Sprawl and Blight

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Abstract

The objective of this paper is to show how the same market failures that contribute to urban sprawl also contribute to urban blight. The paper develops a simple dynamic model in which new suburban and older central-city properties compete for mobile residents. The level of housing services generated by older properties depends on current maintenance or reinvestment expenditures. In this setting, market failures that reduce the cost of occupying suburban locations, thus leading to excessive suburban development, also depress central-city housing prices and undermine maintenance incentives, leading to deficient levels of central-city reinvestment. Corrective policies that shift population from the suburbs to the center result in higher levels of reinvestment in central-city housing, therefore reducing blight.

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1. Introduction

Urban sprawl has become a major public policy issue in recent years, reflecting widespread complaints that the spatial growth of cities is "paving over" the American landscape in a fashion that is undesirable on environmental and aesthetic grounds.[1] In response to these concerns, many cities and states have adopted policies to limit sprawl, including various restrictions on development at the urban fringe, new charges levied on builders, and public purchases of open space.

Many commentators recognize that sprawl is in part caused by the growing populations of US cities and their rising incomes, both of which increase the derived demand for land. In addition, highway investment and growing automobile ownership are viewed as contributing to sprawl by reducing the cost of access to employment centers from suburban locations (see Glaeser and Kahn (2004) and Baum-Snow (2007)).[2] While such fundamental forces naturally lead to urban spatial expansion, economists argue that sprawl can be faulted on efficiency grounds only if the operation of these forces involves market failures. Several market failures have indeed been identified, including unpriced traffic congestion, failure to account for open-space amenities in development decisions, and failure to levy marginal-cost-based infrastructure charges (see Brueckner (2000, 2001)). With unpriced congestion, the social cost of commuting exceeds the private cost, and the equilibrium urban development pattern features excessively long commute trips and thus a city that is too spread out. When open-space amenities are present or when infrastructure is underpriced, the social cost of suburban land development exceeds the private cost faced by builders, again leading to inefficient urban expansion.

The objective of this paper is to show that the same market failures that contribute to urban sprawl also contribute to urban blight. More precisely, our objective is to show that urban sprawl, defined as excessive investment in new suburban properties, and urban blight, defined as deficient reinvestment in older central city properties, result from the same underlying economic process. To achieve this goal, the paper develops a simple dynamic model of an urban economy in which new suburban properties and older central-city properties compete for mobile residents. In the model, the production of housing services from existing structures depends, in part, on maintenance or reinvestment expenditures. Then, with unpriced traffic congestion, underpriced infrastructure provision, or open-space amenities, both sprawl and blight arise from the natural operation of the land market: the cost of suburban living is inefficiently low, which distorts the allocation of population, drawing residents away from the downtown. This population shift in turn depresses housing prices in the center and undermines incentives to maintain or reinvest in existing structures. Under each market failure, the appropriate corrective policy shifts population toward the city center, improving maintenance incentives and reducing urban blight. The analysis thus demonstrates that blight reduction is a beneficial byproduct of policies designed to control urban sprawl.

Early writers on blight and urban renewal were clearly aware of the possibility of complex relationships between central-city and suburban development. For example, Fisher (1942, pp. 334-5), writing in the *American Economic Review*, notes that improvements in transportation technology greatly expanded the land area accessible from the center of a city, resulting in a "great suburban migration" and the emergence of blight:

> This migration embraces not only residential land uses but commercial as well, and there are some indications that industrial land uses are falling into the line of march away from 'downtown' congestion, high taxes, high land 'values,' and restricted areas to the open spaces of 'suburbia' This accelerating march to the periphery of urban areas has greatly accentuated the problem of the areas lying near the center of cities . . . The areas which are thus being drained of their population and purchasing power are lumped together and called 'blighted' . . . the structures in the area are progressively deteriorating . . . new capital is fleeing from or refusing to enter the area.

Similar concerns are expressed by contemporary critics of sprawl; see, for example, Richardson and Gordon (2001) and Nelson et al. (2004).

Several other independent mechanisms also contribute to the problem urban blight. The most obvious, and presumably most important, is central-city poverty. A number of forces make central cities the best location for poor households (see Glaeser, Kahn and Rappaport (2008)), and the resulting low incomes of many central neighborhoods generate low housing qualities via the filtering process (see Braid (1986) and Bond and Coulson (1989)). Another key mechanism, and the dominant issue in early studies of urban renewal, is externalities between properties. These "neighborhood effects," which influence individual maintenance and reinvestment decisions, can lead to a process of contagious neighborhood decline (see Davis (1960), Davis and Whinston (1961)). Bradbury, Downs and Small (1980, pp. 412) summarize the process as follows:

> If one unit is severely damaged ... or left vacant for an extended period ..., it detracts from the desirability of the entire neighborhood. The subsequent demand reduction in turn reduces the profitability of landlords' maintenance efforts. Additional units will run down, and may even be abandoned, and the demand for neighborhood units falls further in response. In this way, physical blight and abandonment are contagious through a process of self-fulfilling expectations.

Under this view, blight arises from the interaction of neighborhood externalities and an exogenous event causing an initial decline in maintenance or reinvestment for some properties.[3] The present analysis argues that such an exogenous trigger is unnecessary, with blight arising instead from the natural operation of the land market in the presence of sprawl-generating market failures. It would be possible to add neighborhood effects to the process that determines maintenance levels in the model, and this change would certainly amplify the impacts of market failures. The model's key difference, however, is that the initial impetus to underinvestment in central properties is an endogenous and inefficient response to market failures emanating from other sectors of the urban economy.

There are two other large literatures that bear on the relationship between suburban development and blight. First, the "spatial mismatch" literature argues that the suburbanization of jobs, coupled with discrimination in suburban labor, housing and mortgage markets, has contributed to the concentration and persistence of minority poverty (and hence blight) in central cities (see Kain (1968, 1992), Ihlanfeldt and Sjoquist (1990)). Second, the "flight from blight" explanation of post-war US suburbanization has emphasized that the process of decentralization was encouraged by a desire on the part of affluent households to escape central-city crime, poverty, racial tensions, schools, taxes, congestion and pollution (see Bradford and Kelejian (1973), Mills and Price (1984), Mieszkowski and Mills (1993), Cullen and Levitt (1999)). In contrast to these views, blight does not cause sprawl in the present model: both are equilibrium responses to more-primitive market failures.

The paper is organized as follows. Section 2 develops the model, assuming the absence of any of the three market failures (referred to henceforth as "distortions"). Section 3 sequentially incorporates each of the distortions, analyzes the divergence between the social optimum and laissez-faire equilibrium, and derives the appropriate corrective policy. These policies are congestion pricing, an open-space amenity tax, and a marginal-cost-based impact fee for infrastructure, and the analysis reveals their impacts on sprawl and blight.[4] Section 4 asks whether a quantity-based anti-sprawl policy (an urban growth boundary) can be used in place of section 3's price-based policies to achieve the social optimum. Section 5 provides some additional comparative-static results, and section 6 concludes.

2. The Model

2.1. The setup

The model has two periods, denoted 1 and 2, and it focuses on a closed city experiencing population growth between the periods. For simplicity, differentiation of space in the city is limited to two zones, with commuting costs homogeneous within each zone. A central zone, whose land area is normalized to unity, contains the CBD at its left end. Since intrazonal commuting is assumed to be costless, the cost of traveling to the CBD from any location within the central zone equals zero. A suburban zone, which contains ℓ worth of potentially developable land, is connected to the central zone by a congestible bridge. Figure 1 illustrates the city's spatial structure.[5]

Since intrazonal commuting is costless, commuting cost from the suburbs to the CBD is simply equal to the cost of crossing the bridge. For each commuter, this cost is given by a function t, which depends on the number of commuters crossing the bridge. In the absence of congestion, this function is simply a constant, with costs independent of the traffic volume. When congestion is introduced in section 3.1 below, the function acquires a traffic-volume argument.

Structures in the city are built at a fixed density, which cannot be altered between periods. However, structures built in period 1 are subject to quality deterioration between the periods, with the extent of this depreciation determined by the level of reinvestment, in the form of maintenance or reinvestment expenditures, in period 2. Specifically, structures provide β square feet of housing per unit of land, with β normalized to one for simplicity. Again for simplicity, the construction cost of c per unit of land, which equals a constant given fixed density, is set at zero. When new, each square foot of housing provides one unit of housing services, while services per square foot equal a in a structure that is one period old. Without any reinvestment expenditure, $a = \underline{a} < 1$. But a higher service level of $a > \underline{a}$ can be achieved by a reinvestment expenditure of $k(a)$ per square foot, where $k(\underline{a}) = 0$, $k' > 0$, and $k'' \geq 0$ (with $\beta = 1$, $k(a)$ also gives reinvestment cost per unit of land). Thus, the cost of quality improvement in an old structure rises with the target housing-service level and does so at a nondecreasing rate. Some level of reinvestment expenditure would presumably achieve like-new quality (with $a = 1$) or perhaps even better-than-new quality ($a > 1$), but the ensuing discussion presumes that a value satisfying $\underline{a} < a < 1$ is chosen. The key observation is that, since urban blight is associated with a low level of building reinvestment, public policies that induce an increase in a serve to reduce blight.

Consumer preferences depend on consumption of a non-housing numeraire good, denoted e, and housing services, denoted Q. Letting q denote housing square footage, $Q = q$ holds in new housing (with services identically equal to floor space), while $Q = aq$ holds for housing in a one-period-old building. Note that the level of q (housing quantity) is chosen by the consumer, while the builder chooses a (which determines housing quality). For simplicity, preferences are assumed to be quasi-linear, with the utility function given by $U(e, Q) \equiv e + v(Q)$, where $v' > 0$ and $v'' < 0$. The consequences of relaxing this assumption, which eliminates income effects on housing consumption, are discussed below.

The entire central zone is assumed to be developed in period 1, with no spillover to the suburban zone. Suburban development then occurs in period 2, following growth in the city's population. Thus, the period-2 city inherits a fixed stock of housing in the central zone, which is subject to quality deterioration. Note that the city's population in period 1 must have the right size in order the exactly fill the central zone, and that the required size might depend on period-2 policies, which can affect period-1 housing prices (and thus period-1 consumption and population density) in an intemporal setting. However, as long as the central zone is fully developed in period 1, these population-size issues are irrelevant to the outcomes in period 2, which are the focus of the analysis. Thus, no further consideration of period 1 is necessary.

2.2. Period-2 analysis

In period 2, the city has a population of N, part of which must accommodated in the suburban zone. The suburban population equals n_s, while the population of the central zone equals n_c, with $n_c + n_s = N$. To characterize suburban development, which first occurs in period 2, let q_s and p_s denote the suburban quantity of housing in period 2 and its price per square foot, with r_s denoting suburban land rent. Let $\overline{r}$ denote the opportunity cost of land. Then, the builder's period-2 profit per unit of land is equal to $\pi_s \equiv \beta p_s - c - r_s = p_s - r_s$, recalling $\beta = 1$ and $c = 0$. Competition among builders forces profit to zero, so that $p_s = r_s$, and competition among landowners forces land rent down to its opportunity cost of $\overline{r}$. The suburban price per square foot of housing then equals this opportunity cost, with $p_s = \overline{r}$.

Letting y denote income, the consumer budget constraint in the suburban zone is given by $e_s + \overline{r}q_s = y - t$, where e_s is suburban non-housing consumption and t is again commuting cost. Eliminating e_s in the utility function $e_s + v(q_s)$, the first-order condition for choice of q_s is then

$$v'(q_s) \quad = \quad \overline{r}. \tag{1}$$

Let q_c and p_c denote the quantity of housing and its price per square foot in the central zone in period 2. With commuting cost equal to zero, the consumer budget constraint is then $e_c + p_c q_c = y$, and utility can be written $y - p_c q_c + v(aq_c)$. Note that, since buildings are now one period old in the central zone, each square foot of housing only generates a units of housing services. Differentiating, the first-order condition for choice of q_c is given by $av'(aq_c) = p_c$, or

$$v'(aq_c) \quad = \quad p_c/a. \tag{2}$$

To interpret (2), note that p_c/a equals the price per unit of housing services in an old dwelling, given by price per square foot (p_c) divided by services per square foot (a). Thus, (2) equates the marginal valuation of housing services in an old dwelling to the relevant price. Observe that, unlike in the suburban zone, the price p_c will reflect an endogenous locational premium and is thus not equal to land's opportunity cost.

Two additional equilibrium conditions provide linkages across the central and suburban zones. The first condition relates the suburban population to housing quantity in the center. Letting n_c denote the period-2 population of the central zone, total land consumption is $n_c q_c$, and setting this expression equal to the zone's unitary land area yields $n_c = 1/q_c$. But since $n_c + n_s = N$, it follows that

$$n_s \quad = \quad N \; - \; 1/q_c. \tag{3}$$

The second condition requires equalization of consumer utilities between the center and suburbs, and it is written $y - p_c q_c + v(aq_c) = y - t - \overline{r} q_s + v(q_s)$, or after canceling the y's,

$$v(aq_c) \; - \; p_c q_c \quad = \quad v(q_s) \; - \; \overline{r} q_s \; - \; t \tag{4}$$

Conditional on a, the conditions (1)–(4) jointly determine equilibrium values of q_s, q_c, p_c, and n_s. However, a is endogenous and set by the builder. In the center, the builder's period-2 profit per unit of land is equal to $\pi_c \equiv p_c - r_c - k(a)$, and a is chosen to maximize this expression (r_c is central land rent). In maximizing, the builder expects that consumers will pay more for better housing quality, with a higher a allowing a higher p_c to be charged. This expectation is based on "utility-taking" behavior, with an individual builder recognizing his inability to influence the equilibrium utility achieved by the consumer. As a result, the builder treats consumer utility as fixed, satisfying $y - p_c q_c + v(aq_c) = u$ for some given u. To reveal $\partial p_c/\partial a$ and thus the consumer's willingness-to-pay for better quality, this condition is differentiated, yielding

$$-\frac{\partial p_c}{\partial a} q_c \; - \; p_c \frac{\partial q_c}{\partial a} \; + v'(aq_c) \left[q_c + a \frac{\partial q_c}{\partial a} \right] \quad = \quad 0. \tag{5}$$

Rearrangement and use of (2) to cancel terms then gives

$$\frac{\partial p_c}{\partial a} \;=\; v'(aq_c). \tag{6}$$

Thus, the builder expects an increase in a to raise p_c at a rate equal to the marginal valuation of housing services, a natural conclusion.

Finally, maximization of the profit expression π_c requires $\partial p_c/\partial a - k'(a) = 0$, and using (6), this condition reduces to

$$v'(aq_c) \;=\; k'(a). \tag{7}$$

Thus, at the optimal a, the marginal valuation of services is set equal to the marginal cost of raising housing quality. Note that since locations within the central zone are homogeneous, builders at all locations choose the same a value. Eq. (7) along with (1)–(4) then together determine equilibrium values for q_s, q_c, p_c, n_s, and a. The final unknown, the central land rent variable r_c, is determined residually by the builder's zero-profit condition. Lastly, suburban development is assumed to leave some open space, with the developed land area (given by n_sq_s) less than the area ℓ of the suburban zone, as shown in Figure 1.

In the standard urban model, housing prices are higher in the center than in the suburbs, and housing consumption is lower. Similar results hold in the present model, adjusted to account for the presence of housing quality differences between the zones. To derive these results, let the utility expresson on the LHS of (4) be rewritten as $y - P_cQ_c + v(Q_c)$, where $P_c = p_c/a$ is the central price per unit of housing services and $Q_c = aq_c$. As seen above, the central-zone consumer maximizes this expression with respect to Q_c (by choosing q_c), satisfying (2). Let the maximized value, which gives the indirect utility function, be written $y + f(P_c)$, where $f'(\cdot) < 0$. Similarly, indirect utility for a suburban resident (the maximized value of the RHS of (4)) is given by $y - t + f(\overline{r})$. Equalization of utilities in (4) then requires $f(\overline{r}) - f(P_c) = t$, which in turn implies $P_c > \overline{r}$ given $f' < 0$. Thus, the price per unit of housing services must be higher in the central zone than in the suburbs (where it equals $\overline{r}$). Since consumption is decreasing in the relevant price from the first-order conditions (1) and (2), it follows that

$Q_c = aq_c < q_s$, so that consumption of housing services is lower in the central zone than in the suburbs. Note that with $a < 1$, this inequality need not imply $q_c < q_s$, and the inequality $P_c > \overline{r}$ similarly need not imply $p_c > \overline{r}$. So housing consumption and price, unadjusted for quality differences, need not exhibit the usual central-suburban variation, although the usual pattern may emerge when a is close to unity.

The analysis so far has been silent about land ownership arrangements in the economy. The reason is that, with quasi-linear preferences, the equilibrium does not depend on the nature of such arrangements. On the one hand, when the land in the two zones is owned by absentee landowners, rental income flows out of the city and thus does not appear in any of the equilibrium conditions. Under resident landownership, by contrast, the rental income from the two zones accrues to the urban residents, with equal per capita division being a common assumption. In this case, consumer income y is supplemented by rental income term R, affecting only eq. (4) among the equilibrium conditions. But since R cancels from both sides of this condition, the resulting equilibrium is the same as in the case of absentee ownership (where $R = 0$). This convenient property of the quasi-linear case is also useful in the ensuing analysis of anti-sprawl policies. As will be seen, the government revenue generated by these policies must be redistributed, and the pattern of such redistribution is irrelevant to their impacts under this form of preferences.

3. Sprawl-Inducing Distortions and the Blight Impacts of Corrective Policies

This section of the paper adds three different sprawl-inducing distortions to the urban economy: unpriced traffic congestion, failure to account for the amenity value of open space, and average- rather than marginal-cost pricing of infrastructure. As explained in the introduction, it is well known that each of these distortions generates urban sprawl, which in the present context means an excessive suburban population, with too few residents in the central zone. In each case, the appropriate corrective policy reduces sprawl by shifting population toward the center. But given presence of endogenous building reinvestment in the model, this population shift will be accompanied by a blight impact. The purpose of the ensuing analysis is to analyze these blight impacts, looking separately at the three different distortions and their associated corrective policies. As noted in the introduction, the three corrective policies are congestion pricing, an open-space amenity tax, and impact fees.

For each distortion, the equilibrium conditions from section 2 are first modified to incorporate the distortion. Then, the planning problem for the urban economy is solved to find the social optimum and the corrective policy required to support it. Comparison of the original, laissez-faire equilibrium and one generated by the corrective policy reveals the ways in which the original urban economy diverges from the social optimum. The analysis shows that, under each distortion, the suburban population is excessive, with imposition of the appropriate corrective policy shifting population toward the central zone. In addition, the analysis reveals inefficiencies in the level of reinvestment in central-city buildings, showing the impact of the corrective policies on urban blight.

3.1. Traffic congestion and the need for congestion pricing

With traffic congestion, the cost of crossing the bridge linking the suburbs to the center, which was previously a constant, becomes a function of the number of suburban commuters. Since this number equals n_s, the suburban population, the cost is written $t(n_s)$, where $t' > 0$. With this change, the laissez-faire equilibrium conditions (1)–(4) and (7) from section 2 are modified in a simple fashion: the constant t in (4) is replaced by $t(n_s)$.

While the laissez-faire equilibrium in the absence of congestion is efficient, congestion creates an externality and thus leads to inefficiency. In particular, when an additional commuter uses the bridge, total commuting cost for existing users rises by $n_s t'(n_s)$ (their number times the increase in individual cost), an external effect that is ignored by each commuter. Intuition suggests that, to maximize social welfare, each commuter should be charged a congestion toll equal to this expression, leading to internalization of the externality.

This intuition can be verified by analyzing the optimization problem that would be solved by a social planner. In period 2, the planner would seek to minimize the city's resource usage subject to achievement of a fixed, common utility level for the central and suburban residents (unequal zone utilities would be unsustainable under free migration). The Lagrangian expression for this problem is

$$
\begin{aligned}
(N - n_s)e_c \ + \ n_s e_s \ + \ \overline{r}(1 + n_s q_s) \ + \ n_s t(n_s) \ + \ k(a) & \\
+ \ \lambda(e_c + v(aq_c) - u) & \\
+ \ \mu(e_s + v(q_s) - u) & \\
+ \ \rho([N - n_s]q_c - 1). & \qquad (8)
\end{aligned}
$$

The second and third lines of (8) contain the utility constraints, while population constraint in the last line replicates (3). The resource expression in the first line of (8) aggregates total non-housing consumption (the first two terms), the opportunity cost of the land used by the city (the third term), total commuting cost, and reinvestment cost. Note that $k(a)$, which gives cost per unit of land, is multiplied by the unitary land area of the central zone.

The first-order conditions for the e variables yield $-\lambda = N - n_s$ and $-\mu = n_s$, and after substitution in the first-order conditions for the q variables, these conditions reduce to (1) and $av'(aq_c) = \rho$, where ρ represents the shadow price of central housing. After eliminating λ and using the last constraint in (8), the first-order condition for a reduces to (7). Finally, the first-order condition for n_s reduces to

$$e_c \ + \ \rho q_c \ = \ e_s \ + \ \overline{r} q_s \ + \ t(n_s) \ + \ n_s t'(n_s), \qquad (9)$$

which says the resource cost of locating an additional individual should be equated across zones. Eliminating e_c and e_s using the utility constraints from (8), and then cancelling the u's and rearranging, (9) reduces to

$$v(aq_c) \ - \ \rho q_c \ = \ v(q_s) \ - \ \overline{r} q_s \ - \ t(n_s) \ - \ n_s t'(n_s). \qquad (10)$$

A comparison with (4) shows that (10) is just a modified version of the laissez-faire equal-utility condition. The modifications are the appearance of ρ in place of p_c on the LHS, and the subtraction of $n_s t'(n_s)$ on the RHS. Since, from above, the remaining optimality conditions coincide with the laissez-faire equilibrium conditions (1), (2), (3) and (7), it follows that the social optimum coincides with the equilibrium generated when consumers are charged an amount $n_s t'(n_s)$ over and above their private cost of commuting, $t(n_s)$. Thus, the social optimum emerges as an equilibrium when commuters are charged the appropriate congestion toll. Note that the shadow price ρ is replaced by the market price p_c in this reinterpretation of the optimality conditions. Observe also that the parametric utility level u from (8), which does not appear in this set of conditions, serves only to determine the levels of nonhousing consumption, e_c and e_s, which are not a focus of the analysis.

3.2. The effects of congestion pricing

To analyze the effects of congestion pricing, ρ is first replaced by p_c in (10), with p_c then replaced by $av'(aq_c)$ using (2). The condition (10) then becomes

$$v(aq_c) \ - \ aq_c v'(aq_c) \ = \ v(q_s) \ - \ \overline{r}q_s \ - \ t(n_s) \ - \ n_s t'(n_s). \tag{11}$$

Note that first-order condition $v'(q_s) = \overline{r}$ from (1), which applies with or without congestion pricing, requires that q_s is equal to some constant, implying that $v(q_s) - \overline{r}q_s$ in (11) equals a common constant in both cases. In the laissez-faire equilibrium, however, $n_s t'(n_s)$ is deleted from the RHS of (11).

Using (11) along with the other conditions, the following results can be established:

Proposition 1. *In the presence of traffic congestion, the laissez-faire equilibrium has a lower level of reinvestment in central-city buildings and a larger suburban population than the social optimum. Thus, imposition of congestion pricing, which achieves the optimum, raises reinvestment (reducing urban blight) while shifting population toward the center.*

Proof: Let the laissez-faire solutions be denoted by hats and the solutions under congestion pricing (at the social optimum) be denoted by stars. Proceeding by contradiction, suppose that $n_s^* \geq \widehat{n}_s$. Then, given $t' > 0$, $t(n_s^*) + n_s^* t'(n_s^*) > t(\widehat{n}_s)$ holds, so that the RHS of (11) is smaller under congestion pricing than in the laissez-faire case. The LHS expression in (11) must then also be smaller, yielding $a^*q_c^* < \widehat{a}\,\widehat{q}_c$ given that $v(z) - zv'(z)$ is an increasing function of z (its derivative is $-zv'' > 0$). But with $n_s^* \geq \widehat{n}_s$, (3) implies $q_c^* \geq \widehat{q}_c$. Then, for $a^*q_c^* < \widehat{a}\,\widehat{q}_c$ to hold, $a^* < \widehat{a}$ must be satisfied as well. Given $k'' \geq 0$, it then follows that $k'(a^*) \leq k'(\widehat{a})$, so that satisfaction of (7) in both cases would require $v'(a^*q_c^*) \leq v'(\widehat{a}\,\widehat{q}_c)$, or $a^*q_c^* \geq \widehat{a}\,\widehat{q}_c$ given $v'' < 0$. This contradition rules out the premise on n_s, establishing $n_s^* < \widehat{n}_s$ and $q_c^* < \widehat{q}_c$, using (3). Then, $a^* > \widehat{a}$ must be satisfied since otherwise $a^*q_c^* < \widehat{a}\,\widehat{q}_c$ would hold, implying $k'(a^*) \leq k'(\widehat{a})$ and $v'(a^*q_c^*) > v'(\widehat{a}\,\widehat{q}_c)$, which rule out satisfaction of (7) in the two cases. Finally, combining (2) and (7) yields $p_c/a = k'(a)$ or $p_c = ak'(a)$, and with $a^* > \widehat{a}$, $p_c^* > \widehat{p}_c$ holds, so that p_c is higher under congestion pricing. ■

Thus, in addition to raising a (lowering n_s) from an inefficiently low (inefficiently high) level, imposition of congestion pricing raises the central housing price p_c. An intuitive explanation for the higher level of reinvestment comes from focusing on this p_c impact. With congestion pricing raising the cost of suburban commuting, population must shift toward the center, and since the total square feet of housing available in the center is fixed, the price per square foot p_c must then rise. Holding a constant, this increase raises the price per unit of housing services p_c/a, which is set equal to the marginal valuation $v'(aq_c)$ of services under consumer optimization. But with the builder setting a to equate this marginal valuation to $k'(a)$ from (7), p_c/a and $k'(a)$ must then be equalized, which requires a higher a given the higher p_c. Thus, the higher housing price per square foot in the center generated by congestion pricing elicits greater spending on housing quality, reducing urban blight.

As noted earlier, the equilibrium under congestion pricing is unaffected by the disposition of the revenue collected from the congestion toll, which equals $n_s^2 t'(n_s)$. If the toll revenue were distributed to consumers in both zones in an equal per capita fashion,[6] the resulting amount would cancel from both sides of the equal-utility condition, leaving the previous results unaffected. Similarly, the same equilibrium would arise if the revenue were retained by the government or paid to absentee landowners.

Another question concerns the extent to which the above results depend on the absence of income effects, which are eliminated by the assumption of quasi-linear preferences. Analysis using a general utility function is difficult, but this question can be addressed by imposing the familiar Cobb-Douglas form for preferences. It can be shown that all of the preceding impacts of congestion pricing again emerge in the Cobb-Douglas case, suggesting that the above results may be robust to relaxation of the key assumption on preferences.[7]

3.3. The effects of an open-space amenity tax

Suppose that traffic congestion is again absent, as in section 2, but that the open space surrounding the city generates an amenity for each resident. Accordingly, let the utility function be rewritten as $e + v(Q) + \theta(\ell - n_s q_s)$, where the amount of open space equals $\ell - n_s q_s$, and the marginal valuation of such space is given by the parameter $\theta \geq 0$.[8] See Bento et al. (2006) for an analysis using this same approach to valuing open space.

With these changes, the planning problem from section 3.1 is modified by inclusion of the additional open-space term in the two utility constraints in the second and third lines of (8). Then, the optimality condition for q_s, which was previously $v'(q_s) = \overline{r}$, becomes

$$v'(q_s) \quad = \quad \overline{r} + \theta N. \tag{12}$$

The reason is that, in addition to its opportunity cost, consumption of an extra unit of suburban land now generates a utility loss of θ for each of the city's N residents. In the optimality/equal-utility condition (10), $\overline{r}q_s$ is replaced by $(\overline{r}+\theta N)q_s$, and the commuting-cost terms are replaced by a constant t (p_c again replaces ρ). Note that, in generating this condition from (9), the open-space utility terms cancel.

The optimum can be decentralized by levying an open-space amenity tax equal to θN on each unit of developed suburban land. A builder's cost per unit of land is then $\overline{r} + \theta N$, and competition forces the suburban housing price p_s down to this level. The RHS of (1), the consumer first-order condition for choice of q_s, then becomes $\overline{r} + \theta N$, matching the optimality condition in (12).

In the open-space amenity case, (11) is rewritten as

$$v(aq_c) \;-\; aq_c v'(aq_c) \quad = \quad v(q_s) \;-\; (\overline{r} + \theta N)q_s \;-\; t, \tag{13}$$

which reduces to the laissez-faire condition (4) when $\theta = 0$. However, since $v'(q_s)$ is set equal to $\overline{r} + \theta N$ in either case, (13) can be rewritten as

$$v(aq_c) \; - \; aq_c v'(aq_c) \;\; = \;\; v(q_s) \; - \; q_s v'(q_s) \; - \; t, \tag{14}$$

which applies both under the amenity tax and at the laissez-faire equilibrium.

Using (14) along with the other conditions, the following results can be established:

Proposition 2. *Suppose the marginal cost of housing quality is increasing. Then, in the presence of open-space amenities, the laissez-faire equilibrium has a lower level of reinvestment in central-city buildings and a larger suburban population than the social optimum. Thus, imposition of an open-space amenity tax, which achieves the optimum, raises reinvestment (reducing urban blight) while shifting population toward the center.*

Proof: As before, let stars denote values under the amenity tax and hats denote the laissez-faire equilibrium. With $v'' < 0$ and $\theta > 0$ under the amenity tax, $q_s^* < \widehat{q}_s$ must hold given (12). Then, the RHS of (14) must be smaller under the amenity tax than in the laissez-faire equilibrium, and since the same conclusion applies to the LHS, $a^* q_c^* < \widehat{a}\,\widehat{q}_c$ follows. But, assuming $k'' > 0$, (7) then yields $a^* > \widehat{a}$, which implies $q_c^* < \widehat{q}_c$ using the previous inequality.[9] From (3), $n_s^* < \widehat{n}_s$ then holds, and $p_c^* > \widehat{p}_c$ follows as before. ■

The intuitive explanation of these results mirrors the congestion-pricing case. When an amenity tax is imposed, the suburban housing price rises, causing population to shift toward the center. The resulting increase in the central price p_c increases the incentive for reinvestment, reducing urban blight. Note that, as in the case of congestion pricing, the disposition of the revenue from the open-space amenity tax is immaterial under the maintained assumption of quasi-linear preferences.[10]

3.4. The effects of impact fees

Suppose now that open-space amenities and traffic congestion are both absent, but that infrastructure investment, which exhibits decreasing returns to scale, is required for suburban housing development.[11] The infrastructure cost per unit of land serviced is given by $I(n_s q_s)$, an increasing function of the developed suburban land area. Total infrastructure cost is then $n_s q_s I(n_s q_s)$.

This cost must be included among the resource expenditures in the first line of the Lagrangean expression (8) for the planning problem. When this modification is made, the optimality condition for q_s becomes

$$v'(q_s) \quad = \quad \overline{r} + I(n_s q_s) + n_s q_s I'(n_s q_s). \tag{15}$$

This condition shows that, not only must infrastructure cost per unit of land (I) be taken into account along with $\overline{r}$ in the determination of q_s, but the increase in cost per unit of land resulting from a higher q_s (and the resulting effect on total cost, $n_s q_s I'$) must be considered as well. Note that this is a congestion effect like that in the traffic case, but one that operates through the suburban land area rather than simply through population.

Traditionally, infrastructure is financed through average-cost pricing, in which case a builder would pay a tax equal to $I(n_s q_s)$ per unit of land to the government. This cost would in turn be passed on to the consumer as part of the housing price p_s, which would then equal $\overline{r} + I(n_s q_s)$. The first-order condition (1) in the laissez-faire case would thus be replaced by

$$v'(q_s) \quad = \quad \overline{r} \ + \ I(n_s q_s). \tag{16}$$

To decentralize the social optimum, bringing the consumer's condition in line with (15), the average-cost-based tax in (16) must be replaced with a tax based on marginal cost, given by last two terms in (15), which would instead be passed on to the consumer. This marginal-cost-based tax is referred to as an "impact fee."

Thus, in the equal-utility condition (11), $\overline{r}$ is replaced by $\overline{r} + I(n_s q_s)$ in the laissez-fare case and by $\overline{r} + I(n_s q_s) + n_s q_s I'(n_s q_s)$ under the impact fee (with congestion absent, a constant t replaces the commuting cost terms). But since these expressions are each set equal to $v'(q_s)$ given (15) and (16), the equal-utility condition is the same in two cases and again can be written as (14). Then, the following results can be established:

Proposition 3. *With decreasing returns in infrastructure provision, the laissez-faire equilibrium* (*with average-cost infrastructure pricing*) *has a lower level of reinvesment* *in central-city buildings and a larger suburban population than the social optimum. Thus, imposition of an impact fee, which reflects marginal costs and hence achieves the optimum, raises reinvestment (reducing urban blight) while shifting population toward the center.*

Proof: The argument proceeds by contradiction, focusing first on the case where $k'' > 0$. Accordingly, suppose that $q_s^* \geq \widehat{q}_s$. Then, by (14), $a^* q_c^* \geq \widehat{a}\,\widehat{q}_c$ must hold, so that satisfaction of (7) requires $a^* \leq \widehat{a}$, which implies $q_c^* \geq \widehat{q}_c$ using the previous inequality. Then, (3) yields $n_s^* \geq \widehat{n}_s$ and thus $n_s^* q_s^* \geq \widehat{n}_s \widehat{q}_s$. But, given $q_s^* \geq \widehat{q}_s$ and $v'' < 0$, (15) and (16) imply $I(n_s^* q_s^*) + n_s^* q_s^* I'(n_s^* q_s^*) \leq I(\widehat{n}_s \widehat{q}_s)$, an impossibility given $I' > 0$. Thus, $q_s^* < \widehat{q}_s$ must hold, implying $a^* q_c^* < \widehat{a}\,\widehat{q}_c$, $a^* > \widehat{a}$ by (7), and $q_c^* < \widehat{q}_c$. Then, (3) yields $n_s^* < \widehat{n}_s$, and $p_c^* > \widehat{p}_c$ holds as before. The $k'' = 0$ case is handled similarly.[12] ■

A final point is that, since impact-fee revenue exceeds the cost of the infrastructure given decreasing returns and marginal-cost pricing, the excess revenue must be redistributed. As before, the disposition of the revenue has no effect on the equilibrium.[13]

4. Alternate Anti-Sprawl Policies

The anti-sprawl policies considered above attack each distortion by eliminating the relevant wedge between the optimality and equilibrium conditions, generating a new equilibrium that is socially optimal. It is interesting, however, to consider whether other policies are capable of supporting the optimum. As has been discussed in the literature, an urban growth boundary (UGB) can sometimes be a perfect substitute for a price-based policy in controlling sprawl. In the present context, a UGB specifies an upper limit on the amount of land in the suburban zone that may be developed for housing, thus constituting a quantity-based policy. Letting L denote this land area, the UGB restriction is therefore

$$n_s q_s \quad \leq \quad L. \tag{17}$$

With the suburban land area restricted, the rent for developed land in the suburban zone will no longer equal the opportunity cost $\overline{r}$, taking instead some larger value. To understand the new equilibrium conditions that determine this value, consider the model with open-space amenities. Letting $\widetilde{r}$ denote the equilibrium suburban land rent, the first-order condition (1) is replaced by

$$v'(q_s) \quad = \quad \widetilde{r}. \tag{18}$$

The new conditions (17) and (18), along with the previous conditions (2)–(4) and (7), determine equilibrium values for the endogenous variables under the UGB, which now include $\widetilde{r}$.

Similarly, in the model with infrastructure costs, the equilibrium condition (16) is replaced by

$$v'(q_s) \quad = \quad \widetilde{r} \ + \ I(n_s q_s). \tag{19}$$

This condition in conjunction with (17), (2)–(4) and (7) determines the UGB equilibrium with infrastructure costs, which may have a different $\widetilde{r}$ value than in the amenity model. The following result establishes that a properly chosen UGB can support the social optimum in both these models:

> **Proposition 4.** *Suppose that, under the UGB policy, L is set equal to $n_s^* q_s^*$ in the presence of either open-space amenities or underpriced infrastructure ($n_s^* q_s^*$ generally differs between the two cases). Then, the UGB equilibrium coincides with the social optimum. Thus, when the UGB enforces development of the socially optimal amount of suburban land under either of these models, the overall equilibrium (including the level of reinvesment in central-city buildings) is efficient.*

Proof: First, note that, since the laissez-faire equilibrium satisfies $\widehat{n}_s \widehat{q}_s > n_s^* q_s^*$ under both models, the constraint (17) will bind in both cases when $L = n_s^* q_s^*$. Then observe that the UGB equilibrium conditions are satisfied by the (model-specific) socially optimal values n_s^*, q_s^*, q_c^*, a^*, and p_c^* when suburban rent equals $\widetilde{r} = \overline{r} + \theta N$ under the amenity model and when $\widetilde{r} = \overline{r} + n_s^* q_s^* I'(n_s^* q_s^*)$ under the infrastructure-cost model. Thus, the suburban rent premium above $\overline{r}$ under the given UGB exactly equals the appropriate wedge from the other price-based policies, accounting for the UGB's efficiency. ■

The proposition thus shows that, in addition to limiting spatial expansion of the city, a UGB also reduces urban blight, raising central-city reinvestment to an efficient level. In contrast to these results, however, a UGB *cannot* support the social optimum in the model with traffic congestion. This conclusion can be seen by recalling that the optimality condition for q_s under traffic congestion is the same as the laissez-faire equilibrium condition (1) ($v'(q_s) = \overline{r}$). Therefore, since the congestion model with a UGB would have (17) as the corresponding equilibrium condition (as in the amenity model), and since land rent $\widetilde{r}$ will exceed $\overline{r}$, the UGB equilibrium cannot be optimal. This conclusion is illustrated by Brueckner (2007) in a more realistic model with a continuum of residential locations. In his model, the UGB achieves only a small fraction of the welfare gain generated by congestion pricing.

The problem with the UGB in the congestion context is its failure to achieve a sufficient increase in central densities, as can be seen in Brueckner (2007). Suburban density is inefficiently increased instead, with q_s falling in the present model via the increase in land rent above the opportunity cost $\overline{r}$. While such suburban densification in not optimal in the congestion case, it is required under the two other models, accounting for the efficiency of the UGB in these cases.

Given suboptimal reinvestment in central-city buildings in the laissez-faire equilibrium, one might wonder whether subsidization of building maintenance could generate the social optimum. Suppose that a subsidy of κ per unit of maintenance where introduced so as to raise the level of a, with (7) then rewritten as $v'(aq_c) = k'(a) - \kappa$. If κ were adjusted to achieve $a = a^*$, it is clear that the associated value of q_c from this modified equilibrium condition will not equal q_c^*. Thus, a reinvestment subsidy cannot generate the social optimum.

5. The Effects of Faster Population Growth

As explained above, the blight impact of the anti-sprawl policies arises through their inducement of a redistribution of the (period-2) population toward the central zone. Given the population distribution's role in determining blight, it is interesting to explore a related issue: the blight impact of faster growth in the city's *overall* population between the initial period and period 2 (namely, a larger in N). The expected impact of a higher N would appear to be a reduction in blight, but an appropriate analysis is required. That analysis consists of a comparative-static exercise involving N, but this exercise may give different answers depending on the absence or presence of a market failure (as well as the nature of the failure). The following analysis assumes that, when a market failure is present, it is uncorrected.

Consider first the impact of an increase in N in the presence of unpriced traffic congestion. In this case, after deleting the term involving t' and using (3), condition (11) can be written

$$v(aq_c) \;-\; aq_c v'(aq_c) \;=\; v(q_s) \;-\; \overline{r}q_s \;-\; t(N - 1/q_c). \tag{19}$$

With q_s constant, this condition and (7) determine a and q_c. Differentiation of these two equations yields

$$\frac{\partial a}{\partial N} \;=\; \frac{aq_c^2 v'' t'}{q_c v'' t' \;-\; (t' - a^2 q_c^3 v'')k''} \;>\; 0, \tag{20}$$

where the inequality uses $v'' < 0$ and $k'' \geq 0$. Thus, the faster population growth associated with a larger N raises central-city reinvestment, reducing blight. It can also be shown that the derivative $\partial q_c/\partial N$ is proportional to $-\partial a/\partial N$ and thus negative (implying that the central population n_c rises). Given these impacts, $\partial p_c/\partial N > 0$ follows from (2) and (7).

If traffic congestion is absent but suburban infrastructure is underpriced, then (16) again applies along with (14) and (7). To find the impact of a larger N, n_s in (16) is eliminated using (3) and the resulting three-equation system is differentiated, yielding

$$\frac{\partial a}{\partial N} \;=\; \frac{q_s^2 v''(q_s) I'}{v''(q_s) - n_s I'} \, \frac{a v''(aq_c)}{q_c v''(q_s) - k''} \left[\frac{q_s^2 v''(q_s) I'}{q_c^2 (v''(q_s) - n_s I')} \;+\; \frac{a^2 q_c v''(aq_c) k''}{q_c v''(aq_c) - k''} \right] \;>\; 0. \tag{21}$$

Therefore, as in the case of unpriced congestion, faster population growth in the presence of underpriced infrastructure reduces blight. The derivative $\partial q_c/\partial N$ is again proportional to $-\partial a/\partial N$ and negative, so that n_c rises, and $\partial p_c/\partial N > 0$ also holds.

When the only distortion is open-space amenities, then (7) and (13) with $\theta = 0$ (same as (4)) jointly determine a and q_c. Since N is absent from these equations, the solutions are independent of N, implying no connection between the city's population growth and blight. In this case, the only effect of a larger N is to raise n_s, from (3). These same conclusions also apply when none of the three market failures is present, given that a and q_c are again determined by (7) and (4).

These latter conclusions may seem counterintuitive, given that a larger population in the standard urban model raises the price of housing in the center, which would elicit more reinvestment in the current framework. However, the two-zone structure of the model leads to a different outcome. As long as congestion in commuting or infrastructure is absent, adding population to the suburbs has no effect on the relative attractiveness of the two zones. Given the interzonal nature of commuting costs, these costs are no higher even though the average suburban resident is farther from the CBD, and a parallel conclusion applies to infrastructure costs. As a result, additional population can be accommodated in the suburbs with no effect on n_c and p_c. By contrast, when congestion is present, a larger suburban population raises commuting or infrastructure costs, making the suburbs less attractive. Therefore, the overall increase in population is accompanied by a shift toward the center, which raises p_c and leads to a reduction in blight.

Summarizing yields

Proposition 5. *Faster population growth* (a *higher* N) *reduces blight in the presence of unpriced traffic congestion or underpriced suburban infrastructure. However, population growth has no blight impact under open-space amenities or in the absence of the three market failures.*

6. Conclusion

This paper argues that urban sprawl and urban blight result from the same underlying economic process, both being responses to fundamental market failures affecting urban land markets. The analysis shows that distortions commonly identified as causes of inefficient spatial expansion of urban areas (unpriced traffic congestion, uninternalized open-space externalities, and underpriced suburban infrastructure) also cause an inefficient shortfall in housing reinvestment and maintenance in the central city. Adoption of corrective policies shifts population toward the center, improving reinvestment incentives and thus reducing urban blight. However, it is important to note that the paper considers only one aspect of a very large issue. Poverty and neighborhood externalities are important causes of the problem of urban blight, and these causes should receive at least equal consideration in policy discussions of the problem.

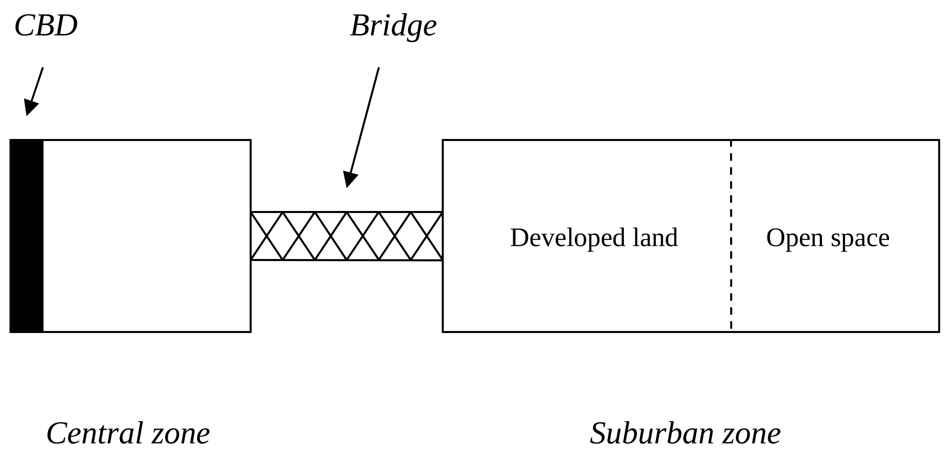

Figure 1. Regional Map

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Footnotes

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[1]See Richardson and Gordon (2001) and Bruegmann (2005) for a critical summary of the arguments.

[2]For a related survey on sprawl that emphasizes public-finance forces in explaining sprawl, see Nechyba and Walsh (2004). For documentation of the sprawl phenomenon using satellite imagery, see Burchfield, Overman, Puga and Turner (2006).

[3]These ideas have been extended in more-recent research on neighborhood dynamics, which includes papers by Bond and Coulson (1989), Aaronson (2001) and Rosenthal (2007).

[4]See Wheaton (1998) and Brueckner (2007) for earlier analyses of the spatial effects of congestion pricing and Brueckner (1997) for an analysis of impact fees.

[5]In their analysis of congestion pricing, Anas and Pines (2007, 2008) assume a similar structure, although in the second paper, the central land area is set at zero, with a bridge effectively connecting the suburbs to the CBD.

[6]Note that in this case, central residents receive toll revenue even though they incur no cost from tolls. A similar outcome occurs in Brueckner's (2007) model, which has a continuum of locations and equal toll redistribution even though toll payments differ across residents. For a related paper, see Wheaton (1998).

[7]With income effects now present, the distribution of the toll revenue as well as landownership arrangements matter in determining the urban equilibrium. The analysis of the Cobb-Douglas case assumes absentee landownership and government retention of the toll revenue (or its distribution to landownwers).

[8]Note that $\ell + 1$ can be viewed as the land area of the region containing the city. Holding consumption fixed, consumers are happier when more of the region consists of open space.

[9]Note that if $k'' = 0$, then $a^* q_c^* < \widehat{a}\,\widehat{q}_c$ is inconsistent with satisfaction of (7), ruling out the case of constant marginal maintenance costs.

[10]It can be shown that Proposition 2 also holds under Cobb-Douglas preferences.

[11]Central infrastructure was built in period 0 and can thus be ignored in the period-1 analysis.

[12]When $k'' = 0$, (7) reduces to $v'(aq_c) = \kappa$ for some constant κ, and it follows that $a^*q_c^* = \widehat{a}\,\widehat{q}_c = z$, where z is some constant. From (14), $q_s^* = \widehat{q}_s$ then holds, and (15) and (16) imply $I(n_s^*q_s^*) + n_s^*q_s^*I'(n_s^*q_s^*) = I(\widehat{n}_s\widehat{q}_s)$. Given $q_s^* = \widehat{q}_s$ and $I' > 0$, it follows that $n_s^* < \widehat{n}_s$. Eq. (3) then implies $q_c^* < \widehat{q}_c$, and since $a^*q_c^* = \widehat{a}\,\widehat{q}_c$, $a^* > \widehat{a}$ must hold. Finally, $p_c^* > \widehat{p}_c$ holds as before.

[13]It can be shown that Proposition 3 also holds under Cobb-Douglas preferences.