).$$

Observe that $\partial p_T^i/\partial G^i > 0$ for any $G^i < B_A^i(G^j)$. In addition, comparing the FOC under autarky (13) with the FOC under trade (15) shows that $\widetilde{B}_T^i\left(G^j\right) < B_A^i(G^j)$ for any feasible G^j. Thus, we have $\underline{\alpha}^i < \overline{\alpha}^i$. Furthermore, since $\partial V_A^i/\partial\alpha^i < 0$, $V_A^i\left(B_A^i\left(G^j\right), G^j; \underline{\alpha}^i\right) > V_A^i\left(B_A^i\left(G^j\right), G^j; \overline{\alpha}^i\right)$ holds. Moreover, we have

$$V_A^i\left(B_A^i\left(G^j\right), G^j; \underline{\alpha}^i\right) > \widetilde{V}_T^i\left(\widetilde{B}_T^i\left(G^j\right), G^j\right) > V_A^i\left(B_A^i\left(G^j\right), G^j; \overline{\alpha}^i\right).$$

The definition of $\underline{\alpha}^i$ and the fact that $V_A^i\left(B_A^i\left(G^j\right), G^j; \underline{\alpha}^i\right) > V_A^i(\widetilde{B}_T^i(G^j), G^j; \underline{\alpha}^i)$ gives the first inequality. Intuitively, when $\alpha = \underline{\alpha}^i \equiv p_T^i(\widetilde{B}_T^i\left(G^j\right), G^j))$, there are no gains from trade and prices are fixed, so that country i improves its payoffs by operating on its best-response function under autarky $B_A^i(G^j)$ and thus increasing G^i all the way to $B_A^i(G^j)$. The validity of the second inequality follows from the definition of $\overline{\alpha}^i$ and the fact that $\widetilde{V}_T^i(\widetilde{B}_T^i\left(G^j\right), G^j\right) > \widetilde{V}_T^i\left(B_A^i\left(G^j\right), G^j\right)$. When $\alpha^i = \overline{\alpha}^i \equiv p_T^i(B_A^i(G^j), G^j)$, country i can improve its payoff by operating on its best-response function under trade, $\widetilde{B}_T^i\left(G^j\right)$, which takes into account the terms-of-trade effect of arming. Since V_A^i is continuously decreasing in α^i, there exists a value of α^i, $\alpha_0^i \in (\underline{\alpha}^i, \overline{\alpha}^i)$, that solves

$$V_A^i\left(B_A^i\left(G^j\right), G^j; \alpha_0^i\right) = \widetilde{V}_T^i\left(\widetilde{B}_T^i\left(G^j\right), G^j\right).$$

Note that this value of α_0^i generally depends on G^j. Furthermore, there exist combinations of G^i and G^j such that $p_T^i(G^i, G^j) = \alpha_0^i$.

Let $\Lambda^i\left(G^j, \alpha\right)$ denote the difference in payoffs when country i operates on $\widetilde{B}_T^i(G^j)$ and when it operates on $B_A^i(G^j)$ given α^i and G^j, for $i \neq j = 1, 2$:

$$\Lambda^i\left(G^j, \alpha^i\right) \equiv \widetilde{V}_T^i\left(\widetilde{B}_T^i\left(G^j\right), G^j\right) - V_A^i\left(B_A^i\left(G^j\right), G^j; \alpha^i\right).$$

This function helps pin down possible discontinuities in player i's constrained best-response function under free trade, $B_T^i\left(G^j;\alpha^i\right)$, and clarifies their dependence on α^i. For any given G^j and α^i, we have

$$B_T^i(G^j;\alpha^i) = \begin{cases} \widetilde{B}_T^i(G^j) & \text{if} \quad \Lambda^i\left(G^j, \alpha^i\right) \geq 0 \\ B_A^i(G^j) & \text{if} \quad \Lambda^i\left(G^j, \alpha^i\right) \leq 0. \end{cases}$$

Since $\partial V_A^i/\partial \alpha^i < 0$ and $\widetilde{V}_T^i$ is independent of α^i, $\partial \Lambda^i/\partial \alpha^i \equiv \Lambda_\alpha^i > 0$. Of course, the sign of $\Lambda^i\left(G^j, \alpha\right)$ also depends on G^j. With the envelope theorem, the negative externality that the rival's arming confers on country i implies $\partial \widetilde{V}_T^i/\partial G^j < 0$ and $\partial V_A^i/\partial G^j < 0$ for feasible G^j. Thus, unless we impose additional structure on the model, we cannot sign $\partial \Lambda^i/\partial G^j \equiv \Lambda_{G^j}^i$. $\Lambda^i\left(G^j, \alpha\right)$ could change signs (positive to negative or vice versa) or not change at all as G^j changes. This last possibility means that the equality $\Lambda^i\left(G^j, \alpha_0^i\right) = 0$ need not imply $B_T^i(\cdot)$ is discontinuous at G^j given $\alpha^i = \alpha_0^i$.[2]

Nevertheless, assuming $\Lambda_{G^j}^i \neq 0$ so that $\Lambda^i\left(G^j, \alpha_0^i\right) = 0$ does imply a discontinuity in $B_T^i(G^j)$, we can be more precise about how values of α^i for $i = 1, 2$ matter for the existence of a pure-strategy equilibrium under trade. Suppose $G^j = G_T^j$, and find the value of α^i (denoted by α_{0T}^i) that implies $\Lambda^i(G_T^j, \alpha_{0T}^i) = 0$. Since $\Lambda_\alpha^i > 0$, we know that $\Lambda^i(G_T^j, \alpha^i) > 0$ for all $\alpha^i > \alpha_{0T}^i$ at $G^j = G_T^j$. Furthermore, for $\alpha^i = \alpha_{0T}^i$, a marginal increase in G^j above G_T^j implies $\Lambda^i(G^j, \alpha^i) \gtreqless 0$ as $\Lambda_{G^j}^i \gtreqless 0$. When α^i rises marginally above α_{0T}^i, the value of G^j that restores the equality $\Lambda^i(G^j, \alpha^i) = 0$ will change according to $dG^j/d\alpha^i|_{\Lambda^i=0} = -\Lambda_\alpha^i/\Lambda_{G^j}^i$: $dG^j/d\alpha^i|_{\Lambda^i=0} \lesseqgtr 0$ iff $\Lambda_{G^j}^i \gtreqless 0$.

To see the implications of these observations, suppose that $\Lambda_{G^j}^i > 0$ in the neighborhood of G_T^j, which implies $\Lambda^i(G^j, \alpha_{0T}^i) > 0$ for G^j marginally above G_T^j, as illustrated in Fig. B.1.[3] In this case, a small increase in α^i above α_{0T}^i implies not only $\Lambda(\alpha^i, G_T^j) > 0$ at G_T^j, but also the value of G^j that restores the equality $\Lambda(\alpha^i, G^j) = 0$ falls. Thus, $B_T^i(G^j)$ shifts back from $B_A^i(G^j)$ to $\widetilde{B}_T^i(G^j)$ for values of G^j just below G_T^j given values of α^i just above

[2]For a given combination of $\alpha = \alpha_0^i$ and G^j that implies $\Lambda^i\left(G^j, \alpha\right) = 0$, it is possible that $\Lambda_{G^j}^i = 0$ for values of G^j in the neighborhood of that point of indifference, in which case country i would remain indifferent between trade and autarky in that neighborhood. Furthermore, it is possible that country i favors trade for all other possible values in G^j, or alternatively that it favors autarky for all other possible values of G^j. Although we cannot rule out such possibilities, we view them as highly unlikely, and focus on cases where $\Lambda_{G^j}^i \neq 0$.

[3]Thus, for G^j above the point of discontinuity (G_T^j), $B_T^i(G^j) = \widetilde{B}_T^i(G^j)$; and for $G^j < G_T^j$, $B_T^i(G^j) = B_A^i(G^j)$. In the case that $\Lambda_{G^i}^i < 0$, $B_T^i(G^j) = \widetilde{B}_T^i(G^j)$ for G^j below the discontinuity and $B_T^i(G^j) = B_A^i(G^j)$ for points above it.

α^i_{0T}, such that $B^i_T(G^j) = \widetilde{B}^i_T(G^j)$ over a larger range of G^j in the neighborhood of G^j_T. Alternatively, when $\Lambda^i_{G^j} < 0$ so that $\Lambda^i(G^j, \alpha_{0T}) > 0$ for G^j just below G^i_T, the marginal increase in α above α^i_{0T} implies that the point of discontinuity is above G^j_T. Then for values of G^j just above G^j_T, $B^i_T(G^j)$ shifts back from $B^i_A(G^j)$ to $\widetilde{B}^i_T(G^j)$. As such, the marginal increase in α^i implies once again that the range of G^js in the neighborhood of G^j_T for which $B^i_T(G^j) = \widetilde{B}^i_T\left(G^j\right)$ expands. Thus, the larger is the degree of comparative advantage α^i for country i relative to α^i_{0T}, the more likely it is that $\widetilde{B}^i_T(G^j_T) \in B^i_T(G^j_T)$. Since this is true for both i, (G^1_T, G^2_T) will be an equilibrium point under trade if $\alpha^i > \alpha^i_{0T}$ for both i.[4] ||

Proof of Proposition A.1. The case of complete symmetry under free trade is clearly a special case of Proposition 2. Here, however, we admit the possibility that trade in world markets can be costly first due to the possible existence of geographic or physical trade barriers that take Samuelson's "iceberg" form. In particular, for each unit of a final good that country i imports (and consumes), country j must ship τ^i (≥ 1) units. Trade across national borders might be costly also due to the existence of import tariffs. Denote country i's (ad valorem) import tariff plus 1 by t^i (≥ 1).[5] Assume that these costs are identical across countries: $\tau^i = \tau \geq 1$ and $t^i = t \geq 1$.

In the presence of such costs, we need to modify the analysis in two principal ways. First, we must account for the effect of tariff revenues on a country i's income, Y^i, under trade.[6] Defining $T^i \equiv \frac{t^i-1}{t^i} \in [0,1)$, one can solve for Y^i to find

$$Y^i = R^i + T^i p^i_j D^i_j = R^i + T^i \gamma^i_j Y^i \quad \Longrightarrow \quad Y^i = \frac{R^i}{1-\gamma^i_j T^i}, \quad i \neq j. \tag{B.9}$$

Substitution of this value of Y^i back into (7), then, gives country i's demand for good j ($j \neq i$) as a function of domestic prices, tariffs, and factor incomes derived from the production of the intermediate good.

Second, we need to distinguish between domestic and world prices. Specifically, let q_i denote the "world" price of country i's exportable good. Competitive forces imply $q_i = p^i_i = c^i$ for $i = 1, 2$. But, the presence of trade costs drives a wedge between this world price and the domestic price of that same good imported by country j (p^j_i): $p^j_i = t^j \tau^j q_i$, $j \neq i$.

The world price of country i's importable good in terms of its exportable good, denoted by $q^i_T \equiv q_i/q_j = c^i/c^j$, is endogenously determined as a function of security policies by both countries, G^i $i = 1, 2$, as well as trade costs, t^i and τ^i, to ensure that product markets clear. Given countries specialize completely in the production of their respective exportable, such

[4] If $\alpha^i < \alpha^i_{0T}$, $G^i_T \neq \widetilde{G}^i_T$ and the equilibrium under free trade will involve mixed strategies and/or it will coincide with autarky.

[5] We abstract from the possible existence of internal transportation costs and export taxes.

[6] We assume the revenues that tariffs generate are distributed to consumers in a lump sum fashion.

an equilibrium requires balanced trade: $\tau^1 q_2 D_2^1 = \tau^2 q_1 D_1^2$, where $D_j^i = \gamma_j^i Y^i / p_j^i$ with Y^i given by (B.9) and $R^i = c^i Z^i$. After rearranging, the condition for balanced trade can be written as

$$q_T^i \equiv \frac{q_j}{q_i} = \frac{\gamma_j^i(\gamma_i^j + \gamma_j^j t^j) Z^i}{\gamma_i^j(\gamma_j^i + \gamma_i^i t^i) Z^j}, \quad i \neq j. \tag{B.10}$$

The corresponding domestic relative price, then, is $p_T^i = t^i \tau^i q_T^i$. Differentiating (B.10) logarithmically, while accounting for the effects of price changes on expenditure shares, gives:

$$\widehat{q}_T^i = \frac{1}{\Delta}\left[\widehat{Z}^i - \widehat{Z}^j - \frac{\gamma_i^i t^i[\sigma\widehat{t}^i + (\sigma-1)\widehat{\tau}^i]}{\gamma_j^i + \gamma_i^i t^i} + \frac{\gamma_j^j t^j[\sigma\widehat{t}^j + (\sigma-1)\widehat{\tau}^j]}{\gamma_i^j + \gamma_j^j t^j}\right], \tag{B.11a}$$

where we assume

$$\Delta \equiv \varepsilon^1 + \varepsilon^2 - 1 > 0, \text{ with } \varepsilon^i \equiv -\frac{\partial D_j^i / \partial p_j^i}{D_j^i / p_j^i} = 1 + (\sigma - 1)\frac{\gamma_i^i t^i}{\gamma_j^i + \gamma_i^i t^i}, \tag{B.11b}$$

for $i \neq j = 1, 2$, or that the Marshall-Lerner condition for stability of equilibrium is satisfied.[7]

World prices can vary only within a certain range such that domestic prices satisfy $p_T^i \in [1/\alpha^j, \alpha^i]$, for $i \neq j = 1, 2$. Recalling that $p_T^i = \tau^i t^i q_T^i$ and noting that $q_T^j = 1/q_T^i$, this range can also be expressed in terms of the world price of country i's importable: $q_T^i \in [t\tau/\alpha^j, \alpha^i/t\tau]$. Comparing this range to that which is applicable in the case of zero trade costs reveals that the boundary conditions are generally stronger in the presence of trade costs. Nevertheless, the general logic regarding the importance of the strength of comparative advantage applies in the presence of trade costs. Thus, in what follows, we abstract from this issue, assuming that $\alpha^i > \alpha_{0T}^i$ for $i = 1, 2$, and focus instead on the salient features of the trade equilibrium under complete symmetry.

Turning to the payoff functions under trade, substitute the expression for Y^i in (B.9) into the indirect utility function. Recalling that $R^i = c^i Z^i$, we find V_T^i is as shown in (11) where now $m_T^i \equiv m_T(p_T^i, t^i) = \frac{\mu(p_i^i, p_j^i) c^i}{1-\gamma_j^i T^i} = \frac{\mu(1, p_T^i)}{1-\gamma_j^i T^i}$ and as before Z^{ie} satisfies (5) and (6). For future reference, observe that differentiating m_T^i logarithmically with respect to p_T^i,

[7] This condition depends on demand elasticities here, because import demand functions coincide with demand functions under complete specialization by both countries. Inspection of (B.11b) reveals that, in the presence of positive tariffs ($t^i > 1$), a sufficient condition for $\Delta > 0$ is that the elasticity of substitution in consumption (σ) is not too small.

accounting for its impact on γ_j^i, gives

$$\frac{\partial m_T^i}{\partial p_T^i}\frac{p_T^i}{m_T^i} \equiv -\Gamma^i = -\gamma_j^i\left[1+(\sigma-1)\frac{\gamma_i^i(t^i-1)}{\gamma_j^i+\gamma_i^i t^i}\right] < 0, \tag{B.12}$$

which shows, with (11), that the effect of an increase in p_T^i alone reduces country i's payoff under trade V_T^i.

With the first expression in (14), (12), and (B.11a) taking trade costs as fixed, we rewrite country i's FOC under trade allowing for trade costs:

$$\frac{1}{m_T^i}\frac{\partial V_T^i}{\partial G^i} = \frac{1}{c^i}\left\{\left[1-\frac{\Gamma^i}{\Delta}-\left(\frac{r^j/c^jZ^{je}}{r^i/c^iZ^{ie}}\right)\frac{\Gamma^i}{\Delta}\right]r^iK_0\phi_{G^i}^i-\left[1-\frac{\Gamma^i}{\Delta}\right]\psi^i\right\}\leq 0, \tag{B.13}$$

where Γ^i and Δ are shown respectively in (B.12) and (B.11b). In the case of complete symmetry, the additional assumption that $G^1=G^2=G$ implies: $Z^{1e}=Z^{2e}=Z^e$, so that $q_T=1$, $p_T=\min[t\tau,\alpha]$, $\gamma_1^1=\gamma_2^2$, and $\gamma_2^1=\gamma_1^2$, which in turn imply for $i\neq j=1,2$

$$\Gamma=\gamma_j^i\left[1+\frac{(\sigma-1)\,(t-1)\,(1-\gamma_j^i)}{t(1-\gamma_j^i)+\gamma_j^i}\right] \quad \text{and} \quad \Delta=1+\frac{2t\,(\sigma-1)\,(1-\gamma_j^i)}{t(1-\gamma_j^i)+\gamma_j^i}. \tag{B.14}$$

Furthermore, the assumption that $G^i=G$ for $i=1,2$ implies $w^i=w$, $r^i=r$, and $\omega^i=\omega$, such that $c^i=c$ and $\psi^i=\psi$ for $i=1,2$. Define $E\equiv\Gamma/\Delta>0$. Then, using the above observations, we can simplify (B.13) as follows:

$$\left.\frac{1}{m_T^i}\frac{\partial V_T^i}{\partial G^i}\right|_{G^i=G} = \frac{1}{c}\left[(1-2E)rK_0\phi_{G^i}^i|_{G^i=G}-(1-E)\psi\right]\leq 0, \tag{B.15}$$

for $i=1,2$, where from our specification in (1), $\phi_{G^i}^i\big|_{G^i=G}=f'(G)/4f(G)\in(0,\infty)$ for $i=1,2$ and all $G\geq 0$. The first term in the expression above reflects the marginal benefit of arming to each country, whereas the second term reflects the marginal cost.

Consider first the marginal cost. Using the expressions for Γ and Δ shown in (B.14) with the definition of $E\equiv\Gamma/\Delta$ and simplifying gives

$$1-E=\frac{\Delta-\Gamma}{\Delta}=\frac{\gamma_i^i}{\Delta}\left\{1+\frac{(\sigma-1)\left[\gamma_j^i+\left(1+\gamma_i^i\right)t\right]}{\gamma_j^i+\gamma_i^i t}\right\},\quad i\neq j=1,2. \tag{B.16}$$

This expression is strictly positive for all $\sigma>\underline{\sigma}\in(\sigma_0,1)$, where $\underline{\sigma}$ satisfies $1-E=0$ and σ_0 implicitly solves $\Delta=0$.[8] Henceforth, we assume $\sigma>\underline{\sigma}$ to rule out negative marginal

[8]Note that the Marshall-Lerner condition of a trading equilibrium is satisfied for all $\sigma>\sigma_0$. Also note that, if trade is totally free (i.e., $t=\tau=1$) then $\sigma_0=0$ and $\underline{\sigma}=\frac{1}{2}$.

costs in the production of guns.

Inspection of the first term in (B.15) reveals that the sign of the marginal benefit of producing an additional gun is determined by the sign of $1 - 2E$. After some algebra, we can show that

$$1 - 2E = \frac{\Delta - 2\Gamma}{\Delta} = \frac{1}{\Delta}\left[\frac{1}{2} - \gamma_j^i + (\sigma - 1)\gamma_i^i\right], \;\; i \neq j = 1, 2. \tag{B.17}$$

Now note the following. If $\sigma = 1$ (i.e., preferences are Cobb-Douglas) then $\gamma_i^i = \gamma_j^i = \frac{1}{2}$ for all $t\tau \in (1, \alpha)$ that ensure positive trade flows. Inspection of (B.17) reveals that, in this case, $1 - 2E = 0$. Since, by our assumptions for the conflict technology (1), $\phi_{G^i}^i\big|_{G^i=G}$ is defined and finite when evaluated at $G = 0$, the marginal benefit to country i of producing an additional gun will be zero. Turning to the case where $\sigma \neq 1$ and $t\tau \in (1, \alpha)$, $\sigma \gtreqless 1$ implies $\gamma_i^i \gtreqless \frac{1}{2}$ and $\gamma_j^i \lesseqgtr \frac{1}{2}$; therefore, equation (B.17) reveals further that $\sigma \gtreqless 1 \Rightarrow 1 - 2E \gtreqless 0$. In short, if $\sigma \in (\underline{\sigma}, 1]$, then neither country has an incentive to produce guns under complete symmetry when trade flows are positive (i.e., $G_T^* = 0$), because the marginal benefit of arming is non-positive.[9] By contrast, if $\sigma \in (1, \infty)$, the marginal benefit is positive, thus satisfying a necessary condition for producing positive quantities of guns.

Focusing now on the case where $\sigma > 1$ ($> \underline{\sigma}$), which implies $1 - E > 0$, we can factor $(1 - E)/c$ out of the RHS of (B.15), implying that each country i's optimizing security policy satisfies the following condition for $G_T^{i*} = G_T^* > 0$:

$$\Omega^i(G_T^*, \tau, t) = ArK_0\phi_{G^i}^i\big|_{G^i=G_T^*} - \psi = 0, \tag{B.18}$$

where

$$A \equiv 1 - \frac{E}{1-E} = \frac{1 - 2\Gamma/\Delta}{1 - \Gamma/\Delta} \in (0, 1). \tag{B.19}$$

The conditions of complete symmetry imply that changes in the common level of guns by the two countries have no effect on the countries' terms of trade and therefore no effect on the countries' expenditure shares. Thus, from (B.14), Γ, Δ and consequently E are independent of $G^i = G$ for $i = 1, 2$, implying that A is independent of G as well.

A comparison of the expression for $\Omega^i(G_T^*, \tau, t)$ with the expression in square brackets of the FOC under autarky shown in (13), after imposing the conditions of complete symmetry $G_A^{i*} = G_A^*$ (i.e., $rK_0\phi_{G^i}\big|_{G^i=G_A^*} - \psi = 0$), reveals that the same conditions which guarantee the existence of a unique, interior solution under autarky ($G_A^* > 0$ for all $\sigma > 0$) also guarantee a unique, symmetric interior solution to $\Omega^i(G_T^*, \tau, t) = 0$ for $\sigma > 1$. That is to say, provided that labor and capital are sufficiently substitutable in the production of arms

[9] In terms of the notation introduced in the main text, $\overline{\sigma}^i = \overline{\sigma} = 1$.

and/or in the production of the intermediate good and $\phi^i_{G^i}(0,\cdot)$ is arbitrarily large, we have a unique, interior solution for arming under trade when $\sigma > 1$: $G^*_T > 0$. What is more, provided $\sigma \in (1,\infty)$ such that $A \in (0,1)$, we have $G^*_T \in (0, G^*_A)$. Combining this result with our earlier finding that $G^*_T = 0$ when $\sigma \leq 1$, whereas $G^*_A > 0$ for all $\sigma > 0$, completes the proof. ||

Proof of Proposition A.2. Under the conditions of complete symmetry and provided $\sigma > 1$, the FOC for an interior solution for G^i under trade is given by $\Omega^i(G^*_T, \tau, t) = 0$ shown in (B.18). Note that, by the uniqueness of equilibrium under trade in the case of symmetry as shown in Proposition A.1 (assuming that labor and capital are sufficiently substitutable in the production of arms and the intermediate good), we have $\Omega^i_G < 0$.

The independence of factor prices from σ and the definition of A in (B.19) imply $\text{sign}\{\Omega^i_\sigma\} = \text{sign}\{A_\sigma\} = -\text{sign}\{E_\sigma\}$. As such, to prove $dG^*_T/d\sigma > 0$ we need only show $E^i_\sigma < 0$ and invoke the implicit function theorem. Clearly, E depends on σ directly and indirectly through the expenditure share γ^i_j. But $\gamma^i_j \equiv (t\tau)^{1-\sigma}/[1+(t\tau)^{1-\sigma}]$ for $i \neq j = 1,2$ under symmetric CES preferences where $t, \tau \geq 1$; therefore, $d\gamma^i_j = -\gamma^i_j(1-\gamma^i_j)\ln(t\tau)\, d\sigma$, which implies $d\gamma^i_j/d\sigma \leq 0$. Keeping this in mind, we differentiate $\ln\Gamma$ and $\ln\Delta$ totally to obtain, for $i \neq j = 1,2$:

$$
\begin{aligned}
d\ln\Gamma &= \frac{(t-1)(1-\gamma^i_j)\gamma^i_j}{\Gamma\left[t(1-\gamma^i_j)+\gamma^i_j\right]}d\sigma + \frac{1}{\Gamma}\left\{1 + \frac{(\sigma-1)(t-1)\left[t(1-\gamma^i_j)^2-(\gamma^i_j)^2\right]}{\left[t(1-\gamma^i_j)+\gamma^i_j\right]^2}\right\}d\gamma^i_j \\
d\ln\Delta &= \frac{2t(1-\gamma^i_j)}{\Delta\left[t(1-\gamma^i_j)+\gamma^i_j\right]}d\sigma - \frac{2t(\sigma-1)}{\Delta\left[t(1-\gamma^i_j)+\gamma^i_j\right]}d\gamma^i_j.
\end{aligned}
$$

With these expressions, noting that $d\ln E = d\ln\Gamma - d\ln\Delta$, one can verify the following:

$$
\begin{aligned}
d\ln E &= -\frac{(1+t)(1-\gamma^i_j)\gamma^i_j}{\Gamma\Delta\left[t(1-\gamma^i_j)+\gamma^i_j\right]}d\sigma + \left\{\frac{2t(\sigma-1)}{\Delta\left[t(1-\gamma^i_j)+\gamma^i_j\right]^2}\right. \\
&\quad \left. + \frac{1}{\Gamma}\left[1 + \frac{(\sigma-1)(t-1)\left[t(1-\gamma^i_j)^2-(\gamma^i_j)^2\right]}{\left[t(1-\gamma^i_j)+\gamma^i_j\right]^2}\right]\right\}d\gamma^i_j,
\end{aligned}
$$

for $i \neq j = 1,2$. Now observe that, assuming complete symmetry and $\sigma > 1$, the definitions of trade costs, $\tau \geq 1$ and $t \geq 1$, imply $1-\gamma^i_j \geq \gamma^i_j$ and $t(1-\gamma^i_j)^2-(\gamma^i_j)^2 \geq 0$ for $i \neq j = 1,2$. Then, since $d\gamma^i_j/d\sigma \leq 0$, inspection of the expression for $d\ln E$ readily reveals that $E_\sigma < 0$.

To prove the limit part of the proof, it is sufficient to show that $\lim_{\sigma\to\infty} E = 0$. We rewrite E using the expressions in (B.14) as follows:

$$E = \frac{\Gamma}{\Delta} = \gamma_j^i N \quad \text{where } N \equiv \frac{1 + \frac{(\sigma-1)(t-1)(1-\gamma_j^i)}{t(1-\gamma_j^i)+\gamma_j^i}}{1 + \frac{2t(\sigma-1)(1-\gamma_j^i)}{t(1-\gamma_j^i)+\gamma_j^i}} = \frac{\frac{1}{\sigma-1} + \frac{(t-1)(1-\gamma_j^i)}{t(1-\gamma_j^i)+\gamma_j^i}}{\frac{1}{\sigma-1} + \frac{2t(1-\gamma_j^i)}{t(1-\gamma_j^i)+\gamma_j^i}},$$

for $i \neq j = 1, 2$. Since $\lim_{\sigma\to\infty} \gamma_j^i = 0$ and $\lim_{\sigma\to\infty} N = \frac{t-1}{2t} \geq 0$ for $t \geq 1$, we have $\lim_{\sigma\to\infty} E = (\lim_{\sigma\to\infty} \gamma_j^i)\,(\lim_{\sigma\to\infty} N) = 0.$ $\quad ||$

Proof of Proposition A.3. From (B.10), the conditions of complete symmetry with identical trade costs imply $q_T^i = 1$ in equilibrium and thus $p_T^i = t\tau q_T^i = t\tau$ for $i = 1, 2$. Accordingly, from (B.12) and (11), m_T and thus V_T rise when either t or τ falls given guns. Establishing the proposition, then, requires only that we show equilibrium arming under trade is increasing in t and τ.

Recall the conditions of complete symmetry, with the assumption $\sigma > 1$, imply that the FOC for an interior solution for G^i under trade is given by $\Omega^i(G_T^*, \tau, t) = 0$ shown in (B.18). The uniqueness of equilibrium under trade in the case of symmetry as shown in Proposition A.1 implies further that $\Omega_G^i < 0$. With definition of A in (B.19), the independence of factor prices from trade costs implies $\text{sign}\{\Omega_\tau^i\} = \text{sign}\{A_\tau\} = -\text{sign}\{E_\tau\}$ and $\text{sign}\{\Omega_t^i\} = \text{sign}\{A_t\} = -\text{sign}\{E_t\}$, for $i = 1, 2$. To demonstrate that globalization ($\tau \downarrow$) and trade liberalization ($t \downarrow$) reduce arming, it is sufficient to show (by invoking the implicit function theorem) that $E_\tau < 0$ and $E_t < 0$, respectively.

To proceed, observe that $\tau \geq 1$ and $t \geq 1$ imply $(1-\gamma_j^i) \geq \gamma_j^i$ and $t(1-\gamma_j^i)^2 - (\gamma_j^i)^2 \geq 0$ for $i \neq j = 1, 2$. Then, differentiating Γ (shown above) logarithmically with respect to τ and t and imposing the conditions of complete symmetry on the resulting expressions yield:

$$\frac{\tau\Gamma_\tau}{\Gamma} = -\frac{(\sigma-1)\gamma_i^i\gamma_j^i \left\{1 + \frac{(\sigma-1)(t-1)}{(\gamma_j^i+\gamma_i^i t)^2}\left[t(\gamma_i^i)^2 - (\gamma_j^i)^2\right]\right\}}{\Gamma} < 0 \tag{B.20a}$$

$$\frac{t\Gamma_t}{\Gamma} = -\frac{(\sigma-1)\sigma(t-1)\gamma_i^i\gamma_j^i\left[t(\gamma_i^i)^2 - (\gamma_j^i)^2\right]}{\Gamma(\gamma_j^i + \gamma_i^i t)^2} < 0, \tag{B.20b}$$

for $i \neq j = 1, 2$. Similarly, differentiating Δ logarithmically and imposing the assumptions of complete symmetry give:

$$\frac{\tau\Delta_\tau}{\Delta} = \frac{2(\sigma-1)^2 t\gamma_i^i\gamma_j^i}{\Delta(\gamma_j^i + \gamma_i^i t)^2} > 0 \tag{B.21a}$$

$$\frac{t\Delta_t}{\Delta} = \frac{2(\sigma-1)\sigma t\gamma_i^i\gamma_j^i}{\Delta(\gamma_j^i + \gamma_i^i t)^2} > 0, \tag{B.21b}$$

for $i \neq j = 1, 2$. The proposition then follows from (B.20) and (B.21) along with the definition of E ($\equiv \Gamma/\Delta$). $\quad ||$

Proposition B.1 (*Pareto-improving globalization.*) *Under the conditions of Proposition A.2 and provided the intensities of resource use in the guns and intermediate goods sectors are not too different, reciprocal reductions in trade barriers improve the welfare of both labor and capital owners in each country.*

Proof: We abstract from tariffs ($t = 1$) and consider only physical trade barriers ($\tau > 1$).[10] Furthermore, to highlight the distributional effects of globalization, we focus on the per unit payoffs to labor (v_L^i) and to capital owners (v_K^i), given respectively by

$$\begin{aligned} v_L^i &= \mu\left(1, p_T^i\right) w^i / p_i^i = \mu\left(1, p_T^i\right) w^i / c^i \\ v_K^i &= \mu\left(1, p_T^i\right) r^i / p_i^i = \mu\left(1, p_T^i\right) r^i / c^i, \end{aligned}$$

where we have used the competitive pricing relationship that requires $p_i^i = c^i \equiv c(w^i, r^i)$. This focus makes sense, since factor owners receive their rewards regardless of where they are employed. It does, however, abstract from the gains of globalization that derive from lower arming and thus greater production and consumption of final goods (i.e., higher Z production), and as such tends to understate the benefits realized by factor owners.[11]

To proceed, recall that $p_T^i = \tau q_T^i$ and observe from (B.10) in the symmetric trade equilibrium with $t = 1$ (which implies $Z^i = Z^j$ and $\gamma_j^i = \gamma_i^j$) that $q_T^i = 1$ for $i = 1, 2$. Then, logarithmic differentiation of the payoff functions above gives:

$$\widehat{v}_L^i = -\gamma_j^i \widehat{p}_T^i + \theta_{KZ}^i \widehat{\omega}^i = \gamma_j^i(-\widehat{\tau}) + \theta_{KZ}^i \widehat{\omega}^i \tag{B.22a}$$

$$\widehat{v}_K^i = -\gamma_j^i \widehat{p}_T^i - \theta_{LZ}^i \widehat{\omega}^i = \gamma_j^i(-\widehat{\tau}) - \theta_{LZ}^i \widehat{\omega}^i, \tag{B.22b}$$

where $\theta_{KZ}^i \equiv r^i c_r^i / c^i$ and $\theta_{LZ}^i \equiv w^i c_w^i / c^i = 1 - \theta_{KZ}^i$ are the cost shares of capital and labor respectively in the production of Z^i in country $i = 1, 2$ and $\omega^i \equiv w^i / r^i$. Equation (B.22) reveals that reciprocal reductions in trade costs generate both direct and indirect effects on capital and labor payoffs. The first term in each line reflects the direct effect that arises as reciprocal reductions in τ cause the domestic cost of purchasing the country's importable to fall, a benefit enjoyed by all factor owners (i.e., as consumers). The indirect effect, represented by the second term in each line, operates through the effect of reciprocal reductions in τ on the country's' arming, which influences the equilibrium relative wage ω^i

[10] While it is possible to conduct the analysis in the presence of tariffs (making an assumption about the disposition of tariff revenues), we abstract from this complication to keep the analysis simple.

[11] Our abstraction from tariffs reinforces this understatement. Specifically, in the presence of tariffs, reductions in physical trade barriers would increase trade flows and thus tariff revenues that would be distributed to factor owners, implying an additional benefit for both labor and capital owners.

and thus the respective factor owners' purchasing power (in terms of the good in which the country has a comparative advantage).

We first evaluate the indirect effect of a given reduction in τ on the payoffs to labor and capital owners as it depends on the intensity of labor in the guns sector relative to that in the intermediate goods sector, and then combine that indirect effect with the direct effect to evaluate the signs of $\widehat{v}_L^i$ and $\widehat{v}_K^i$ shown in (B.22). Evaluating the indirect effect requires that we first identify the effect of a given change in arming on the wage-rental ratio. Imposing complete symmetry (which implies $G^i = G$ and $\phi^i = \frac{1}{2}$) and dropping country superscript i for simplicity, we differentiate the factor market-clearing condition in (6) to obtain

$$\widehat{\omega} \equiv \left(\frac{\theta_{LG} - \theta_{LZ}}{\theta_{LG}}\right) \lambda \widehat{G}, \tag{B.23}$$

where $\lambda \in (0,1)$ is defined in (A.6). The expression in (B.23) shows that equal increases (decreases) in the countries' arming induce an increase (decrease) in the market-clearing relative wage if labor is employed intensively in the production of guns; that is, if $\theta_{LG} - \theta_{LZ} > 0$. Exactly the opposite is true, if $\theta_{LG} - \theta_{LZ} < 0$.

Next, we identify the effect of a given change in τ on arming, which allows us to identify the magnitude of the indirect effect on payoffs. From (B.18), the FOC under trade for an interior symmetric solution ($\sigma > 1$), when $t = 1$, can be written as

$$ArK_0 \phi_{G^i}^i \Big|_{G^i = G_T^*} - \psi = 0,$$

where now from (B.14) and (B.19)

$$A \equiv \frac{\Delta - 2\Gamma}{\Delta - \Gamma} = \frac{2\sigma\gamma_i^i - 1}{(2\sigma - 1)\gamma_i^i},$$

for $i = 1, 2$. Since $\gamma_i^i \in [\frac{1}{2}, 1)$ for $\tau \geq 1$ and $\sigma \in (1, \infty)$, we have $A \in (0,1)$. Keeping in mind the assumption of complete symmetry and our focus on reciprocal changes in τ imply $dG^i = dG^j = dG$ and $\phi_{G^iG^j}^i = 0$, logarithmic differentiation of the FOC gives

$$\widehat{A}^i - \eta\widehat{G} + \widehat{r} - \widehat{\psi} = 0,$$

where $\eta \equiv -G\phi_{G^iG^i}^i/\phi_{G^i}^i + G\phi_{G^iG^j}^i/\phi_{G^j}^i = -G\phi_{G^iG^i}^i/\phi_{G^i}^i = Gf'/f - Gf''/f' > 0$ for $i = 1, 2$.[12] One can easily verify that $\widehat{A} = \frac{(\sigma-1)\gamma_j^i}{(2\gamma_i^i\sigma - 1)}\widehat{\tau}$, where $2\gamma_i^i \geq 1$ for $\tau \geq 1$ and $\sigma > 1$ for $i \neq j = 1, 2$; therefore, $\widehat{A}/\widehat{\tau} > 0$. Finally, observe that $\widehat{r} - \widehat{\psi} = -\theta_{LG}\widehat{\omega}$, where $\theta_{LG} \equiv w\psi_w/\psi$

[12] The last equality can be confirmed using the specification in (1). That $\eta > 0$ holds follows from our assumptions that $f' > 0$ and $f'' < 0$.

denotes the cost share of labor in the production of arms. Using these findings in the above expression allows us to transform it as follows:

$$\frac{(\sigma-1)\,\gamma_j^i}{\left(2\gamma_i^i\sigma-1\right)}\widehat{\tau}-\eta\widehat{G}-\theta_{LG}\widehat{\omega}=0. \tag{B.24}$$

To identify the impact of changes in τ on G and ω we must solve (B.23) and (B.24) simultaneously. Doing so yields

$$\begin{aligned}\widehat{G} &= -\frac{(\sigma-1)\,\gamma_j^i}{\left(2\gamma_i^i\sigma-1\right)\left[\eta+\left(\theta_{LG}-\theta_{LZ}\right)\lambda\right]}\left(-\widehat{\tau}\right)\\ \widehat{\omega} &= -\left(\frac{\gamma_j^i}{\theta_{LG}}\right)\frac{(\sigma-1)\left(\theta_{LG}-\theta_{LZ}\right)\lambda}{\left(2\gamma_i^i\sigma-1\right)\left[\eta+\left(\theta_{LG}-\theta_{LZ}\right)\lambda\right]}\left(-\widehat{\tau}\right).\end{aligned}$$

As one can verify, the condition that ensures uniqueness of equilibrium under autarky (more generally in the case of asymmetries as well as in the case of complete symmetry) is analytically equivalent to requiring $\eta+\left(\theta_{LG}-\theta_{LZ}\right)\lambda>0$.[13] Under symmetry, this condition also ensures uniqueness of equilibrium under trade with the additional assumption that comparative advantage is sufficiently strong. Since globalization reduces arming, the condition for globalization to reduce (raise) the relative wage in each country is that labor be employed intensively in the production of guns (intermediate good).

Next, define

$$\Theta\equiv\frac{(\sigma-1)\left(\theta_{LG}-\theta_{LZ}\right)\lambda}{\left(2\gamma_i^i\sigma-1\right)\left[\eta+\left(\theta_{LG}-\theta_{LZ}\right)\lambda\right]}.$$

Using this definition in $\widehat{\omega}$ implies $\widehat{\omega}=-\left(\gamma_j^i/\theta_{LG}\right)\Theta\left(-\widehat{\tau}\right)$. Then, substituting this expression in (B.22a) and (B.22b) and simplifying gives

$$\widehat{v}_L^i = \gamma_j^i(-\widehat{\tau})\left[1-\left(\frac{\theta_{KZ}}{\theta_{LG}}\right)\Theta\right] \tag{B.25}$$

$$\widehat{v}_K^i = \gamma_j^i(-\widehat{\tau})\left[1+\left(\frac{\theta_{LZ}}{\theta_{LG}}\right)\Theta\right]. \tag{B.26}$$

Clearly, if the intensities of resource use in guns and the intermediate good do not differ, then $\theta_{LG}-\theta_{LZ}=0$ and $\Theta=0$. Under these conditions, the relative wage ω^i in each country will not change with globalization and thus globalization will necessarily be Pareto-improving. Next, suppose $\theta_{LG}-\theta_{LZ}>0$, which implies $\Theta\in(0,1)$ and thus that the equilibrium relative wage falls with globalization. Obviously, capital owners will benefit in this case. A sufficient condition for labor to also benefit is $\theta_{KZ}/\theta_{LG}\leq 1$. It follows

[13] See the part of the proof of Proposition 1 presented above in the Supplementary Appendix (B.1).

that globalization will be Pareto-improving, if $\theta_{LG} \geq \max[\theta_{LZ}, \theta_{KZ}]$. In the case that $\theta_{LG} - \theta_{LZ} < 0$ (but $\eta + (\theta_{LG} - \theta_{LZ})\lambda > 0$), the relative wage rises with globalization to make labor unambiguously better off. As one can verify, a sufficient condition for capital owners to be better off as well is that $\theta_{LZ} < \sqrt{\eta\theta_{LG}/\lambda + \theta_{LG}^2}$. ||

Proposition B.2 (*Relative appeal of free trade with a third friendly country.*) *If comparative advantage is sufficiently weak, then free trade can imply strictly lower payoffs for the two contending countries than does autarky.*

Proof. To illustrate the possibility that a shift from autarky to free trade in the setting of Section 6 can reduce welfare, we focus on cases where the gains from trade are relatively small, which we know are those cases where the world market-clearing price is close to the autarky price. The proof builds on the feature of the model that, as in the analysis with only two countries, the world price is bounded by the two autarky prices: $p_T \in [1/p_A^3, p_A]$. To keep matters simple but without loss of generality, we suppose that $a_1^1 = a_1^2 = a_2^3 = 1$ and $a_2^1 = a_2^2 = a_1^3 = \alpha > 1$, so that countries 1 and 2 have an identical comparative advantage in the production of good 1, whereas country 3 has a comparative advantage in the production of good 2; and, furthermore, comparative advantage is symmetric across countries 1 and 2 on the one hand and country 3 on the other, with $\alpha > 1$ indicating the strength of that advantage. Given these simplifications, $p_A^i = \alpha$ for $i = 1, 2$ and $p_A^3 = \alpha$. To simplify a little more, suppose in addition that preferences are Cobb-Douglas ($\sigma = 1$) and identical across countries.[14] Then, the condition of balanced trade implies

$$p_T = \begin{cases} 1/\alpha & \text{if} \quad \pi \leq 1/\alpha \\ \pi & \text{if} \quad 1/\alpha < \pi < \alpha \\ \alpha & \text{if} \quad \alpha \leq \pi, \end{cases}$$

where $\pi \equiv \frac{\gamma_2}{\gamma_1}\left[\frac{Z^{1e}+Z^{2e}}{Z^{3e}}\right]$. Since the expenditure shares ($\gamma_1 = 1 - \gamma_2$ for all three countries) are constant, the expression above shows that p_T remains fixed for combinations of Z^1 and Z^2 that imply (given Z^{3e}) a constant sum $Z^{1e} + Z^{2e}$.

We assume further (for clarity) that guns are produced with labor only (so that $\psi^i = w^i$), which implies that $Z^i = Z(L - G^i, K + \phi^i K_0)$, where $Z(\cdot)$ now represents a standard constant returns to scale production function that gives the output of the intermediate good Z^i; henceforth, we drop the "e" superscript on Z^i.[15] Then, the relevant upper bound on the

[14] The analysis does not change substantively if we allow $\sigma > 1$. In any case, note that we are not restricting preferences here to be symmetrically defined across the two goods.

[15] The assumption that guns are produced with labor alone is not crucial, but simplifies the analysis and guarantees uniqueness of equilibrium under autarky and free trade (assuming sufficiently large comparative advantage in the latter trade regime.

world price can be written as

$$\sum\nolimits_{i=1,2} Z^i = \sum\nolimits_{i=1,2} Z(L - G^i, K + \phi^i K_0) = \frac{\gamma_1}{\gamma_2}\alpha Z^3 \equiv \overline{Z},$$

which defines combinations of guns such that $p_T = \alpha$.[16] This constraint is depicted in Fig. B.2, which also shows country 1's best-response function under autarky $B^1_A(G^2)$ and its best-response function under free trade that ignores the constraint on p_T, $\widetilde{B}^1_T(G^2)$.[17] Combinations of guns above the $p_T = \alpha$ constraint imply $p_T < \alpha$, whereas combinations on and below the constraint imply $p_T = \alpha$ (or equivalently $\sum_{i=1,2} Z^i \geq \overline{Z}$). Starting at points where the constraint binds, changes in guns by either country would cause p_T to fall only if the new point is beyond the $p_T = \alpha$ constraint.

The shape of this constraint, as shown in the figure, has three key properties:

(i) The constraint has a slope of -1 where it intersects the 45° line, as can be confirmed by evaluating $dG^2/dG^1|_{p_T=\alpha}$ at $G^2 = G^1$.

(ii) It is not possible for both G^1 and G^2 to increase as we move rightward along the constraint away from the 45° line towards $\widetilde{B}^1_T(G^2)$.[18]

(iii) The properties above in turn imply that the $p_T = \alpha$ constraint must intersect (or approach) $\widetilde{B}^1_T(G^2)$ at some point C' below and to the right of its intersection with the 45° line.[19]

Larger values of α imply that the $p_T = \alpha$ constraint lies closer to the origin, without affecting the positioning of $B^1_A(G^2)$ and $\widetilde{B}^1_T(G^2)$.[20]

Now consider the value of G^2 denoted by G^2_C such that $G^1_A = B^1_A(G^2_C)$ implies $\sum_{i=1,2} Z^i > \overline{Z}$ and, more importantly, such that $G^1_T = \widetilde{B}^1_T(G^2_C)$ implies $\sum_{i=1,2} Z^i = \overline{Z}$. For $G^2 = G^2_C$, a

[16] A similar constraint can be written for the lower bound of $p_T = 1/\alpha$: $\sum_{i=1,2} Z^i = Z^3\gamma_1/(\gamma_2\alpha) \equiv \underline{Z}$, with $\underline{Z} < \overline{Z}$. However, it is the upper bound that is relevant here.

[17] That $B^1_A(G^2) < \widetilde{B}^1_T(G^2)$ for given G^2 reflects the effect of trade to augment a country's incentive to arm against another country when the two compete in the same export market, as emphasized in the text.

[18] This property follows from the effects of changes in arming on $\sum_{i=1,2} Z^i$. Starting at the point where the $p_T = \alpha$ constraint intersects $B^1_A(G^2)$ in Fig. B.2, let G^1 rise while keeping G^2 fixed. The FOC under autarky (13) implies $\partial V^1_A/\partial G^1 < 0$ for $G^1 > B^1_A(G^2)$, so that $dZ^1/dG^1 < 0$; since $dZ^2/dG^1 < 0$ holds too, the increase in G^1 causes $\sum_{i=1,2} Z^i$ and thus p_T to fall, implying that the new combination of guns is above the $p_T = \alpha$ constraint. Repeated applications of this logic establish that G^1 and G^2 cannot both increase as we continue to move along the constraint approaching $\widetilde{B}^1_T(G^2)$). That is to say, the $p_T = \alpha$ constraint cannot be U-shaped to the right of the 45° line. However, it is possible that $dG^2/dG^1|_{p_T=\alpha} \to \infty$ somewhere along the constraint as we move further towards or beyond $\widetilde{B}^1_T(G^2)$—a possibility illustrated in Fig. B.2. There exist no combinations of G^1 and G^2 beyond this critical point, whether it is located to the left or right of $\widetilde{B}^1_T(G^2)$, where the price constraint binds.

[19] C' can be arbitrarily close to point 0, but that does not matter for our argument to follow.

[20] The $p_T = \alpha$ constraint likewise moves closer to the origin when either Z^3 or $\gamma_1 = 1 - \gamma_2$ increases, but such changes could also affect the positioning of $\widetilde{B}^1_T(G^2)$. Focusing on how the positioning of the $p_T = \alpha$ constraint depends on α allows us to show most clearly how the price constraint relates to the location of the discontinuity of the best-response function under free trade as derived below.

shift from autarky to free trade (shown in Fig. B.2 as a movement from point C to point C') implies $p_T = p_A = \alpha$. With no gains from trade given $G^2 = G_C^2$, country 1 prefers to stay on its best-response function under autarky that simply maximizes its output of Z^1. That is to say, $V_A^1(B_A^1(G_C^2), G_C^2) > \widetilde{V}_T^1(\widetilde{B}_T^1(G_C^2), G_C^2)$. Next consider a larger value of G^2 denoted by G_D^2 such that $G_A^1 = B_A^1(G_D^2)$ implies $\sum_{i=1,2} Z^i = \overline{Z}$ and furthermore $G_T^1 = \widetilde{B}_T^1(G_D^2)$ implies $\sum_{i=1,2} Z^i < \overline{Z}$. In this case as shown in Fig. B.2, a shift from autarky to free trade implies $p_T < \alpha$ and induces country 1 to increase its arming in a move from $B_A(G_D^2)$ (point D) to $\widetilde{B}_T(G_D^2)$ (point D'), with $\widetilde{V}_T^1(\widetilde{B}_T^1(G_D^2), G_D^2) > V_A^1(B_A^1(G_D^2), G_D^2)$. By continuity, there exists a value of arming by country 2, denoted by $G_0^2 \in (G_C^2, G_D^2)$, such that $V_A^1(B_A^1(G_0^2), G_0^2) = \widetilde{V}_T^1(\widetilde{B}_T^1(G_0^2), G_0^2)$. This value of G_0^2 defines the discontinuity in country 1's best-response function under (free) trade, denoted by $B_T^1(G^2)$:

$$B_T^1(G^2) = \begin{cases} B_A^1(G^2) & \text{if } G^2 \leq G_0^2; \\ \widetilde{B}_T^1(G^2) & \text{if } G^2 \geq G_0^2. \end{cases}$$

Since the location of the $p_T = \alpha$ constraint depends on the strength of comparative advantage, moving closer to the origin as α increases, the location of the discontinuity depends on α as well.

We flesh out the possible welfare implications here, with the help of Fig. B.2. Point A on the 45° line where the best-response functions of the two (identical) countries under autarky would cross (so that $G_A^{1*} = G_A^{2*} = G_A^*$) represents the unique, symmetric equilibrium under autarky. Point T also on the 45° line shows where the unconstrained best-response functions under trade would cross so that $G_T^{1*} = G_T^{2*} = G_T^*$. Provided point T lies above the $p_T = \alpha$ constraint, it represents an equilibrium under free trade. A movement along the 45° line from A to T implies no change in the distribution of K_0, but higher security costs. Whether these added security costs are greater than or less than the gains from trade depends on the location of the discontinuity of the constrained best-response function under trade, $B_T^1(G^2)$.

Suppose that the discontinuity occurs at $G_0^2 = G_A^*$, depicted as point A. By the definition of the discontinuity, the payoffs to country 1 under autarky and trade will be equal at this level of arming by country 2: $V_A^1(B_A^1(G_A^*), G_A^*) = \widetilde{V}_T^1(\widetilde{B}_T^1(G_A^*), G_A^*)$. We now show that a movement along $\widetilde{B}_T^1(G^2)$ from $(\widetilde{B}_T^1(G_A^*), G_A^*)$ in the direction of point T reduces country 1's payoff in the 3-country case. By the envelope theorem, since we are moving along country 1's best-response function under trade, we need only to consider the welfare effects of a change in G^2 as it influences country 1's optimal production of the intermediate input and p_T. Keeping in mind the effects a change in G^2 on both countries' optimizing production,

dZ^1/dG^2 and dZ^2/dG^2 shown in (12), we use (16) with $\Delta = \sigma = 1$ and rearrange to find:

$$\frac{1}{V_T^1}\frac{\partial V_T^1}{\partial G^2} = \left[1 - \nu^1\gamma_2\right]\frac{dZ^1/dG^2}{Z^1} - [\nu^2\gamma_2]\frac{dZ^2/dG^2}{Z^2}. \tag{B.27}$$

To evaluate the sign of (B.27), observe that, for points on $B_T^1\left(G^2\right)$ where $G^2 < B_T^2\left(G^1\right)$, the net marginal benefit of arming for country 2 is positive:

$$\frac{1}{V_T^2}\frac{\partial V_T^2}{\partial G^2} = [1 - \nu^2\gamma_2]\frac{dZ^2/dG^2}{Z^2} - [\nu^1\gamma_2]\frac{dZ^1/dG^2}{Z^1} > 0.$$

This inequality can be rewritten as

$$-[\nu^2\gamma_2]\frac{dZ^2/dG^2}{Z^2} < -\left[\frac{\nu^1\nu^2\left(\gamma_2\right)^2}{1 - \nu^2\gamma_2}\right]\frac{dZ^1/dG^2}{Z^1},$$

and then combined with (B.27) (using the fact that $\nu^1 + \nu^2 = 1$) to find

$$\frac{1}{V_T^1}\frac{\partial V_T^1}{\partial G^2} < \left[\frac{1 - \gamma_2}{1 - \nu^2\gamma_2}\right]\frac{dZ^1/dG^2}{Z^1}.$$

Since $\nu_2, \gamma_2 < 1$ and $dZ^1/dG^2 < 0$, the RHS of the expression above is negative, which in turn implies that $\partial V_T^1/\partial G^2 < 0$.[21] As such, $V_T^*(G_T^*, G_T^*) < V_A^*(G_A^*, G_A^*)$ when $G_0^2 = G_A^*$.

By continuity, there exist higher values of α (with the $p_T = \alpha$ constraint and the discontinuity in $B_F^1(G^2)$ moving towards the origin), such that the gains from trade continue to be less than the higher security costs under trade, implying that a shift from autarky to free trade is welfare reducing. Of course, increases in the strength of comparative advantage eventually imply sufficiently large gains from trade that swamp the higher security costs and thus render free-trade Pareto preferred to autarky.[22] ||

B.2 Asymmetric Distributions of Secure Resources, Arming and Welfare

In this appendix, we provide more details on the consequences of asymmetric distributions of secure resources for arming incentives and the balance of power under both autarky and trade, along with the welfare implications. For this analysis, it is helpful to start by assessing the impact of redistributions of secure resource endowment on the FOCs under both autarky and trade, starting initially from the benchmark where secure resources are distributed

[21] A sufficient (but not necessary) condition for this result to hold more generally under CES preferences is that $\sigma \geq 1$.

[22] Note that for smaller α, the discontinuity in $B_T^1(G^2)$ moves between points A and T. In such cases, both points A and T represent pure-strategy equilibria, with autarky being Pareto preferred to free trade. For α sufficiently small to push the $p_T = \alpha$ constraint beyond point T, the only pure-strategy equilibrium is autarky.

identically across the two countries. Such an analysis indicates how the relevant best-response functions shift as resources are reallocated away from the symmetric distribution. Here, we focus on the following three types of redistribution:

(i) $d\vartheta^i = dK^i$;
(ii) $d\vartheta^i = dL^i$; and,
(iii) $d\vartheta^i = dL^i$ and $dK^i = (c_r/c_w)\, dL^i$,

where $d\vartheta^j = -d\vartheta^i$ for $j \neq i$. The first two types involve the redistribution of a single secure resource (capital or labor) from one country to the other. The third involves a special redistribution of both secure labor and capital from one country to the other that, as discussed in the main text, implies $k_Z^i = k_Z^j$ for given guns $G^i = G^j$. From the market-clearing condition (6), relative factor prices remain unchanged by this sort of redistribution if arming remains unchanged.

For each of the three types of redistribution, differentiation of the appropriate FOC, using the definition $F^i(K_Z^i, L_Z^i) \equiv Z^{ie}$ with $K_Z^i = K^i + \phi^i K_0 - \psi_r^i G^i$ and $L_Z^i = L^i - \psi_w^i G^i$, and noting that changes in country i's secure endowments do not directly affect $F^j \equiv Z^{je}$ give the following:

Autarky:

$$\left.\frac{d\left(\partial V_A^i/\partial G^i\right)}{V_A^i}\right|_{G^i=B_A^i} = \frac{F^i_{G^i\vartheta^i}}{F^i} d\vartheta^i. \tag{B.28}$$

Free Trade:

$$\begin{aligned}\left.\frac{d\left(\partial V_T^i/\partial G^i\right)}{V_T^i}\right|_{G^i=B_T^i} &= (1-\gamma_j/\sigma)\left[\frac{F^i F^i_{G^i\vartheta^i} - F^i_{G^i} F^i_{\vartheta^i}}{(F^i)^2}\right] d\vartheta^i \\ &\quad + (\gamma_j/\sigma)\left[\frac{F^j F^j_{G^i\vartheta^j} - F^j_{G^i} F^j_{\vartheta^j}}{(F^j)^2}\right] d\vartheta^j \\ &\quad - \left(\frac{\partial\gamma_j/\partial p_T^i}{\sigma}\right)\left(\frac{F^i_{G^i}}{F^i} - \frac{F^j_{G^i}}{F^j}\right)\left[\left(\frac{\partial p_T^i}{\partial\vartheta^i}\right) d\vartheta^i + \left(\frac{\partial p_T^i}{\partial\vartheta^j}\right) d\vartheta^j\right].\end{aligned} \tag{B.29}$$

A comparison of (B.28) with (B.29) shows that assessing the implications of a redistribution of secure resources for the FOCs under trade is considerably more complex than that under autarky. Under autarky, the effects of a redistribution in secure endowments operate only through their effect on the country's own marginal value $F^i_{G^i}$. Under trade, the effects are transmitted through three channels: (a) through their impact on the marginal values for both countries, $F^i_{G^i}$ and $F^j_{G^i}$; (b) through their impact on the output levels of the intermediate goods, $F^i = Z^{ie}$ and $F^j = Z^{je}$; and, (c) through the impact of the world market clearing price p_T^i on expenditure shares γ_j.

To evaluate the effects of the three sorts of redistribution outlined above on the FOCs, first recall $F^i_{K^i} = Z^{ie}_K = r^i/c^i$ and $F^i_{L^i} = Z^{ie}_L = w^i/r^i$. In addition, the following expressions reveal the effects of a given redistribution on the marginal values $F^i_{G^i}$ and $F^j_{G^i}$:

$$
\begin{aligned}
F^i_{G^iK^i} &= -w^i_{K^i}\left[\left(c^i_w F^i_{G^i} + \psi^i_w\right)/c^i\right] < 0 && \text{(B.30a)}\\
F^j_{G^iK^j} &= -w^j_{K^j}\left[c^i_w F^j_{G^i}/c^j\right] > 0 && \text{(B.30b)}\\
F^i_{G^iL^i} &= -w^i_{L^i}\left[\left(c^i_w F^i_{G^i} + \psi^i_w\right)/c^i\right] > 0 && \text{(B.30c)}\\
F^j_{G^iL^j} &= -w^j_{L^j}\left[c^i_w F^j_{G^i}/c^j\right] < 0, && \text{(B.30d)}
\end{aligned}
$$

whereas an application of the implicit function theorem to $Z^{ie}_\omega(\omega^{ie},\cdot) = 0$ shows

$$
\begin{aligned}
w^i_{K^i} &= -\frac{Z^{ie}_{Kw}}{Z^{ie}_{ww}} = \frac{w^i c^i_w/c^i}{c^i_{rw}F^i + \psi^i_{rw}G^i}\\
&= \frac{w^i r^i \theta^i_{LZ}}{\sigma^i_Z\theta^i_{LZ}\theta^i_{KZ}c^iF^i + \sigma^i_G\theta^i_{LG}\theta^i_{KG}\psi^iG^i} > 0 && \text{(B.31a)}\\
w^i_{L^i} &= -\frac{Z^{ie}_{Lw}}{Z^{ie}_{ww}} = -\frac{w^i c^i_r/c^i}{c^i_{rw}F^i + \psi^i_{rw}G^i}\\
&= -\frac{\left(w^i\right)^2\theta^i_{KZ}}{\sigma^i_Z\theta^i_{LZ}\theta^i_{KZ}c^iF^i + \sigma^i_G\theta^i_{LG}\theta^i_{KG}\psi^iG^i} < 0. && \text{(B.31b)}
\end{aligned}
$$

Finally, we identify the effects on expenditure shares and world prices, which are relevant for our analysis under free trade:

$$
\begin{aligned}
\partial\gamma_j/\partial p^i_T &= -\left(\sigma-1\right)\gamma_j\left(1-\gamma_j\right)/p^i_T && \text{(B.32a)}\\
\partial p^i_T/\partial K^i &= \left(p^i_T/\sigma\right)\left(Z^{ie}_{K^i}/Z^{ie}\right) = \left(p^i_T/\sigma\right)\left(r^i/c^iF^i\right) && \text{(B.32b)}\\
\partial p^i_T/\partial L^i &= \left(p^i_T/\sigma\right)\left(Z^{ie}_{L^i}/Z^{ie}\right) = \left(p^i_T/\sigma\right)\left(w^i/c^iF^i\right) && \text{(B.32c)}\\
\partial p^i_T/\partial K^j &= -\left(p^i_T/\sigma\right)\left(Z^{je}_{K^j}/Z^{je}\right) = -\left(p^i_T/\sigma\right)\left(r^j/c^jF^j\right) && \text{(B.32d)}\\
\partial p^i_T/\partial L^j &= -\left(p^i_T/\sigma\right)\left(Z^{je}_{L^j}/Z^{je}\right) = -\left(p^i_T/\sigma\right)\left(w^j/c^jF^j\right). && \text{(B.32e)}
\end{aligned}
$$

Observe from (B.30), that the effects of a change in K^i or of L^i (or both) on $F^i_{G^i}$ operate solely through the wage (or, more precisely, the relative wage $\omega^i = w^i/r^i$ given r^i). Thus, any feasible redistribution of both labor and capital that leaves $k^i_{ZA} = k^*_{ZA}$ and thus factor prices unchanged (resulting in new allocations of secure resources within $\mathcal{S}_0$) also leaves $F^i_{G^i} = (r^i\phi^i_{G^i}K_0 - \psi^i)/c^i$ unchanged on net given the countries' arming choices. Accordingly from (B.28), such redistributions do not affect arming incentives under autarky. By contrast, deviations from $\mathcal{S}_0$ due to changes in either K^i or L^i do shift the best-response functions under autarky. In particular, because $F^i_{G^iK^i} < 0$ as shown in (B.30a), if $dK^i = -dK^j > 0$

and $dL^i = dL^j = 0$ for $i \neq j$, then B_A^i decreases whereas B_A^j increases given the opponent's arming choice, implying that country i (j) becomes less (more) powerful. Furthermore, since $F^i_{G^iL^i} > 0$ as shown in (B.30c), exactly the opposite is true when $dL^i = -dL^j > 0$ and $dK^i = dK^j = 0$. Of course, what these effects mean for equilibrium arming depend also on the cross-guns effects. Still, when we start from the symmetric allocation of resources such that $G^i = G^j$ initially, the above shows the direction of the effect on arming incentives. We return to these implications below.

Turning to the case of free trade, we use (B.29), (B.30), (B.31) and (B.32) to find the following expressions for the changes in secure endowments considered for the three types of redistribution (i)-(iii) outlined above.

Case (i): $d\vartheta^i = dK^i > 0$, implying an effect on w^i and w^j.

$$\left.\frac{\partial^2 V_T^i}{\partial G^i \partial K^i}\right|_{G^i=B_T^i} = -V_T^i\left(\frac{F^i_{G^i}}{F^i}\right)\left\{\left[\frac{r^i}{c^iF^i}+\frac{r^j}{c^jF^j}\right]\left[\frac{1-\gamma_j+(\sigma-1)\gamma_j}{\sigma}\right]\right.$$
$$\left.+\left(1-\frac{\gamma_j}{\sigma}\right)\left[\overset{(+)}{w^i_{K^i}}\left(\frac{c^i_wF^i_{G^i}+\psi^i_w}{c^iF^i_{G^i}}\right)+\overset{(+)}{w^j_{K^j}}\left(\frac{c^j_w}{c^j}\right)\right]\right\}<0. \tag{B.33}$$

Since from the FOC under trade (15) $F^i_{G^i} > 0$, a redistribution of capital from country j to country i for given guns tends to reduce (increase) country i's (j') incentive to arm. This is similar, but not identical, to what happens under autarky.

Case (ii): $d\vartheta^i = dL^i > 0$, again implying an effect on w^i and w^j.

$$\left.\frac{\partial^2 V_T^i}{\partial G^i \partial L^i}\right|_{G^i=B_T^i} = -V_T^i\left(\frac{F^i_{G^i}}{F^i}\right)\left\{\left[\frac{w^i}{c^iF^i}+\frac{w^j}{c^jF^j}\right]\left[\frac{1-\gamma_j+(\sigma-1)\gamma_j}{\sigma}\right]\right.$$
$$\left.+\left(1-\gamma_j/\sigma\right)\left[\overset{(-)}{w^i_{L^i}}\left(\frac{c^i_wF^i_{G^i}+\psi^i_w}{c^iF^i_{G^i}}\right)+\overset{(-)}{w^j_{L^j}}\left(\frac{c^j_w}{c^j}\right)\right]\right\}. \tag{B.34}$$

As can be seen in this expression, and in contrast to the case of autarky, the effect of a redistribution in labor under trade is ambiguous. Below, we proceed by imposing more structure on the technologies for guns and the intermediate goods.

Case (iii): $d\vartheta^i = dL^i$ and $dK^i = (c_r/c_w)\, dL^i$, implying no direct effect on wages (given r^i as well as the countries' arming choices). Noting that

$$dV^i_{G^i} = \frac{\partial^2 V_T^i}{\partial G^i \partial K^i} dK^i + \frac{\partial^2 V_T^i}{\partial G^i \partial L^i} dL^i,$$

and that, by construction, factor prices do not change given arming ($G^i = G^j$) and are identical across countries, we use (B.33) and (B.34) to find:

$$\left.\frac{\partial^2 V_T^i}{\partial G^i \partial L^i}\right|_{G^i=B_T^i} = -V_T^i \left(\frac{F_{G^i}^i}{F^i}\right)\left[\frac{w^i}{c^i F^i} + \frac{w^j}{c^j F^j}\right]\left[\frac{1-\gamma_j + (\sigma-1)\gamma_j}{\sigma}\right] < 0.$$

This expression shows that, in contrast to the case of autarky, a redistribution of both labor and capital that leaves factor prices unchanged will change arming incentives. In particular, the recipient (donor) country will have a smaller (larger) incentive to arm, given the other country's arming.

B.2.1 Arming Incentives under Free Trade vs. Autarky Revisited

As discussed in the text, understanding the possible dependence of the slope of each country's best-response function under autarky in the neighborhood of the autarkic equilibrium on the distribution of secure factor endowments helps to identify how a shift from autarky to trade influences arming decisions. Specifically, we know that, if both countries' security policies under autarky are strategic complements (as they are for secure endowment distributions in the neighborhood of $\mathcal{S}_0$, defined by distributions that imply $k_{ZA}^i = k_{ZA}^*$ for $i = 1, 2$), then trade always results in lower arming and hence benefits both sides. Still, one has to wonder about the limits of these central results. We know that strategic complementarity of both countries' security policies can extend beyond the immediate neighborhood of $\mathcal{S}_0$. Furthermore, we know that strategic complementarity in the neighborhood of the autarkic equilibrium is sufficient, but not necessary for trade to induce both countries to arm by less than they would under autarky.[23]

To proceed, let us consider more precisely how secure factor endowments matter for arming decisions under autarky. In particular, start at some initial allocation in $\mathcal{S}_0$, and suppose that a small quantity labor is reallocated from country 2 to country 1 so that $dL^2 = -dL^1 < 0$.[24] The impact of this reallocation on the equilibrium quantities of guns can be found by differentiating the FOCs under autarky and simplifying as follows

$$\begin{pmatrix} dG_A^1 \\ dG_A^2 \end{pmatrix} = \frac{1}{|J|}\begin{pmatrix} \frac{\partial^2 V_A^2}{(\partial G^2)^2} & -\frac{\partial^2 V_A^1}{\partial G^1 \partial G^2} \\ -\frac{\partial^2 V_A^2}{\partial G^2 \partial G^1} & \frac{\partial^2 V_A^1}{(\partial G^1)^2} \end{pmatrix}\begin{pmatrix} -\frac{\partial^2 V_A^1}{\partial G^1 \partial L^1} dL^1 \\ -\frac{\partial^2 V_A^2}{\partial G^2 \partial L^2} dL^2 \end{pmatrix}, \tag{B.35}$$

where as defined earlier

$$|J| = \frac{\partial^2 V_A^1}{(\partial G^1)^2}\frac{\partial^2 V_A^2}{(\partial G^2)^2} - \frac{\partial^2 V_A^1}{\partial G^1 \partial G^2}\frac{\partial^2 V_A^2}{\partial G^2 \partial G^1},$$

[23] As argued in the text, at least one country's best-response function must be positively sloped in that neighborhood.

[24] We can repeat this exercise for reallocations of capital using a similar logic.

which is positive as required for uniqueness of equilibrium under autarky. Observe that $\partial^2 V_A^i/(\partial G^i)^2 < 0$ for $i = 1, 2$ as shown in the proof of existence of equilibrium under autarky (see the proof of Proposition 1); furthermore, from (B.28), (B.30c), and (B.31b), we have

$$\frac{\partial^2 V_A^i}{\partial G^i \partial L^i} = m_A^i F_{G^i L^i}^i = -m_A^i w_{L^i}^i \left[\left(c_w^i F_{G^i}^i + \psi_w^i \right) / c^i \right] > 0,$$

for $i = 1, 2$. Imposing our assumption that $dL^2 = -dL^1 < 0$ and simplifying expressions allows us to rewrite (B.35) as follows

$$\begin{aligned} dG_A^1/dL^1 &= \frac{1}{|J|} \left[-\overset{(-)}{\frac{\partial^2 V_A^2}{(\partial G^2)^2}} \overset{(+)}{\frac{\partial^2 V_A^1}{\partial G^1 \partial L^1}} - \overset{(?)}{\frac{\partial^2 V_A^1}{\partial G^1 \partial G^2}} \overset{(+)}{\frac{\partial^2 V_A^2}{\partial G^2 \partial L^2}} \right] \\ dG_A^2/dL^1 &= \frac{1}{|J|} \left[\overset{(?)}{\frac{\partial^2 V_A^2}{\partial G^2 \partial G^1}} \overset{(+)}{\frac{\partial^2 V_A^1}{\partial G^1 \partial L^1}} + \overset{(-)}{\frac{\partial^2 V_A^1}{(\partial G^1)^2}} \overset{(+)}{\frac{\partial^2 V_A^2}{\partial G^2 \partial L^2}} \right]. \end{aligned}$$

In each line, the first (second) term inside the square brackets captures the impact of an increase (decrease) in L^1 (L^2). The direct effect of a change in country i's (j's) labor on its own guns is straightforward: an increase (decrease) in L^1 (L^2) causes G^1 (G^2) to increase (decrease). However, the sign of the indirect effect of a change in L^i (the one involving a movement along country j's best-response function) depends on whether the rival's guns are strategic complements or substitutes, which depends on the extent to which countries differ in their secure endowments.

Since we are starting from an allocation in $\mathcal{S}_0$, we know that initially $G_A^{i*} = G_A^*$ so that $\phi_{G^i G^j}^i = 0$ for $i \neq j = 1, 2$. Thus, from (A.7b) along with uniqueness of equilibrium, arms are strategic complements for both countries, and the direct effect of a small labor reallocation from country 2 to country 1 dominates the indirect effect. Country 1 expands its production of guns while country 2 reduces it.[25] But while $\partial^2 V_A^1/\partial G^1 \partial G^2 > 0$ continues to hold for additional reallocations of labor in the same direction, country 2's guns could at some point become negatively related to country 1's guns (i.e., $\partial^2 V_A^2/\partial G^2 \partial G^1 < 0$).[26]

The central problem here is to characterize the set of secure resource allocations for which a shift from autarky to free trade induces less arming by both countries, and this requires that we first identify the sets of resource allocations that imply $\partial B_A^i/\partial G^j < 0$ and conversely. One thing is clear: if we were to (arbitrarily) expand L^1, keeping L^2 fixed, the implied clockwise rotation of B_A^1 would cause this function to reach and eventually cross the

[25] Furthermore, one can verify that $dG^1/dL^1 = -dG^2/dL^1 > 0$ for labor reallocations that start in $\mathcal{S}_0$.

[26] An interesting consequence is that it is possible for both countries to arm less heavily under autarky as the secure endowments become increasingly asymmetric.

stationary peak of B_A^2 (defined by the condition that $\partial^2 V_A^2/\partial G^1 \partial G^2 = 0$). However, when this expansion in L^1 is due to a labor reallocation ($dL^2 = -dL^1 < 0$), the simultaneous and clockwise rotation of B_A^2 changes the location of its peak, as illustrated in Fig. B.3. Nevertheless, it should be clear that G^2 will eventually become negatively related to G^1 if we reallocate enough labor from country 2 to country 1.

To proceed, we take a closer look at the determinants of the sign of $\partial^2 V_A^2/\partial G^1 \partial G^2$. We have already shown in the proof of Proposition 1 that the sign of $\partial B_A^i/\partial G^j$ is determined by the sign of $H^i = H_1^i/H_2^i$ in (A.8) or, equivalently, (since $H_2^i > 0$) by the sign of H_1^i:

$$H_1^i \equiv -\frac{\phi^i \phi^i_{G^i G^j}}{\phi^i_{G^i} \phi^i_{G^j}} + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i} \theta^i_{LZ}. \tag{B.36}$$

With the specification of the conflict technology (1) and the definition of λ^i in (A.6) and using (B.4), the above expression can be transformed into

$$\begin{aligned} H_1^i &= \frac{\phi^i}{\phi^j} - 1 + \frac{\phi^i \lambda^i}{\phi^i_{G^i} G^i} \theta^i_{LZ} \\ &= \frac{\phi^i}{\phi^j} - 1 + \frac{\theta^i_{LG} \theta^i_{LZ} \left(r^i \phi^i K_0 \right)}{\sigma^i_G \theta^i_{LG} \theta^i_{KG} \psi^i G^i + \sigma^i_Z \theta^i_{LZ} \theta^i_{KZ} c^i Z^{ie}}. \end{aligned}$$

This expression reveals that $H_1^i > 0$ when $B_A^i(G^j) = G^j$, as is the case for allocations of secure endowments in $\mathcal{S}_0$ where $k_Z^i = k_{ZA}^*$ for $i = 1, 2$. Furthermore, by continuity, the positive sign of H_1^i is preserved for sufficiently small deviations from such allocations. For our purposes, though, knowing how the sign of H_1^i can change when other values of G^j and endowment distributions are considered is important. Knowing this requires us to identify the dependence of the third term of H_1^i on guns, resource endowments and the technologies for G^i and Z^i. In the general case, where σ_G^i and σ_Z^i are endogenous, the problem is intractable.[27] Thus, making further progress requires we impose more structure on the model.[28]

Let us assume that labor is the only input in the production of guns (i.e., $\psi^i = w^i$ so that $\theta^i_{KG} = 0$ and $\theta^i_{LG} = 1$), and that the technology for the intermediate input Z^i is CES, symmetric in inputs, and identical across countries with $\sigma_Z^i = \sigma_Z$ for $i = 1, 2$.[29] Then, solving for G^i and Z^i simultaneously from (3a) and (3b), and using the resulting expression

[27] That is to say, economic theory provides us no guidance on the sign of the third derivatives of unit cost functions.

[28] The imposition of more structure onto the basic model necessarily narrows the scope of our analysis. Although we aim to produce general results, the best we can do now is to identify instances under which endowment asymmetries give rise to strategic substitutability for one country.

[29] Another approach (which we do not pursue here for the sake of brevity) is to assume the technologies for G^i and Z^i are CES production functions. The analysis simplifies a bit under the additional assumptions that $\sigma_Z = \sigma_G = 1$, but the key insights remain intact.

for Z^i implies $c^i Z^i = r^i \left(K^i + \phi^i K_0\right) / \theta^i_{KZ}$, which in turn implies

$$H_1^i = H_1^i \left(G^i, G^j; \cdot\right) = \frac{\phi^i}{\phi^j} - 1 + \frac{\phi^i K_0}{\sigma_Z \left(K^i + \phi^i K_0\right)},$$

where $\partial H_1^i / \partial G^i > 0$ and $\partial H_1^i / \partial G^j < 0$. For given secure capital endowments, $H_1^2 = 0$ implicitly defines a relationship between the two countries' guns such that $\partial B_A^2 / \partial G^1 = 0$ with

$$\frac{dG^2}{dG^1}\Big|_{dH_1^2=0} = -\frac{\partial H_1^2/\partial G^1}{\partial H_1^2/\partial G^2} = -\frac{\phi^2_{G^1}}{\phi^2_{G^2}} > 0.$$

This relationship is illustrated in Fig. B.3 by the pink-dashed schedule. Given the specialization of the model just introduced, the location of this schedule does not depend on residual labor endowments. Instead, it depends solely on K^2, K_0, and σ_Z. Note especially that reductions in K^2 cause it rotate clockwise.[30]

Let $(\overline{L}, \overline{K})$ denote the vector of total secure capital and labor supplies. The green dotted curve in Fig. B.3 going through points A, A' and O depicts the Nash equilibria in security policies that arise when we reallocate labor from the symmetric allocation $(L^1, K^1) = (\overline{L}/2, \overline{K}/2)$ all the way to $L^1 = \overline{L}$. For any given capital allocation $K^1 + K^2 = \overline{K}$ between the two countries, there exists a unique pair $(\overline{L}^1, \underline{L}^2)$ satisfying $\overline{L}^1 + \underline{L}^2 = \overline{L}$ and $\overline{L}^1 > \underline{L}^2$, such that $\partial B_A^2 / \partial G^1 \gtreqless 0$ for $L^1 \lesseqgtr \overline{L}^1$. (A similar argument applies for the sign of $\partial B_A^1 / \partial G^2$.)

Starting at $(\overline{L}^1, \underline{L}^2)$, now reallocate more capital from country 2 to country 1. Our analysis above implies that, to ensure $\partial B_A^2 / \partial G^1 = 0$ as we reallocate capital from country 1 to country 2, we must also reallocate more labor from country 2 to country 1; that is, $\overline{L}^1$ will have to increase. This relationship gives rise to the positively sloped schedule $B_2 B_2$ in Fig. B.4 that has the following property: $\partial B_A^2 / \partial G^1 < 0$ ($\partial B_A^2 / \partial G^1 > 0$) for all resource allocations to the right (left) of $B_2 B_2$. A similar argument establishes that

[30] That the $H_1^2 = 0$ schedule is shown as a straight line follows from our specification of ϕ^i shown in (1), when $f(G^i) = (\delta + G^i)^b$ with $b \in (0, 1]$ and $\delta \geq 0$. Specifically, since $\phi^1 = 1 - \phi^2$, the $H_1^2 = 0$ condition results in (at most) a quadratic equation in ϕ^2, for which there exists a solution that depends on K_0, K^2 and σ_Z. Let us denote that solution by h. Using our specification for ϕ^i in (1) which implies $f(G^2) = f(G^1)h/(1-h)$ and assuming $f(G^i) = (\delta + G^i)^b$ in turn gives us the relationship between G^2 and G^1 along the $H_1^2 = 0$ schedule:

$$G^2 = -\delta + (\delta + G^1)\left[\frac{h}{1-h}\right]^{1/b}.$$

This expression shows that the value of G^2 that solves $H_1^2 = 0$ depends linearly on G^1. Note that as $\delta \to 0$, the schedule will go approximately through the 0 point in the figure. Furthermore, one can verify that, assuming $\sigma_Z = 1$, as $K^2 \to 0$, the schedule implied by $H_1^2 = 0$ converges to the horizontal axis, suggesting that $B_A^2(G^1)$ will be positively sloped at all feasible gun levels. (In contrast, B_A^1 will not coincide with the vertical axis.) Thus, the arming of a country that has no secure capital endowment is a strategic complement for the rival's arming.

there also exists a positively sloped $B_1 B_1$ schedule such that $\partial B_A^1/\partial G^2 < 0$ ($\partial B_A^1/\partial G^2 > 0$) for all secure allocations left (right) of $B_1 B_1$. The blue shaded region in Fig. B.4 captures the set of allocations that ensure the countries' security policies under autarky are strategic complements. Thus, for these allocations, we know that trade induces less aggressive behavior by both countries, and the welfare implications summarized in Lemma 2 apply.

However, recall that strategic complementarity of security policies is only a sufficient condition for trade to reduce arming by both countries. We now illustrate, with numerical simulations, that the set of secure resource allocations that imply $G_T^{1*} \leq G_A^{1*}$ is considerably larger. Define

$$O^i \equiv \{\left(L^i, K^i\right) \mid G_A^{j*} < G_T^{j*} \text{ for } L^i + L^j = \overline{L} \text{ and } K^i + K^j = \overline{K},\ i \neq j = 1, 2\},$$

as the set of international allocations of secure resources that make country i's opponent (i.e., country j) *more* aggressive under trade than autarky. These sets (O^i for $i = 1, 2$) are depicted as the pink shaded areas in Fig. B.4. They were computed under the following parameter values: $\sigma_Z = 1$, $\theta_{LZ} = 0.2$, $\overline{K} = \overline{L} = 2$, $K_0 = 2$, and $\sigma = 1.2$.[31] To see how the elasticity of substitution in consumption (σ) matters, we recalculated O^1 and O^2 for $\sigma' = 4$. Fig. B.4 also shows that the boundaries of these sets change from the solid to the dashed curves. Clearly, these sets are relatively small.

What about the welfare implications for resource allocations in O^i? For each allocation of secure factor endowments $\left(L^i, K^i\right) \in O^i$ there exists a sufficiently small set of comparative advantage values, with α_{0T}^i (defined in the proof of Proposition 2) being the infimum of this set, such that country i is worse off under trade (i.e., $V_T^{i*} < V_A^{i*}$) for all α^i in the set. To see the intuition, consider an allocation $\left(L^i, K^i\right) \in O^i$ so that $G_A^{j*} < G_T^{j*}$ ($i \neq j = 1, 2$). As before, α_{0T}^i is defined so that (G_T^{i*}, G_T^{j*}) qualifies as a pure strategy equilibrium for α^i close to but larger than α_{0T}^i. If the value of α^i we choose is sufficiently close to α_{0T}^i, then the gains from trade for country i (which will be infinitesimally small) will be outweighed by the welfare losses due to the fact that its rival behaves more aggressively under trade.

B.2.2 The Balance of Power and Trade

Above, we have seen that, starting from a symmetric allocation of resources, a redistribution of capital from one country to the other dampens the enlarged country's incentive to arm under autarky and trade. In addition, a redistribution of labor from one country to the other amplifies the enlarged country's incentive to arm under autarky; however, its effect under trade is generally ambiguous. Moreover, we have yet to see how trade can influence the

[31] An appealing feature of our specialized model is that, when $\sigma_Z = 1$, the quantities of each country i's labor endowment allocated to G^i and Z^i are always positive.

distribution of power relative to autarky and how that influence depends on the distribution of secure resources.

To address these issues, we continue to use the specialized version of the general model, where we assume (i) that guns are produced only with secure labor, implying $\psi^i = w^i$ and $\theta^i_{LG} = 1$ and $\theta^i_{KG} = 0$; and, (ii) the technology for Z^i in each country is described by a symmetric CES production function with $\sigma^i_Z = \sigma_Z$. The assumptions imply that the impact of endowment redistributions on the wage rate (B.31) are

$$\begin{aligned} w^i_{K^i} &= \frac{w^i c^i_w / c^i}{c^i_{rw} F^i} = \frac{w^i r^i}{\sigma^i_Z \theta^i_{KZ} c^i F^i} > 0 \\ w^i_{L^i} &= -\frac{w^i c^i_r / c^i}{c^i_{rw} F^i} = -\frac{\left(w^i\right)^2}{\sigma^i_Z \theta^i_{LZ} c^i F^i} < 0. \end{aligned}$$

In addition, from (B.34) and (B.31b) we have

$$\begin{aligned} \left.\frac{\partial^2 V^i_T}{\partial G^i \partial L^i}\right|_{G^i = B^i_T} = \; & V^i_T \left\{ \left(\frac{F^i_{G^i}}{F^i} \right) \left[\frac{w^i}{c^i F^i} + \frac{w^j}{c^j F^j} \right] \left[\frac{(\sigma - 1)(1 - \gamma_j)}{\sigma} - \frac{(\sigma_Z - 1)(1 - \gamma_j/\sigma)}{\sigma_Z} \right] \right. \\ & \left. + (1 - \gamma_j/\sigma) \left[\frac{\left(w^i / c^i F^i\right)^2}{\sigma_Z \theta^i_{LZ}} \right] \right\}. \end{aligned}$$

Inspection of the above expression reveals that if $\sigma \geq \sigma_Z$ then $\partial^2 V^i_T / (\partial G^i \partial L^i) > 0$, such that an increase in a country's labor endowment increases its incentive to arm under trade given the opponent's arming. Clearly, this inequality also holds true if $\sigma_Z = 1$. Incidentally, for the specialized version of the model under autarky, we have:

$$\left.\frac{\partial^2 V^i_A}{\partial G^i \partial L^i}\right|_{G^i = B^i_A} = V^i_A \left(F^i_{G^i L^i} / F^i\right) = V^i_A \frac{\left(\psi^i / c^i F^i\right)^2}{\sigma_Z \theta^i_{LZ}} > 0.$$

Keeping in mind that the comparative statics of endowment redistributions are fairly general under autarky, we now use this model to shed additional light on the question of power with the help of Figs. B.5, B.6, and B.7. These figures are derived from a numerical analysis of the specialized model assuming the same set of assumptions regarding parameter values used to construct Fig. B.4.

Fig. B.5 identifies the allocations of secure endowments that ensure the two countries are equally powerful under autarky (the AA schedule) and under free trade (the TT schedules), and sheds some light on the question of how the trade regime in place influences the balance of power. The solid (dashed) TT schedule is drawn for σ is close to (further away from) 1.[32] Allocations to the left of/above (right of/below) the AA schedule ensure country 2 (1)

[32]Since arming under autarky is independent of σ, so is the positioning of the AA schedule. Note that this schedule simply gives $\mathcal{S}_0$, the set of secure allocations satisfying the condition $k^i_{ZA} = k^*_{ZA}$ for $i = 1, 2$.

is more powerful under autarky. Similarly, allocations to the left of/above (right of/below) the TT schedules imply country 2 (1) is more powerful under free trade. Interestingly, the figure suggests that a shift from autarky to trade could tilt the balance of power towards one country, especially the smaller country (i.e., where the allocation of secure resources is closer to the origin for country 1 or country 2). More precisely, as exemplified by the resource allocations in the shaded regions in this figure, trade can reverse the balance of power as compared with autarky. For example, an allocation in the blue shaded region implies that country 1 is more powerful than country 2 under autarky, but is less powerful than country 2 under autarky. The figure also suggests that the value of the elasticity of substitution in consumption σ plays an important role in this context. Specifically and as one would expect, the larger is σ, the smaller are the effects of trade (as compared with autarky) on the balance of power.

Fig. B.6 provides a different perspective on how the introduction of trade influences power relative to autarky. Specifically, it shows how trade changes a country's equilibrium power relative to the autarkic equilibrium, for all possible allocations of secure resource endowments. Both the solid blue curve and the dashed pink curve depict secure endowment distributions that imply each country's power is independent of trade regimes—i.e., $\phi_A^i = \phi_T^i$ for $i = 1, 2$. The difference between these two curves is that the solid blue one is drawn from σ close to 1, whereas the dashed pink one is drawn for a larger value of σ. Trade reduces (enhances) country 1's (2's) power for resource allocations above this schedule (conditioned on the value of σ). Exactly the opposite is true for resource allocations below the appropriate schedule. Thus, the figure confirms, once again, the tendency for trade to enhance the relatively small country's power.

Finally, Fig. B.7 combines Figs. B.5 and B.6 (assuming the value of σ close to one) to offer one more perspective on trade and the balance of power. In particular, it shows how for some distributions of secure endowments, country i is more powerful than its opponent under both free trade and autarky (i.e., $\phi_A^i > \phi_A^j$ and $\phi_T^i > \phi_T^j$), but free trade tends to diminish its power relative to what it enjoys under autarky (i.e., $\phi_T^i < \phi_A^i$ and thus $\phi_T^j > \phi_A^j$). Such rankings emerge for country 1 in the blue shaded region of Fig. B.7 (see, for example, point C). Looking at this figure along with Fig. B.4 shows secure distributions within only a very small part of this region (including extremely large values of L^1) induce the other country (country 2) to arm more heavily under trade than autarky. Thus, the conditions that imply $G_A^{2*} > G_T^{2*}$ are not sufficient for $\phi_A^1 > \phi_T^1$. Furthermore, the two figures show that $G_A^{2*} > G_T^{2*}$ is not necessary either.

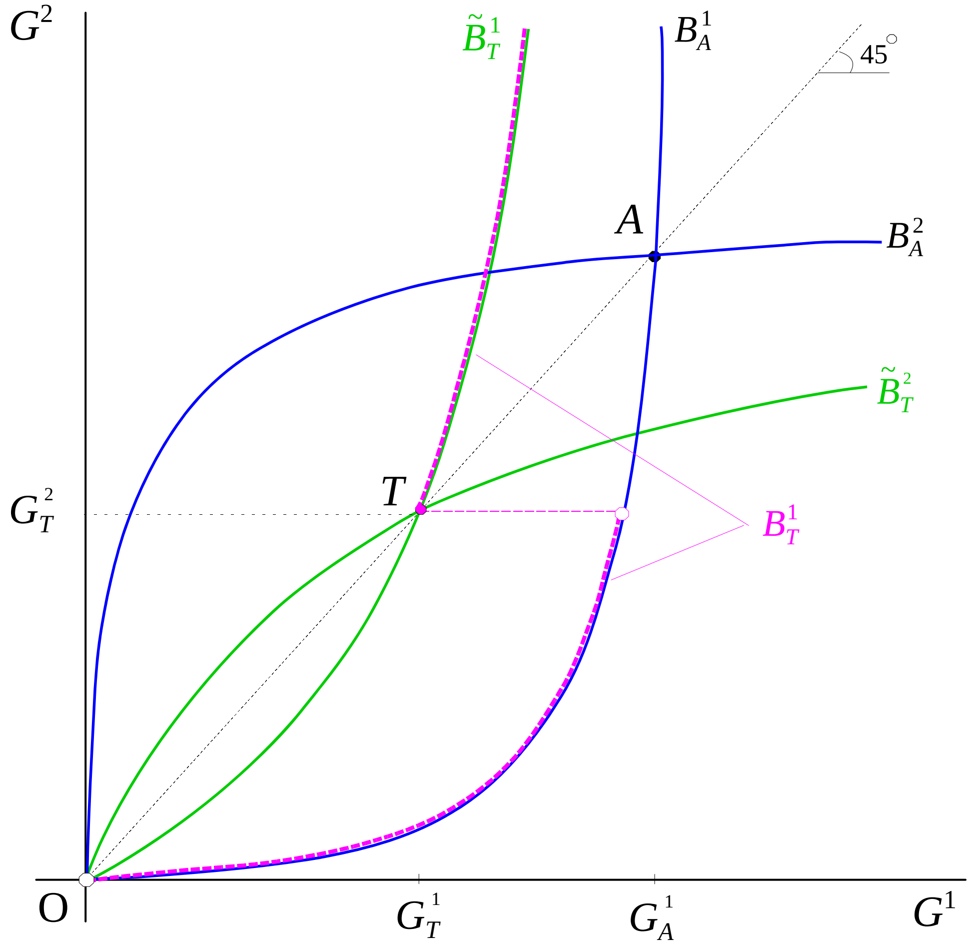

Figure B.1: Derivation of Best-Response Function under Trade and Nash Equilibria in Security Policies in the Two-Country Case

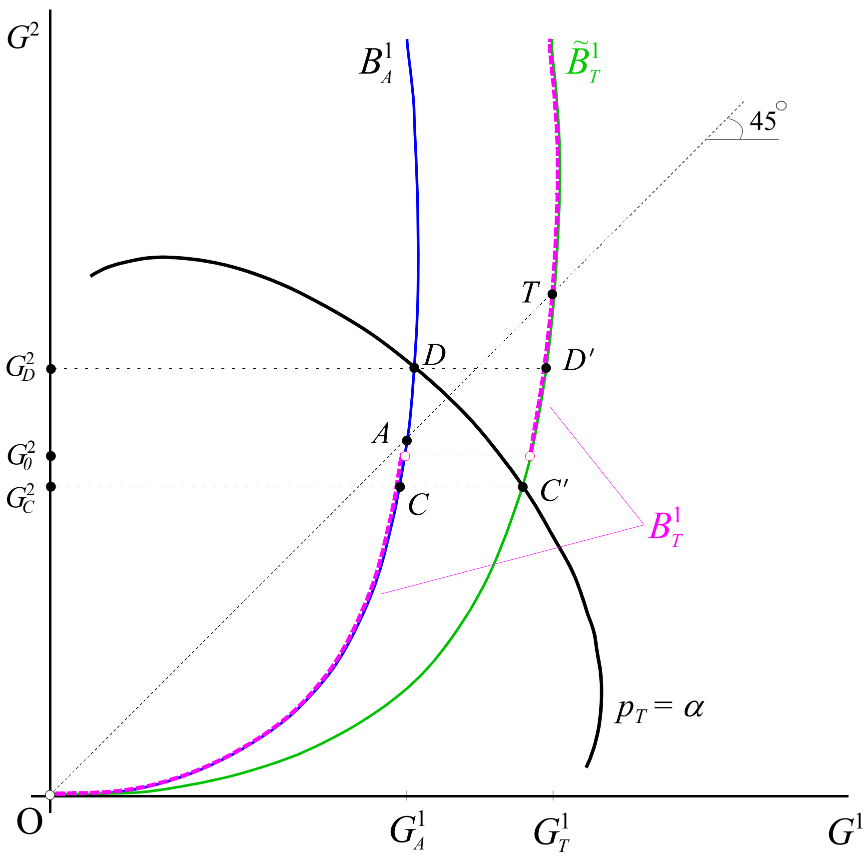

Figure B.2: Derivation of Best-Response Function under Free Trade and Comparison of Nash Equilibria in Security Policies in the Three-Country Case

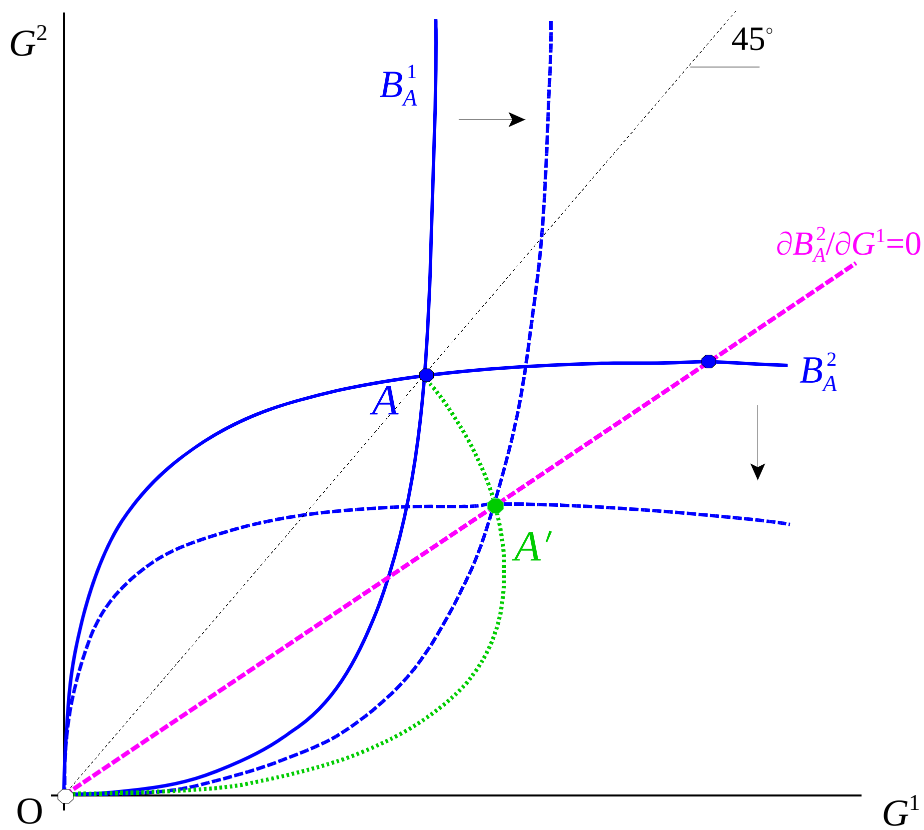

Figure B.3: Autarkic Best-Response Functions and Their Dependence on the Distribution of Secure Resources Endowments

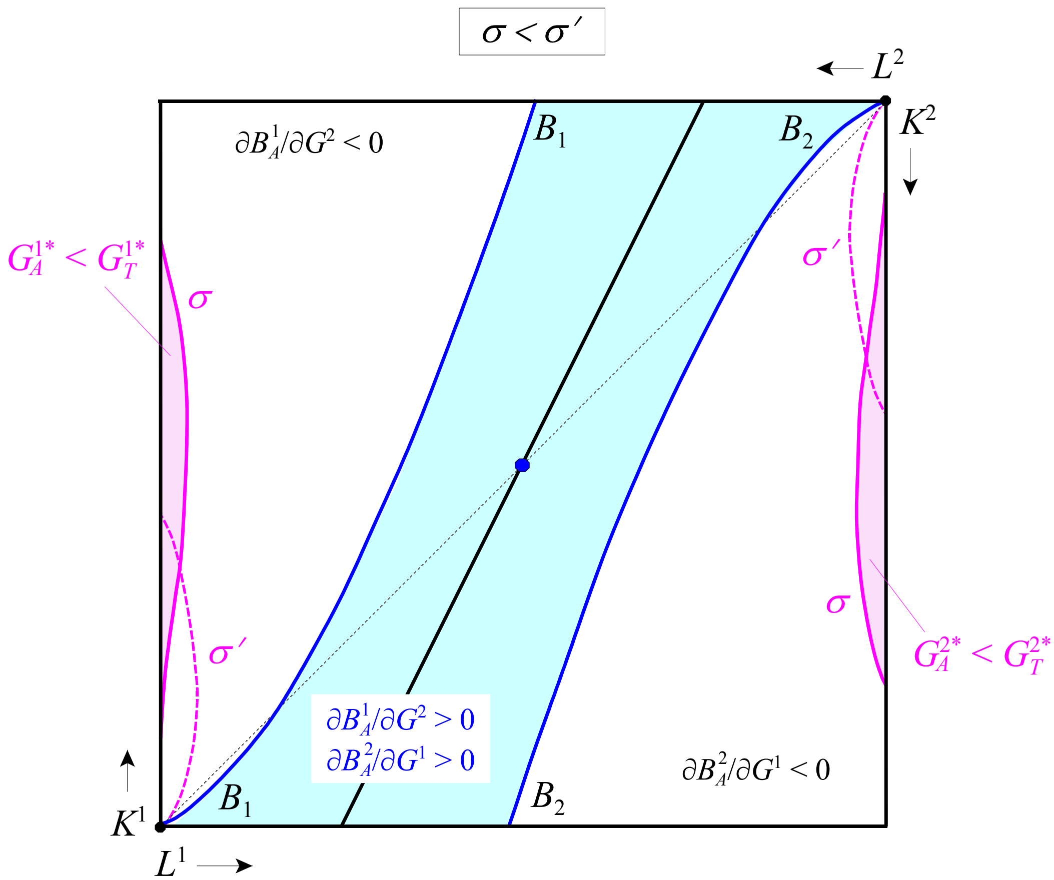

Figure B.4: The Distribution of Secure Resources Endowments and Strategic Complementarity/Substitutability in Security Policies

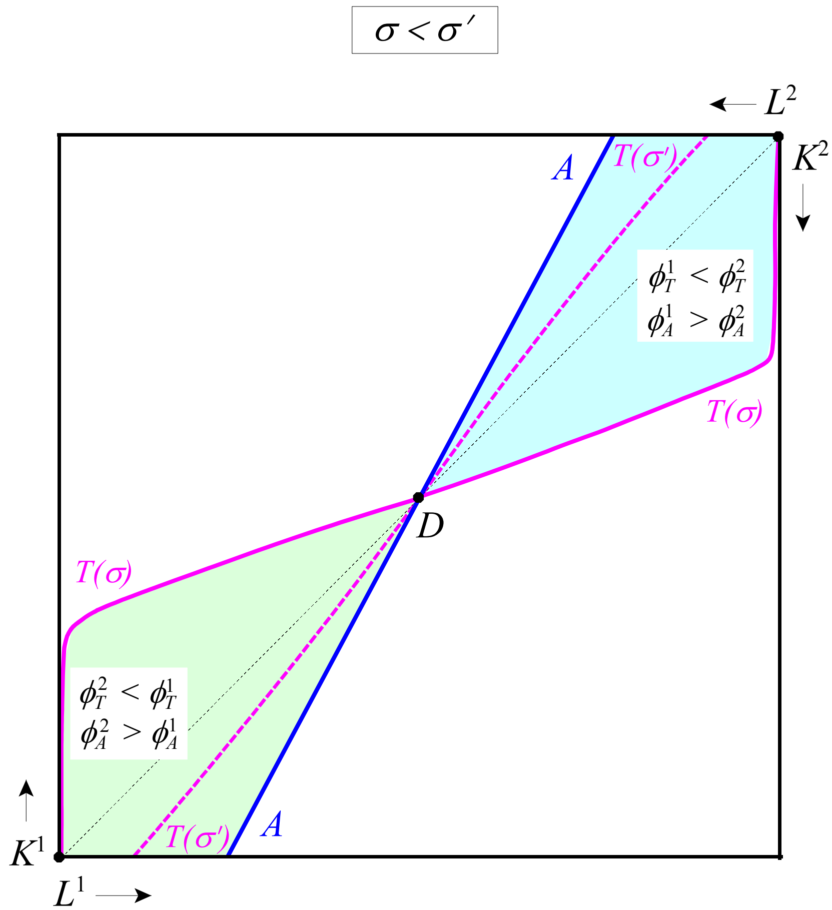

Figure B.5: The Distribution of Secure Resource Endowments and Relative Power Reversals under Trade

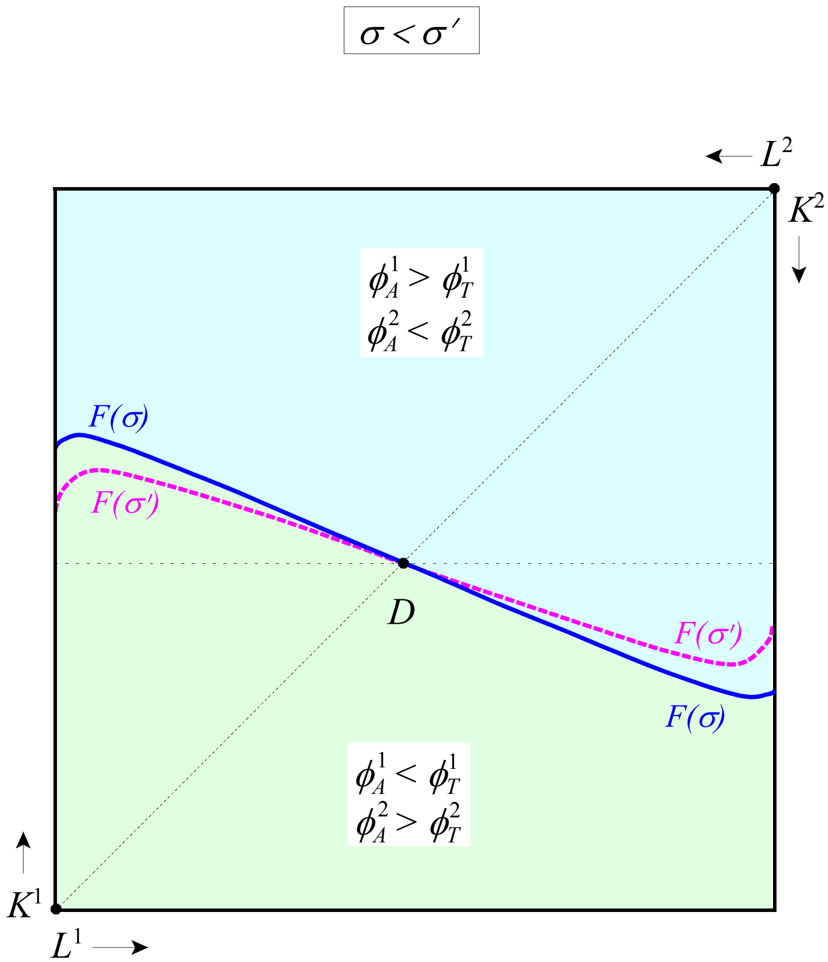

Figure B.6: Superiority in Power, Trade Regimes and the Distribution of Secure Resource Endowments

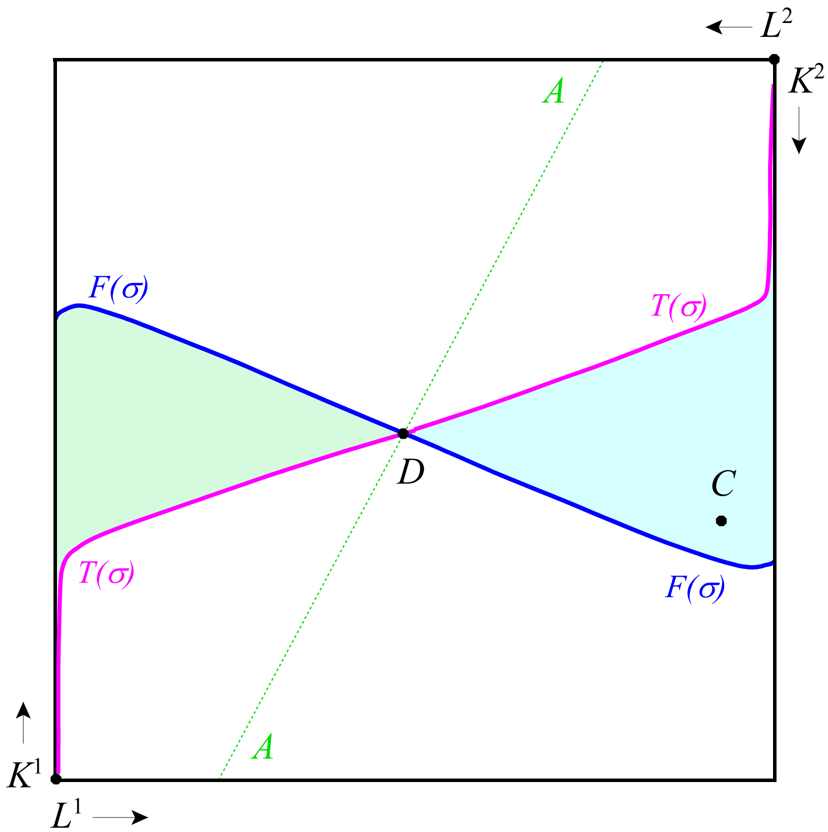

Figure B.7: Power, Trade Regimes and the Distribution of Secure Resource Endowments