TECHNOLOGIES OF CONFLICT

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ABSTRACT: We explore theoretical foundations and issues in the empirical estimation of conflict technologies. Such technologies are probabilistic choice functions that depend on the military capacities of adversaries, where the military capacities themselves depend on economic inputs via ordinary production functions. Different classes of functional forms can be derived stochastically or axiomatically. The additive form, in particular, (which includes both the logit and ratio functional forms) has both stochastic and axiomatic foundations. Issues in the empirical estimation that we explore include concerns with data, endogeneity, structural breaks, and model comparison.

1 Introduction

Warfare is a costly economic activity that combines inputs as varied as those employed in any ordinary economic activity. Modern warfare requires, for example, many different types of labor: common soldiers, officers, tank and airplane technicians, electronics specialists, skilled pilots, complex logistic experts, nuclears scientists and engineers, spies and other specializations that have no counterparts in the civilian economy. The weapons systems and the means through which they are designed and manufactured can be similarly either routine or highly specialized compared to ordinary production. Labor, capital, and land – the three general factors of production – are all employed in warfare. This is not just true of modern warfare. After the agricultural revolution and the emergence of recognizable states, warfare became a highly organized activity, with logistics, engineering, and hierarchies that often superseded anything comparable in civilian production. The level of sophistication of ancient siege machinery, for instance, was highly complex and in ways that were rarely similar in civilian projects (Landels, 1978).

Contrary to ordinary production in which the final outputs are typically concrete and measurable, however, the final output of warfare is less clearly definable. The weapons and manpower of militaries are themselves inputs to what can be thought of as *military capacity*, but that in itself can only be considered an intermediate input. In turn, the military capacities of rivals, unlike the case of ordinary economic production in which inputs are combined cooperatively, are combined *adversarially*. The ultimate, final output of warfare in which military capacities are adversarially combined, can be thought of as wins and losses, each one with a probability that depends on the rivals' military capacities.

With the building of military capacities following the rules of ordinary economic production, it is how military capacities of adversaries translate into probabilities of wins and losses that is the focus of this chapter. Hirshleifer (1989) was, to our knowledge, the first to call such functions *technologies of conflict*, and this is the term that we adopt in this chapter as well. Technologies of conflict are examples of probabilistic choice functions that were first examined in the case of individual choice by Luce (1959) and, independently, by econometricians in the 1970s (e.g., McFadden, 1974). For applications in games of not only warfare but also of any situation in which inputs are combined adversarially – with applications from sports to litigation and lobbying and rent-seeking – these functions have been called *contest* or *contest success functions*.[1] The theory of such functions is becoming mature. However, there is very little empirical research estimating

[1]For overviews of the theory of contests and conflict, see, respectively, Konrad (2009) and Garfinkel and Skaperdas (2007).

such functions in the case of warfare or, more generally, of contests.

In section 2 we first introduce some commonly employed functional forms and then provide an overview of two different types of foundations: stochastic and axiomatic. In section 3, we discuss some of the technical issues in econometrically estimating technologies of conflict.

2 Functional forms and theoretical foundations

Our purpose in this section is to introduce and discuss the properties of different functional forms of technologies of conflict and explore theoretical foundations on which they might be based. The two main types of theoretical foundations that we will consider are those we call *stochastic* and those that are typically called *axiomatic*.Stochastic foundations are based on assumptions about how performance in warfare – in the sense of probabilities of winning and losing – might be a noisy function of military capacities. Axiomatic foundations are derived from general properties (or, axioms) that a technology of conflict might be expected to have and the implications of combinations of such properties would be expected to have for functional forms.

Consider two adversaries, labeled 1 and 2. Denote their choice of military capacities as M_1 and M_2. We suppose that military capacities are the outputs of production functions of different inputs. These production functions can be the same for the adversaries or they can be different. Associated with them are cost functions $c^1(M_1)$ and $c^2(M_2)$. Since in this section we are solely concerned with how pairs of military capacities translate into probabilities of wins and losses and not how military capacities might be chosen, we will keep these cost and production functions in the background. For any given combination of military capacities, each rival has a probability of winning and a probability of losing. (The probability of an impasse or "draw" is considered to be zero, but we do discuss the case when this assumption does not hold below.) Denote the probability of party $i = 1$ winning as $p_1(M_1,M_2)$ and the probability of party $i = 2$ winning as $p_2(M_1,M_2)$.

For the $p_i's$ to be probabilities, they need to take values between 0 and 1, and add up to 1: $p_2(M_1,M_2) = 1- p_1(M_1,M_2) \geq 0$. Moreover, we can expect an increase in one party's military capacity to increase that party's winning probability and reduce the winning probability of its opponent; that is, we should have $p_1(M_1,M_2)$ be strictly increasing in M_1 (when $p_1(M_1,M_2) < 1$) and strictly decreasing in M_2 (when $p_1(M_1,M_2) > 0$).

A class of technologies that has been widely examined takes the following additive form:

$$p_1(M_1, M_2) = \begin{cases} \frac{f(M_1)}{f(M_1)+f(M_2)} & \text{if } \sum_{i=1}^{2} f(M_i) > 0; \\ \frac{1}{2} & \text{otherwise,} \end{cases} \tag{1}$$

where $f(\cdot)$ is a non-negative, strictly increasing function. This class has been employed in a number of fields, including in the economics of advertising (Schmalensee, 1972, 1978), sports economics (Szymanski, 2003), rent-seeking (Tullock, 1980; Nitzan, 1994), as well as contests in general (Konrad, 2009).

One unique and appealing feature of the class of conflict technologies in (1) is that it naturally extends to the case of conflict between more than two parties. Thus, if there were n parties to the conflict, denoting the military capacity of rival i by M_i, and the vector of gun choices by all other agents $j \neq i$ by M_{-i}, the winning probability of i would be as follows:

$$p_i\left(M_i, M_{-i}\right) = \begin{cases} \frac{f(M_i)}{\sum_{j=1}^{n} f(M_j)} & \text{if } \sum_{j=1}^{n} f(M_j) > 0; \\ \frac{1}{n} & \text{otherwise.} \end{cases} \tag{2}$$

The most commonly used functional form is the one in which $f(M_i) = M_i^{\mu}$,[2] where $\mu > 0$ (and often, for technical reasons of existence of pure-strategy Nash equilibrium, $\mu \leq 1$), so that

$$p_1(M_1, M_2) = \frac{M_1^{\mu}}{M_1^{\mu} + M_2^{\mu}} = \frac{(\frac{M_1}{M_2})^{\mu}}{(\frac{M_1}{M_2})^{\mu} + 1}. \tag{3}$$

This functional form, sometimes referred to as the "power" form or as the "ratio" form, is that which was employed by Tullock (1980) and the ensuing voluminous literature on rent-seeking. This is also the workhorse functional form used in the economics of conflict. As Hirshleifer (1989) has noted, the probability of winning in this case depends on the *ratio* of military capacities, $\frac{M_1}{M_2}$, of the two parties.

Another well-known functional form is the following "logit" specification, in which $f(M_i) = e^{\mu M_i}$, where $\mu > 0$, so that,

$$p_1(M_1, M_2) = \frac{e^{\mu M_1}}{e^{\mu M_1} + e^{\mu M_2}} = \frac{1}{1 + e^{\mu(M_2 - M_1)}}. \tag{4}$$

Again as Hirshleifer (1989) has noted and as is evident from the expression following the second equal sign in (4), by this specification, the probability of winning depends on the *difference* in guns between the two parties. As we shall see, the general additive form as well as the two specific functional forms have both stochastic and axiomatic foundations.

2.1 Stochastic foundations

The outcome of a particular battle or war can be thought of as a noisy function of the two rivals' military capacities. In particular, we can posit that each rival's "performance" on the battlefield, denoted by Y_i, is a function

[2] A variation on this form is $f(M_i) = M_i^{\mu} + b$ where $\alpha = b > 0$. Amegashie (2006) examined the properties of this form.

of military capacity and noise so that $Y_i = h(M_i, \theta_i)$ where θ_i represents a random variable and $h(\cdot, \cdot)$ is a function of the two variables. Then, the probability of side 1 winning can be represented by the probability that its performance is higher than that of its adversary so that:

$$p_1(M_1, M_2) = prob[Y_1 > Y_2] = prob[h(M_1, \theta_1) > h(M_2, \theta_2)] \tag{5}$$

From this stochastic perspective, each side's probability of winning depends not only on the military capacities of of both sides, but also on the functional form of $h(\cdot, \cdot)$ and the distribution of the θ_i's .

The most commonly used form of $h(\cdot, \cdot)$ is the linear form so that $h(M_i, \theta_i) = M_i + \theta_i$. In that case, when the θ_i's are independently identically distributed according to the normal distribution, the resultant probabilities of winning and losing for the two sides are described by the *probit* (see, e.g., Train, 2003; Albert and Chib, 1993),

$$p_1(M_1, M_2) = \Phi(M_1 - M_2), \tag{6}$$

where Φ is the cumulative distribution function of the standard normal distribution.

Most likely because there is no analytical functional form representing the probit, it has not been used as much in the case of conflict games as for (2) or its more general form in (1).

Still when $h(M_i, \theta_i) = M_i + \theta_i$ but the θ_i's are independently identically distributed according to the extreme value distribution, the logit form is obtained (McFadden, 1974) :

$$p_i(M_i, M_{-i}) = \frac{e^{\mu M_i}}{\sum_{j=1}^{n} e^{\mu M_j}}. \tag{7}$$

The cumulative distribution function of the (type I) extreme value distribution is

$$G_{\theta_i} = \exp(-\exp(-z)),$$

which has been known as the double exponential distribution (Yellott, 1977; Luce, 1977) or the log-Weibull distribution.

Motivated by the derivations of the probit and logit forms, Jia (2008b) provides a stochastic interpretation of the ratio form (3) by assuming the performance function $h(\cdot, \cdot)$ has the multiplicative form $h(M_i, \theta_i) = M_i\theta_i$. This result extends to the n-party conflict model:

$$p_i(M_i, M_{-i}) = \frac{M_i^{\mu}}{\sum_{j=1}^{n} M_j^{\mu}}. \tag{8}$$

Jia (2008b) shows that, for $n > 2$, the conflict model has the generalized ratio form (8) if and only if the independent random shocks $\{\theta_i\}_{i=1}^{n}$ have a specific distribution, which is known as the *Inverse Exponential Distribution.*

Specifically, a random variable belongs to the *Inverse Exponential Distribution with Parameters* α *and* μ, $\alpha, \mu > 0$ [$IEXP(\alpha, \mu)$ for short] if and only if its probability density function (p.d.f.) has the form:

$$g(z) = \alpha\mu z^{-(\mu+1)} \exp(-\alpha z^{-\mu}) I_{[z>0]}, \tag{9}$$

where I is the indicator function which is equal to 1 when $z > 0$ and 0 otherwise. Accordingly, the cumulative distribution function (c.d.f.) of $IEXP(\alpha, \mu)$ is

$$G(z) = \int_0^z h(s)ds = \exp(-\alpha z^{-\mu}).$$

For an $IEXP(\alpha, \mu)$ distributed random variable, one can verify that neither its expectation nor variance exist, and its mode is located at $(\frac{\mu\alpha}{\mu+1})^{1/\mu}$. When α increases, its p.d.f. becomes flatter, and more and more mass is being pushed to the right. The parameter μ plays an opposite role. When μ decreases, the p.d.f. becomes flatter.

Jia (2008b) interprets the parameter μ in (7) and (8) as the "noise" of the conflict. Common to all contestants, the parameter μ captures the marginal increase in the probability of winning caused by a higher resources expenditure. Conflicts with low μ can be regarded as poorly discriminating or "noisy" conflicts. When μ converges to zero, the conflict outcome converges to a random lottery with no dependence upon the military capacities of the conflicting parties. Conflicts with high μ can be regarded as highly discriminating; as μ approaches infinity, the conflict outcome is determined by an all-pay auction (in which the side with the highest military capacity wins with probability 1).

A lucid interpretation of the power μ, and an alternative derivation of (8) from (9), has been given by Fu and Lu (2008). Fu and Lu notice that when $\mu = \alpha = 1$, the corresponding conflict model has the conventional Tullock form (Tullock, 1980) as shown in equation (8) where $f(M_i) = M_i$.

Now, consider μ independent draws[3] from the distribution $IEXP(1, 1)$ and name them $\theta_1, \theta_2, \ldots, \theta_\mu$, i.e., $\theta_i \overset{i.i.d.}{\sim} \text{IEXP}(1, 1)$, or $G_{\theta_i}(z) = \exp(-z^{-1})$. Taking the logarithmic transformation of these θ_i's yields a new set of random variables $\{\zeta_i\}_{i=1}^n$, where $\zeta_i = \ln \theta_i$. One can derive the distribution of ζ_i and find its c.d.f. $G_{\zeta_i}(z) = \exp(-\exp(-z))$, which, as mentioned previously, has been known as a "double exponential distribution"(Yellott, 1977; Luce, 1977) or type I extreme-value (maximum) distribution (McFadden, 1974). The latter name reveals its property: Consider the maximum of these μ random variables and denote it $\zeta_{(\mu)}$. Then for any $z > 0$, the cumulative

[3]Here, we assume μ is an integer.

distribution function of the random variable $\zeta_{(\mu)}$ is given by

$$\Pr(\zeta_{(\mu)} < z) = \Pr(\zeta_i < z, \forall 1 \leq i \leq n) = \prod_{i=1}^{\mu} \Pr(\zeta_i < z) = \prod_{i=1}^{\mu} G_{\zeta_i}(z)$$
$$= \prod_{i=1}^{\mu} \exp(-\alpha z^{-1}) = \exp(-\mu\alpha z^{-1}).$$

One can then show that $\zeta_{(\mu)} \sim \text{IEXP}(\alpha, \mu)$. That is to say, a contest with contest success function (8) can be viewed as follows. Each contestant i competes against others by producing better performance. An M_i amount of effort allows contestant i to have a number μ of independent attempts. Each attempt allows the contestant to produce a performance with a random value $\theta \overset{i.i.d.}{\sim} \text{IEXP}(1,1)$. If the contest selects the highest realized performance as a contestant i's entry in the contest, then the corresponding contest success function takes the form (8).

The stochastic approach developed in Jia (2008b) can be easily extended to the general conflict model (2). As such the logit form (7) is isomorphic to the ratio or power form (8) up to a logarithmic transformation. In addition, asymmetric functional form such can be rationalized by relaxing the assumption that all the random variables θ_i's are identically distributed.

Jia (2009) extends the stochastic approach further to allow for the possibility of a draw (or, stalemate) in conflict, which corresponds to the situation that no party can force a win. This is done by introducing a "threshold" c into the performance comparing process. The intuition is simple. A draw can arise if every performance comparison is decided by estimates of the difference in the adversaries' performances with error and is a draw if this difference is smaller than a "threshold" value $c > 0$. Indeed, in most conflicts, the outcomes are not determined by each party's performance, but by measures of their performances, which is the process of estimating the magnitude of all parties' performances against some unit of measurement. Adopting the assumptions that (1) adversaries' performances are determined by their military capacities and some random variables θ_i's, and (2) the random variables are independently and identically distributed with an *inverse exponential distribution*, Jia derives the following functional forms

$$p_i(M_i, M_{-i}) = \frac{f(M_i)}{f(M_i) + c\sum_{j\neq i} f(M_j)}, \quad c > 1, \tag{10}$$

and

$$p_i(M_i, M_{-i}) = \frac{f(M_i)}{(n-1)c + \sum_{j=1}^{n} f(M_j)}, \quad c > 0. \tag{11}$$

Again, by relaxing the i.i.d. assumption to require only independence, one can easily obtain more general asymmetric forms.

2.2 Axiomatic foundations

Luce (1959) first axiomatized probabilistic choice functions such as those in (2) in relation to utility theory, while Skaperdas (1996) provides an axiomatization in relation to contests and conflict. Key to both axiomatizations is an *Independence of Irrelevant Alternatives* property. In the context of conflict, this property requires that the outcome of a battle between any two parties depend only on military capacities of these two parties and not on the military capacities of any third parties to the battle.

The particular "ratio" form, where $f(M_i) = M_i^{\mu}$, has the property of *homogeneity of degree zero* in military capacities, or $p_i(tM_i, tM_{-i}) = p_i(M_i, M_{-i})$ for all $t > 0$. This is an analytically convenient property and likely accounts for the popularity of this functional form in applications.

The "logit" form, where $f(M_i) = e^{\mu M_i}$, can be derived under the property that each adversary's probability of winning is invariant to the addition of a constant D to the military capacity of each adversary (i.e., $p_i(M_i + D, M_{-i} + D) = p_i(M_i, M_{-i})$ for all D such that $M_j + D > 0$ for all j).[4] Though the logit form also has analytical advantages, it has been not used as much as the power form shown in (3). The reason is that, for a number of well-specified models, no pure-strategy Nash equilibrium exists.

Thus, both the "ratio" functional form in (3) and the "logit" form in (4) can be derived axiomatically as well as stochastically.

The class in (1) and the specific forms in (3) and (4) have the property of symmetry or anonymity, in the sense that if the military capacities of two adversaries were switched, their probabilities of winning would switch as well. Consequently when two adversaries have the same military capacities, they have equal probabilities of winning and losing. There are circumstances, however, in which one party might be favored over another even though they might have the same levels of military capacity. An obvious setting conducive to such an asymmetry is where one party is in a defensive position vis a vis her opponent (see, for example, Grossman and Kim, 1995). The defender typically, but not always, has the advantage. A simple way to extend (1) to take account of such asymmetries is the following form:

$$p_1(M_1, M_2) = \frac{a_1 f(M_1)}{a_i f(M_1) + a_2 f(M_2).} \tag{12}$$

where a_1 and a_2 are positive constants. Note that when the adversaries have the same military capacities, $M_1 = M_2$, 1's probability of winning equals $\frac{a_1}{a_1+a_2}$ and 2's probability of winning is $\frac{a_2}{a_1+a_2}$. Consequently, when $a_1 > a_2$, 1 has an advantage, whereas when $a_1 < a_2$, 2 has the advantage. Clark and Riis (1998) have axiomatized this asymmetric form for the case of the ratio

[4] Hirshleifer (1989) , Hirshleifer (1995) and Hirshleifer (2000) provide many insightful discussions of technologies of conflict and comparisons of the functional forms in (3) and (4).

form (i.e., where $f(M) = M^{\mu}$). Rai and Sarin (2009) have provided more general axiomatizations of this that also allow for the function $f(\cdot)$ to be one of many inputs and not just of military capacity (which we have assumed, in general, to be of other inputs as well). Finally, Münster (2009) provided a reinterpretation and extension of the axioms in Skaperdas (1996) and Clark and Riis (1998) by allowing adversaries to be members of groups.

For functional forms that allow for the possibility of a draw, Blavatskyy (2010) axiomatized the following extension:

$$p_1(M_1, M_2) = \frac{f_1(M_1)}{1 + f_1(M_1) + f_2(M_2)}, \tag{13}$$

where $f_1(\cdot)$ and $f_2(\cdot)$ are non-negative strictly increasing functions. Note that this is essentially the 2-player asymmetric version of (11) derived stochastically by Jia (2009).[5] That is, "1" in the denominator of (13) does not have any special significance because if we were to multiply its numerator and denominator by any positive number we would get an equivalent functional form. One way of thinking about (13) is to consider a third party, say "Nature," that has a constant military capacity, M', which is defined by $f(M') = 1$ (where $f(\cdot)$ is non-negative and increasing). When Nature "wins," there is a draw. Blavatskyy (2010) has extended (13) to more than 2 adversaries but not in the straightforward way that (2) extends (1).

2.3 The difference form

The literature on contests has employed or derived some other functional forms that have not been used in the peace and conflict literature. One such class of functional forms is the "difference" form:

$$p_1(M_1, M_2) = \alpha + h_1(M_1) - h_2(M_2) \tag{14}$$

where $\alpha \in (0, 1)$ and the functions $h_1(M_1)$ and $h_2(M_2)$ are suitably constrained so that $p_1(M_1, M_2) \in [0, 1]$. Contest games under specific cases of this class of functions have been explored by Baik (1998) and Che and Gale (2000). Skaperdas and Vaidya (2010) have derived this class in a Bayesian framework in which an audience (for example, a judge) makes a decision based on "evidence" produced by two contestants, with $h_1(M_1)$ and $h_2(M_2)$ being probabilities. That is, the difference class of functions has been derived for non-violent cases of conflict such as litigation, lobbying, and political campaigning. Corchón and Dahm (2010) also derive a particular class of the difference form (similar to that examined by Che and Gale (2000) in an axiomatic setting in which the contest success functions is thought of as a share instead of as a probability.

[5] Just define $f_i(M_i) \equiv \frac{1}{(n-1)c} f(M_i)$

The technologies of conflict we have reviewed in this section have been derived mostly either axiomatically or stochastically. They are thus comparable in terms of foundations to production functions and utility functions, and the theoretical research in the area is rather mature. We next turn to issues of empirically estimating such functions.

3 Some Issues in Empirical Estimation

There is only a small body of literature devoted to empirical estimates and tests of conflict technologies. Perhaps one reason for that is limited data on conflict and, in particular, on battles. Unlike sports economics and the tournament literature in labor economics, conflict data are far less available and are more difficult to obtain and construct. Moreover, since all conflict technologies are nonlinear comparing different functional forms becomes a difficult task. In this section, we review the few empirical studies on the topic and discuss four main issues in empirical estimation.

3.1 Data and recent empirical estimates

Most empirical research on conflict is based on one of two main data sets. They are the battalion-level data set (HERO)[6] authored by the Institute of Dupuy and the interstate and civil wars data set of the Correlates of War (COW) project[7]. The data on battles are the most appropriate for the type of conflict technologies we have discussed up to this point. The HERO data on battles include all major battles that took place from 1600 to 1989. Dozens of aspects of each battle were evaluated and systematically coded by military historians on the basis of primary and secondary sources; their judgements were then evaluated critically by members of the U.S. government. Although the data have some peculiarities, they have been used to predict battle outcomes (Dupuy, 1985), and this data set is the only cross-temporal, large-sample, quantitative data set on individual battles in existence. It is worth noting that the HERO data set considers only battalion-level engagements, which were usually at most of one day in length.

In principle, the data set provides information about quantitative and qualitative aspects of the armies involved in each battle, such as personnel strengths and numbers of artillery units, tanks, and close air support sorties. The data set also contains information on experts' assessments of qualitative factors including initiative, morale, technology, logistics, intelligence, and leadership. Features of the HERO data set that may be of concern include the following. First, the data are all based on ex-post judgements.

[6]This data set is available at `http://www.dupuyinstitute.org/dbases.htm`.
[7]This data set is available at `http://www.correlatesofwar.org/datasets.htm`

The military historians, of course, knew the outcome of the battles when they made the codings. Thus, a legitimate concern is that these data are contaminated with the bias of hindsight. For example, coders might be more likely to infer that one side had superior morale because it won the battle. However, given the mission of the HERO project was to build a complex model of battle outcomes accounting for an array of material and intangible factors, one would reasonably believe that measures had been take to avoid such biases. Reiter and Stam (1998), moreover, present evidence that such biases are not severe. They argue that if this bias were true, one would observe extremely strong correlations between the determinants of victory (morale, initiative, and so on) and outcomes, where in fact only few significant relationships are found.

The second potential problem with the HERO data set is that, except for personnel strengths, there are many missing observations on certain variables, especially for variables on heavy equipment (artillery, tanks) and support systems. This is of course partly due to their non-existence in the relevant subperiods (such as the Thirty Years War between 1618 and 1648), and partly due to missing information.

Third, as the main body of the data set, the World War II data include only the *ratio* of combat powers rather than actual numbers. This feature reduces the usefulness of the data set and also makes model comparison more difficult. For instance, as showed in (4), in the logit form of CSFs the probability of winning depends on the difference of military capacities of the two parties. The logit model, therefore, cannot be estimated by the HERO data. The same logic applies to the more general probit model too. It is easy to show that the HERO data cannot be fitted to the probit model either.

A number of scholars have used the HERO data set. For instance, Reiter and Stam (1998) show that political regimes play an important role in determining battlefield success. Specifically, they argue that armies of democratic states fight with higher military effectiveness on the battlefield by showing statistically that the armies of democratic states tend to fight with substantially better initiative, and superior leadership. By estimating an ordered probit model, Reiter and Stam first identify six key indicators of battlefield success, which are logistics, intelligence, technology, initiative, leadership, and morale. They categorize the first three as the organizational aspects of an army, and the latter three indicators as the soldiering factors of the army. By further estimating a set of bivariate regressions models, using a state's level of democracy as an independent variable and these six key indicators of battlefield success as the dependent variables, Reiter and Stam claim that, on the one hand, relative democracy is not associated either substantially or statistically with any of the organizational aspects of battlefield success; on the other hand, the relationship between two of the three soldiering factors (initiative and leadership) and democracy appears to be quite strong. Their results are summarized in the following Table 1.

Table 1: Relationship between Democracy and Indicators of Success *

Independent Variable	*Dependent Variable*					
	Logistics	*Intelligence*	*Technology*	*Initiative*	*Leadership*	*Morale*
Democracy	.016	.0028	.0059	.086	.093	.011
	(1.42)	(.16)	(.66)	(3.31)	(3.33)	(.56)
Constant	-.0064	.039	-.044	-.16	-.31	-.00006
	(-.09)	(.45)	(-.77)	(-1.15)	(-2.13)	(.001)
Adjusted R^2	.02	.001	.003	.11	.12	.01
Probability $> F$	.16	.87	.51	.001	.001	.56

* Cell values are ordinary least squares estimates on 9-point scales. T-values are reported in parentheses.

Rotte and Schmidt (2002) is a similar study aiming to identify the determinants of battlefield success. Using the same data set, Rotte and Schmidt find that, contrary to the emphasis on technology, the numerical superiority has retained its crucial role for battlefield performance throughout history, together with other human elements of warfare such as leadership, morale, and surprise. Their conclusions come from the estimates of the parameters of two probit models. The first one (Model 1) takes force ratio, posture, surprise, leadership, training, morale, logistics, intelligence, and technology as explanatory variables. And the second model (Model 2) replaces the force ratio variable by the square of it, while keeping all the other variables unchanged. They evaluate the marginal effect for every explanatory variable, which is calculated as the difference in predicted probabilities for the corresponding variable changing from 0 to 1 at the mean of the force ratio (2.1641). Their results are summarized inTable 2.

3.2 Endogeneity

Among the problems of estimating conflict technologies, potential endogeneity calls for special attention. In an econometric model, a variable is said to be endogenous when there is a correlation between the variable and the error term. Generally, a loop of causality between the independent and dependent variables of a model leads to endogeneity. As highlighted in the political economy literature (e.g., see Miguel, Satyanath, and Sergenti, 2004) and more generally, endogeneity generates inconsistent and biased estimates of the unknown parameters, which adversely affects the explanatory power of the model. (See Kennedy, 2008, for more detailed discussions). For our purposes here, endogeneity mainly comes in three possible ways.

Table 2: Marginal Effects of the Determinants of Attacker's Battle Success

Marginal Effects	*Model 1*	*Model 2*
Force ratio	.0516	.0953
	(4.01)	(2.42)
Posture	-.0153	-0255
	(.28)	(.46)
Surprise	.1423	.1412
	(2.76)	(2.72)
Leadership	.4530	.4630
	(9.17)	(9.32)
Training	-.0181	.0046
	(.27)	(.07)
Morale	.2683	.2607
	(5.25)	(5.02)
Logistics	.2019	.1996
	(2.53)	(2.49)
Intelligence	.2585	.2584
	(3.39)	(3.36)
Technology	-.0616	-.0852
	(.53)	(.72)

[a] T-values are reported in parentheses.

One channel through which endogeneity affects conflict is through measurement error. Estimating a conflict technology requires one to measure the resources both parties have devoted to the conflict. However, measuring such resources could be a very difficult task, if it is possible at all. The main difficulty arises because some resources are unquantifiable. For example, morale, intelligence, and logistics can be key factors in determining battlefield success, but since these factors are difficult to quantify, they are sometimes excluded from the data. To overcome this difficulty, researchers propose to use various proxies to the real resources spent. For example, Collier and Hoeffler (1998) suggest using casualties to approximate the resources devoted to civil wars. Hwang (2009) proposes using personnel strengths as a proxy of resources. All of these practices introduce measurement error and hence a possible endogeneity problem.

Comparing conflict technologies to production functions provides another perspective on the potential endogeneity problem involved. As we discussed in the previous section, the military capacity of each adversary, M_i, can be thought of as a production function F that employs m inputs, $(x_1, x_2, \ldots, x_m)$. Scholars from von Clausewitz (1990) to Dupuy (1990), have identified military success or failure throughout history to involve factors such as numbers, morale, technology, and logistics. These elements are typically interpreted as initial conditions, which come to bear in the process of fighting a battle, and which ultimately lead to its outcome: victory or defeat. Although total military capacity cannot be directly observed, one can regard it as a latent variable, and assume the actual outcome of the battle, victory or defeat, is determined as we have done in the previous section by comparing the adversaries' military capacities. However, even with an observable military capacity, the estimation of the "production function" is still questionable from a statistician's perspective. For example, Marschak and Andrews (1944) argue that in a simple "production function,"

$$M = F(x_1, x_2, \ldots, x_m), \tag{15}$$

the independent variables $x_1, x_2, \ldots, x_m$ are not "independent" because they are not controlled by the firm owner who is a profit maximizer. That is to say, the quantities $x_1, \ldots, x_m$ are determined endogenously by the profit-maximizing choices of the firm, and therefore the economist is confronted not only with the single equation (15) in which she may be interested; but with a system of equations which consists of m profit-maximizing conditions. This simultaneity problem generated by the relationship between productivity and input demand has been a major subject in the literature. Applied researchers have spent much effort addressing the econometric problem this simultaneity creates (e.g., see Levinsohn and Petrin, 2003; Olley and Pakes, 1996). Due to the similarity between the production functions and conflict technologies, it is easy to see that conflict technologies also share the simultaneity problem, which introduces another source of endogeneity to our models.

The third possible cause of endogeneity comes from the possibility that the dependent variable, battle outcomes, may have some causal effect on the explanatory variable such as resources invested in the battlefield. Battlefield success is achieved through a series of strategic interactions. During this period, each adversary keeps updating the information about its battlefield situation, such as position and casualties. It then makes the corresponding moves to reinforce its possibility of winning. Clearly, then, both adversaries' total resources invested in the battlefield could be affected by the contemporary combat result. To be more specific, there are two potential channels through which this feedback effect could operate. (1) One party has gained overwhelming advantage so as to secure the final success of the combat. In this case, the leading party may pull back in effort and resources (manpower, artillery power, and so on) in the final stage of a combat or even in the final stage of an entire warfare. (2) One party has fought so poorly that it loses any hope of achieving a final success. In such a case, it is also likely that this party will give up in the following campaigns. This effect can also explain one of the common beliefs of many military experts: when an adversary suffers more than a 50% casualty rate, it is generally unable to keep fighting and should retreat from the battlefield. In both cases, the contemporaneous combat results affect the current and future resources devoted to the battlefields, which suggests a reciprocal causal relationship may exist, thereby giving rise to a possible endogeneity problem. This problem need not be present when we consider the larger environment of a battle, whereby the adversaries just enter into battle with the aim of winning. The possible adjustments to inputs in response to expected wins or losses would be more likely to occur in smaller battles, or sub-battles. That is, endogeneity is less likely to be a problem in larger battles and more likely in smaller ones.

Unless the endogeneity among economic variables is adequately addressed, establishing a convincing causal relationship is difficult. Endogeneity often leads to biased estimates of parameters of interest and false policy implications. In order to remove the endogeneity problem from estimates of conflict technologies, researchers are advised to do two things: (1) USe instrumental variables (IV) to alleviate the potential measurement error problem and (2) carefully scrutinize the data and remove observations which could be possibly affected by the feedback effect.

Generally speaking, an instrumental variable is one that does not itself belong in the explanatory equation and is correlated with the endogenous explanatory variables, conditional on the other explanatory variables. Formal definitions of instrumental variables, using counterfactuals and graphical criteria, are given in Pearl (2000). Heckman (2008) also gives a thorough discussion about the relationship between instrumental variables method and causality in econometrics. As a rule of thumb, there are two main requirements for using an IV:

1. The instrument must be correlated with the endogenous explanatory variables, conditional on the other explanatory variables.

2. The instrument cannot be correlated with the error term in the explanatory equation, that is, the instrument cannot suffer from the same problem as the original predicting variable.

The second treatment, data cleansing, requires the researchers to identify the potential feedback effect between the error terms and the explanatory variables. In the current context, any conflict observation involving strategic interactions between two rivalry parties should be removed from the data set.

3.3 Structural Breaks

Since military technology has been changing over time it is important to take account of those changes. Dupuy (1990) and Dupuy and Dupuy (1993) provide an overview of the basic technological developments and changes in warfare since the beginning of the 17th century. These two papers provide guidelines to divide the era of modern warfare into several subperiods based on their technological characteristics. For example, Rotte and Schmidt (2002) propose that there should be seven distinct periods of prevalent combat technology since 1600 AD with major consequences for tactical doctrine, organization, and other key variables (1600-1700, 1701-1779, 1780-1849, 1850-1889, 1890-1929, 1930-1960,1961-present).

There are other ways to divide the modern history of warfare into subperiods. For example, Snickersbee (2006) divides the era of warfare since 1600 AD into two subperiods: The gunpowder era (1600-1800) and the industrial period (1800 to the present). According to Snickersbee (2006), the gunpowder era "sees a renewal of centralization, the final defeat of the threat from the steppes, the obsolescence of fortifications, and a major innovation as cannon are placed on ships to provide the basis of colonial trading empires not based on migration." The industrial period differs from the gunpowder era in the sense that it "sees the employment of various forms of mechanical power to warfare that places overwhelming force at the disposal of industrialized powers. This makes warfare a case of either mass slaughter of soldiers and civilians, or a matter of local resistance."

Identifying the structural breaks in the modern history of warfare appears a somewhat subjective exercise. If the timing of the breaks is unknown, special treatment may be needed to identify the breaks. Although treatments like CUSUMS and Hansen tests are available in the econometrics literature (see, e.g., Greene, 2008), they are known to suffer from lack of power and generally work for only linear regressions models. Given that conflict technologies are highly non-linear, one needs to develop new econometric tools to identify the unknown structural breaks in the conflicts data. To our best knowledge, such studies on conflict technologies are missing.

3.4 Model Specification

As discussed in the previous section, there a number of classes of functional forms for conflict technologies. The most well-known are the generalized ratio model (3), the logit model (4), and the probit model (6). From an econometric perspective, as already explicated in section 2.1, the main difference among these three models lies in the assumptions regarding the distributions of the error terms. It is therefore natural to wonder which model best captures the characteristics of a particular conflict and gives the most accurate predictions.

The effort to choose among these three models in areas other than conflict research has been frustrated by the fact that despite that the three models have quite different theoretical consequences, they are practically indistinguishable with data in other areas of research in which the three functional forms have been estimated. For instance, Burke and Zinnes (1965) compared a probit model ($\mathcal{T}$) and a logit model ($\mathcal{L}$) and claim:

> Unfortunately, the nature of the solutions makes it very difficult to design an experiment for deciding between the theories... For the Gulliksen-Tukey (1958), Guilford (1954), and Thurstone (1959) data, the $\mathcal{T}$ predictions are considerably better than the $\mathcal{L}$ predictions.

Yet Hohle (1966) found that

> (a) neither model provided uniformly satisfactory representations for the data, and (b) (for) all six sets of data were more accurately represented by Model II ($\mathcal{L}$) than by Model I ($\mathcal{T}$).

In a comprehensive survey, Batchelder (1983) concluded that, achieving any reasonable power in testing between the models statistically, it would require an unrealistic amount of data . Stern (1990) used the Gamma distribution to approximate the probit and logit models and compares their performances against the game results from the 1986 National League baseball season. He also finds it is disturbing to see so little difference between these two models.

This problem arises because of two reasons. Firstly, all three models are highly nonlinear, and yet a commonly accepted goodness-of-fit measure is unavailable to achieve a convincing conclusion. Secondly, the generalized ratio and the logit models are isomorphic up to a logarithmic transformation, hence are nested together. Classical econometric theory fails to deal with these two problems.

One possible remedy is provided by Bayesian Econometrics. In contrast to Classical methods, the Bayesian approach treats any two candidate models as hypotheses. Rather than artificially designing some goodness-of-fit statistic, Bayesians choose a natural criterion, the Bayes factor, to compare alternative models. The Bayes factor for model A_1 versus model A_2 can be defined as

$$B_{12} = \frac{\Pr(y|A_1)}{\Pr(y|A_2)},$$

where

$$\Pr(y|A_i) = \int_{\Theta_i} \Pr(\theta_i|A_i) \Pr(y|\theta_i, A_i) d\theta_i$$

is the probability of observing y, given that model A_i is correst, or the marginal likelihood of model i, $i = 1, 2$. $\Pr(\theta_i|A_i)$ represents the prior information about the *parameters of interest* θ_i given the candidate model A_i, and $\Pr(y|\theta_i, A_i)$ represents the *data generating process* (see Kass and Raftery, 1995).

The interpretation of the Bayes factor is given by Jeffreys (1961, Appendix B), and Kass and Raftery (1995). Jeffreys (1961) suggests the following criterion as the "order of magnitude" interpretation of B_{12}:

$$\begin{array}{lclcll} 1 & < & B_{12} & < & \infty, & \text{evidence supports } A_1, \\ 10^{-1/2} & < & B_{12} & \leq & 1, & \text{very slight evidence against } A_1, \\ 10^{-1} & < & B_{12} & \leq & 10^{-1/2}, & \text{slight evidence against } A_1, \\ 10^{-2} & < & B_{12} & \leq & 10^{-1}, & \text{strong evidence against } A_1, \\ 0 & < & B_{12} & \leq & 10^{-2}, & \text{decisive evidence against } A_1. \end{array}$$

The key step in the Bayesian model comparison is computing a good approximation to the marginal likelihoods. For our nonlinear regression models (generalized ratio, logit, probit), the main difficulty is that the marginal likelihood functions cannot be expressed directly as some posterior moments, and consequently the computation cannot be interpreted directly as a special case of the simulation-consistent approximation of posterior moments. Fortunately, there are computational methods specifically tailored to overcome this kind of problem. These methods, though widely used in Bayesian econometrics, have not appeared in the conflict literature. A brief introduction of these powerful tools may be informative to some interested readers. In the Appendix we compute the marginal likelihoods of probit and logit models.

The idea of the Bayesian model comparison is quite intuitive. It is a simulation-based approach that is computationally intensive which is becoming easier and less-time consuming. Jia (2008a) uses this approach to compare the generalized ratio, the logit, and the probit forms of contest success functions using NBA data. He shows that the probit form is most favored by the NBA data. The same method could be applied to conflict data.

Furthermore, Jia (2010) proposes an alternative classical method to compare the generalized ratio and the logit functional forms. In Corollary 2 of his paper, he derives the following unified model

$$p_i(M_i, M_{-i}) = \frac{\exp\left[\frac{\alpha\gamma}{1-\gamma}\left(\eta + \frac{\beta}{\gamma}M_i\right)^{1-\gamma}\right]}{\sum_{j=1}^{n}\exp\left[\frac{\alpha\gamma}{1-\gamma}\left(\eta + \frac{\beta}{\gamma}M_j\right)^{1-\gamma}\right]}, \tag{16}$$

which takes both the generalized ratio and the logit forms as limiting cases. For instance, if $\eta = 0$, $\beta = \gamma^{-\gamma/(1-\gamma)}$, and $\gamma = 0$, (16) becomes the logit form; if $\eta = 0$, $\beta = \gamma^{-\gamma/(1-\gamma)}$, and $\gamma = 1$, (16) becomes the generalized ratio model.

This result suggests that instead of statistically comparing the goodness-of-fit measures of the generalized ratio and the logit forms, one can estimate the unified model (16) and conclude which form better captures the data by examining the parameter γ: if γ is close to 1, the ratio form fits the data better; if γ is close to 0, the difference form is more plausible for the conflict.

Hwang (2009) applies a similar treatment to the HERO data, and concludes that the generalized ratio form of conflict models fit the data of seventeenth century European wars.

4 Concluding Remarks

Warfare and other forms of conflict are typically uncertain but the outcome does depend on the resources expending by adversaries. We have considered conflict technologies to be probabilistic choice functions that depend on the military capacities of adversaries, with the military capacities themselves depending on economic inputs via ordinary production functions. We have examined both the theoretical foundations of technologies of conflict and various issues in estimating them empirically.

Different classes of functional forms can be derived either stochastically or axiomatically. The additive form, in particular, (which includes both the logit and ratio specific functional forms) has both stochastic and axiomatic foundations. We have explored the various problems in empirically estimating technologies of conflict. They include concerns with data, endogeneity, structural breaks, and model comparison. Whereas research on theoretical foundations is rather mature, there is not much empirical research on the topic and that is clearly a promising area for future research.

A Appendix

A.1 Evaluating the Marginal Likelihood in the Probit Model

The method of evaluating the marginal likelihood of probit model starts with the conditional probability formula:

$$\Pr(\beta|y, A) = \frac{\Pr(\beta|A)\Pr(y|\beta, A)}{\Pr(y|A)}, \tag{17}$$

where A represents the specified model, and $\Pr(\beta|A)$ and $\Pr(\beta|y, A)$ are the prior of parameters of interest β before observing the data y and the posterior of β after y being observed, respectively. The other two terms in (17), $\Pr(y|\beta, A)$ and $\Pr(y|A)$, are the likelihood (or, *data generating process*) and the marginal likelihood of data y.

For computational convenience, after taking a logarithmic transformation on both sides, (17) can be rearranged as

$$\ln\Pr(y|A) = \ln\Pr(\beta|A) + \ln\Pr(y|\beta, A) - \ln\Pr(\beta|y, A). \tag{18}$$

Evaluating the marginal likelihood of a model A is now equivalent to evaluating the right-hand-side of (18). Since the prior is preassigned, and the likelihood functions $\Pr(y|\beta, A)$ is known to be a density function of a normal distribution, the left-hand-side of (18) then boils down to evaluate the posterior $\ln[\Pr(\beta|y, A)]$. If the posterior can be easily determined by choosing an appropriate prior, which is called a conjugate prior, one can easily generate draws from all three functions in the left-hand-side of (18) and assess them numerically. This is exactly the method we treat the probit model.

To be specific, assume in a probit model A, the observables are represented as a $T \times K$ matrix of covariates $X = [x_1, \ldots, x_T]'$ and a corresponding set of T binary outcomes y, with

$$\begin{aligned} \Pr(y_t = 0|x_t, A) &= 1 - \Phi(\beta' x_t) = \Phi(-\beta' x_t), \\ \Pr(y_t = 1|x_t, A) &= \Phi(\beta' x_t). \end{aligned}$$

In a conflict context, the matrix X includes all factors that determine the outcome of the conflict, and the observation y_t represents the outcome of conflict t, with $y_t = 1$ being a preassigned party achieves a victory in conflict t, and $y_t = 0$ otherwise. If we introduce a latent variables $\tilde{y}_t = \beta' x_t + \epsilon_t$, $\epsilon_t \sim \mathcal{N}(0, 1)$, then the first outcome, $y_t = 0$, corresponds to $\tilde{y}_t \leq 0$ and the second to $\tilde{y}_t > 0$. Let $\mathfrak{I}$ be an index function with

$$\mathfrak{I}(\tilde{y}_t) = \begin{cases} 1 & \text{if } \tilde{y}_t \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

It is clear that $\mathfrak{I}(\tilde{y}_t)$ and y_t should be identical. Denote the conditionally conjugate prior distribution by $\beta|X, A \sim \mathcal{N}(\underline{\beta}, \underline{H}^{-1})$, a normal distribution with mean $\underline{\beta} = 0_{K\times 1}$ and covariance matrix $\underline{H}^{-1} = c \times I_{K\times K}$, where $I_{K\times K}$ is the identity matrix of order K and c is any preassigned positive constant[8]. The reasons for using a proper, but a rather flat, prior are the following: (1) a proper prior guarantees that the posterior is also proper, which ensures the convergence of the marginal likelihood; and (2) the flat p.d.f indicates that the prior is of very poor precision, which avoids the potential critique of choosing a specific informative prior.

After some manipulation, one can show that the posterior

$$\beta|\tilde{y}, \mathfrak{I}, A \sim \mathcal{N}(\overline{\beta}, \overline{H}^{-1}),$$

is normally distributed with mean and covariance matrix

$$\begin{aligned} \overline{H} &= \underline{H} + X'X, \\ \overline{\beta} &= \overline{H}^{-1}(\underline{H}\underline{\beta} + X'\tilde{y}). \end{aligned}$$

In addition, in the distribution of $\tilde{y}|\mathfrak{I}, \beta, X, A$, the elements $\tilde{y}_t$, known as probits, are independent. These conditional posterior distributions are the

[8]The constant c should be relatively large (e.g., greater than 1000). This specification corresponds to a rather noninformative prior.

basis for a simple Gibbs sampling algorithm, which was first proposed in Albert and Chib (1993).

One can now follow Chib (1995) and extract the marginal likelihood from the Gibbs output after the posterior simulator has been constructed.

Because Expression (18) always holds, one can evaluate the log-marginal likelihood, the right-hand-side of (18), at a certain point β^*. Given the Gibbs sampling output $\{\beta^{(l)}\}_{l=1}^{L}$, where L is the number of iterations, let β^* be the simulated posterior mean, i.e.,

$$\beta^* = \frac{1}{L}\sum_{l=1}^{L}\beta^{(l)}.$$

Equation (18) can now be expressed as

$$\ln \Pr(y|A) = \ln \Pr(\beta^*|A) + \ln \Pr(y|\beta^*, A) - \ln \Pr(\beta^*|y, A).$$

For the probit model under consideration, the first term on the right-hand-side, $\Pr(\beta^*|A) = \phi(\beta^*|\underline{\beta}, \underline{H}^{-1})$, is a normal density of the prior, the second term is a log-likelihood function evaluated at β^*. The last term, under some regularity conditions, can be approximated by

$$\widehat{\Pr}(\beta^*|y, A) = L^{-1}\sum_{l=1}^{L}\phi(\beta^*|\beta^{(l)}, \overline{H}^{-1}).$$

By the ergodic theorem (see, e.g., Tierney, 1994), when L becomes sufficiently large, $\widehat{\Pr}(\beta^*|y, A)$ converges to $\Pr(\beta^*|y, A)$ almost surely. Therefore, by the Slusky theorem, $\ln[\widehat{\Pr}(\beta^*|y, A)] \overset{a.s.}{\to} \ln[\Pr(\beta^*|y, A)]$. Putting all three pieces together, the approximated marginal likelihood of the probit model can be computed.

A.2 Evaluating the Marginal Likelihood in the Logit Model

The logit model differs from the proit model by its likelihood function. In particular, the likelihood of logit model is given by

$$\Pr(y|\beta, A) = \prod_{t=1}^{T}\Pr(y_t|\beta, X, A) = \prod_{t=1}^{T}[\frac{\exp(\beta' x_t)}{1+\exp(\beta' x_t)}]^{y_t}[\frac{1}{1+\exp(\beta' x_t)}]^{1-y_t},$$

where T is the number of observations. For such a likelihood function, a conjugate prior doesn't exist, which means the posterior $\beta|y, A$ has no closed analytical form. Therefore, one cannot directly apply Equation (18) to evaluate the marginal likelihood of a logit model, an alternative route need to be taken.

Again by Bayes theorem, one has

$$\Pr(y|A) = \int_{\Theta_A} \Pr(\beta|A) \Pr(y|\beta, A) d\beta, \tag{19}$$

so evaluating the marginal likelihood, or the let-hand-side of (19), is identical to numerically assessing the integral on the right-hand-side. A method known as the "density ratio approximation method," which is proposed by Gelfand and Dey (1994), can be used to serve our purpose.

To be more specific, we again assign $\beta|A \sim \mathcal{N}(\underline{\beta}, \underline{H}^{-1})$ as the prior for comparison purpose. The "density ratio approximation method" requires random samples from the posterior distribution $\Pr(\beta|y, A)$. Because this posterior has no analytical representation, directly sampling from it is almost impossible. Fortunately, statisticians propose a sampling scheme, which has been called the Metropolis-Hasting sampling procedure, to overcome this difficulty[9]. Generally speaking, the Metropolis-Hastings algorithm is a Markov chain Monte Carlo method for obtaining a sequence of random samples from a probability distribution for which direct sampling is difficult. This sequence can be used to approximate the distribution (i.e., to generate a histogram), or to compute an integral (such as an expected value).

Notice although the posterior distribution cannot be analytically identified, Bayes theorem indicates that:

$$\Pr(\beta|y, A) \propto \Pr(\beta|A) \Pr(y|\beta, A).$$

That is to say, if one makes many draws from the prior $\Pr(\beta|A)$ and the likelihood $\Pr(y|\beta, A)$, the shape of the posterior distribution can be determined by the products of these draws. But in order to identify the posterior distribution, a normalization factor, which guarantees the probabilities sum up to one, is often extremely difficult to compute. A major virtue of the Metropolis-Hastings algorithm is its ability to generate a sample without knowing this constant of proportionality. Notice the draws from a Metropolis-Hasting sampling are serially correlated, since each draw depends on the previous draw. This serial correlation needs to be considered when using these draws. That is why one should calculate the mean of all the simulations after many iterations of the Metropolis-Hasting procedure and call it a single draw from the posterior $\Pr(\beta|y, A)$[10]. With this draw in hand, one can then proceed with another draw from the Gibbs sampling. In other words, a Metropolis-Hasting sampling procedure needs to be embedded into each Gibbs sampling iterations. In particular, for numerically evaluating the marginal likelihood function of a logit model, all the posterior simulates have to be recorded.

[9] Chib and Greenberg (1995) provide an excellent description of the Metropolis-Hasting algorithm as well as an explanation of why it works.

[10] For a more detailed discussion, see Train (2003).

Given the output of a posterior simulator, $\beta^{(l)} \sim \Pr(\beta|y, A)$, evaluations of the prior density $\Pr(\beta|A)$ and data density $\Pr(y|\beta^{(l)}, A)$, one can follow Geweke (1999) and approximate (19) by

$$L^{-1} \sum_{l=1}^{L} \frac{\Pr(\beta^{(l)})}{\Pr(y|\beta^{(l)}, A) \Pr(\beta^{(l)}|A)} \stackrel{a.s.}{\rightarrow} [f(y|A)]^{-1}, \tag{20}$$

where L is the number of iterations. Obviously, the term $\Pr(\beta^{(l)}|A)$ is the prior, which can be evaluated in a density function of a normal distribution, and

$$\Pr(y|\beta^{(l)}, A) \prod_{t=1}^{T} [\frac{\exp(\beta^{(l)\prime} x_t)}{1 + \exp(\beta^{(l)\prime} x_t)}]^{y_t} [\frac{1}{1 + \exp(\beta^{(l)\prime} x_t)}]^{1-y_t} \tag{21}$$

can be computed to machine accuracy rapidly. The only term left unknown on the left-hand-side of (20) is $\Pr(\beta^{(l)})$, a p.d.f. constructed from the posterior simulator output. By theorem 8.1.2 of Geweke (2005), it can be constructed as follows:

$$\widehat{\Pr}(x, \alpha) = \frac{(2\pi)^{-K/2} |\Sigma_{(L)}|^{-1/2}}{1 - \alpha} \exp[-\frac{1}{2}(x - \mu^{(L)})' \Sigma_{(L)}^{-1} (x - \mu^{(L)})] I_{X_{\alpha}^{(L)}}(x), \tag{22}$$

where $\mu^{(L)}$ and $\Sigma_{(L)}$ are sample mean and variance respectively, α is a predetermined number between 0 and 1, and $X_{\alpha}^{(L)}$ is the truncated highest density region of size $1 - \alpha$.

Putting together all three pieces, the prior $\Pr(\beta^{(l)}|A)$, the likelihood (21), and (22), the marginal likelihood of the logit model can then be evaluated numerically. In particular, one can choose $\alpha = 5\%$ and $L = 1000$.

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