# High Compensation Creates a Ratchet Effect

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**Abstract**

We consider a firm which pays a worker for his effort. The more the firm pays in one period, the wealthier the worker is in following periods, and so the more he must be paid for a given effort. Recognizing this wealth effect, in period 1 the firm may pay the worker little. For related reasons, the worker may choose efort which is higher than the firm prefers.

Keywords: Principal-agent, Compensation, Moral hazard, Wealth effects, Ratchet effects

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# 1 Introduction

Firms often profit from hiring the same worker over multiple periods. The benefits may arise from the worker acquiring firm-specific human capital, from the firm learning about the worker's characteristics (which allows it to allocate or design tasks to increase productivity), or from the use of efficiency wages which give a worker an incentive to perform better. Moreover, a worker may prefer to work for the same firm over multiple periods; reasons can include the cost of searching for a job or of moving from one job to another.

Under such conditions, a firm should recognize that the amount it pays to a worker in the current period can affect the amount it must pay in future periods. Our paper focuses on one such connection—the wealth effect. We examine the problem by assuming perfect information and extending the standard principal-agent model. The essential problem the firm faces is that the more the worker earns in period 1, the higher his initial wealth in period 2. This increased wealth both increases the worker's reservation utility (and so requires the firm to pay more in period 2 for any given level of effort by the worker), and changes the worker's marginal rate of substitution between effort and income (thereby increasing the firm's marginal cost of increasing effort).

# 2 Literature

The problem we consider builds on principal-agent models, and more particularly on the ratchet effect. The ratchet effect considers a worker who may be unwilling to work hard today, fearing that the employer may infer that the worker has a low cost of effort, and so will offer a lower wage in the future. For example, in Lazear (1986) and Gibbons (1987) the worker has private information about the firm (such as the difficulty of a job), which he is reluctant to reveal. In Aron (1987) and Kanemoto and MacLeod (1992) the worker's private information concerns a worker-specific attribute such as ability.

# 3 The model

We consider a two-period model. The worker's effort in period $i$ is $e_i$, his observable effort in period $i$ is $B_i$, and his initial wealth is $w$. The worker's income from the firm in period $i$ is $y_i$. The upper bound on the worker's effort in each period is $T$. To make income in period 1 affect behavior in period 2, let all goods be durable—a good bought in period 1 provides the same services in periods 1 and 2.

The two-period incentive problem between a firm and a worker is given as follows

- Period 1
  - The firm observes wealth $w$ and offers the worker a monetary incentive schedule $y_1 = C_1(B_1)$. The worker is paid $y_1$ if he produced observable output $B_1$ in period 1.
  - The potential worker (the agent ) accepts or rejects the contract.
  - If the agent accepts $C_1(B_1)$ he exerts efforts $e_1$, producing output $B_1(e_1)$.
  - The firm and the worker observe $B_1$ and the worker is paid according to the monetary incentive schedule $C_1(B_1)$. The worker's utility in that period is $U_1(w + y_1, T - e_1)$.
- Period 2
  - The firm observes the worker's wealth , $w + y_1$, and offers him a monetary incentive schedule $y_2 = C_2(B_2)$, with the same interpretation as in period 1.
  - The agent accepts or rejects the contract.
  - If the agent accepts $C_2(B_2)$ he exerts effort $e_2$.
  - The firm observes $B_2$ and the worker gets paid according to $C_2(B_2)$. The worker's utility in that period is $U_2(w + y_1 + y_2, T - e_2)$.

Let the worker's utility function be Cobb-Douglas, with the parameter $\alpha$ satisfying

$0 < \alpha < 1$. We then suppose that

$$B_i(e_i) = b_i e_i \text{ with } b_2 \geq b_1 > 0 \quad (1)$$

$$U_1(w + y_1, T - e_1) = (w + y_1)^\alpha (T - e_1)^{1-\alpha} \quad (2)$$

$$U_2(w + y_1 + y_2, T - e_2) = (w + y_1 + y_2)^\alpha (T - e_2)^{1-\alpha}. \quad (3)$$

This specification supposes that the worker cannot borrow in period 1 to smooth consumption over time. The constraint on borrowing follows naturally from the unobservability of effort and thus from the inability of workers to pledge credible repayments of loans. To avoid the analysis of saving decisions we assume that the worker solves his consumption and saving decisions by buying durable consumption goods. We normalize the price of such goods to 1. Note that both assumptions are made for tractability. Essentially, we require that consumption increase over the lifetime. It is natural to assume that $b_2 \geq b_1$; the increase can reflect technical progress or learning experience of workers. Lastly, we assume that firms can only write one–period contracts and we neglect discounting.[1]That is, the firm and the worker each has a discount factor of 1. The worker's utility over the two periods is thus

$$U_1(w + y_1, e_1) + U_2(w + y_1 + y_2, e_2).$$

The firm's profits in period $i$ are $\Pi_i$. The firm maximizes

$$\Pi = \Pi_1 + \Pi_2 = b_1 e_1 - y_1 + b_2 e_2 - y_2.$$

We simplify the formal exposition by assuming a tie-breaking rule that a worker who is indifferent between different effort levels chooses the profit-maximizing level.

In the following we distinguish two cases, differing by the worker's confidence that he will be hired in the following period. When the worker fears that he will be hired only in one period, the firm must offer the worker at least his reservation utility in each period. We can say that the firm has incomplete monopsony power. When the worker expects to be hired over two periods, or when the firm has complete monopsony power, the firm can attract the worker by offering him over two periods the same utility as if the worker enjoyed his reservation utility over two periods. Which assumption is more appropriate depends on the environment, and is discussed below.

1 Though, conceptually, discounting can easily be considered, the optimal contracts become cumbersome, without generating further insights.

# 4 Incomplete monopsony power

We examine first optimal incentive contracts for a firm with incomplete monopsony power. Optimal incentive contracts maximize the firm's profits subject to the participation and incentive constraints of the agent. We derive optimal incentive schedules by working backwards.

## 4.1 Period 2

The firm's contractual problem in period 2 is

$$\max_{y_2=C_2(b_2e_2)} \{\Pi_2 = b_2e_2 - y_2\}$$
$$\text{s.t. } (w+y_1+y_2)^{\alpha}(T-e_2)^{1-\alpha} \geq (w+y_1)^{\alpha}\, T^{1-\alpha}, IC$$

The first constraint is the participation constraint (PC). The incentive constraint (IC) is fulfilled whenever the participation constraint is satisfied, and thus we need not consider the incentive constraint explicitly. In the Appendix we show:

**Proposition 1**

*(i) The firm offers the incentive contract*

$$y_2^0 = C_2(b_2e_2) = (w+y_1)T^{\frac{1-\alpha}{\alpha}}(T-e_2)^{\frac{\alpha-1}{\alpha}} - w - y_1.$$

*(ii) The worker chooses*

$$e_2^0 = T - \left(\frac{w+y_1}{b_2}\right)^{\alpha} T^{1-\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}.$$

*(iii) The firm's profits are*

$$\Pi_2^0 = Tb_2 + w + y_1 - (w+y_1)^{\alpha}\, b_2^{1-\alpha}\, T^{1-\alpha}\left\{\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\frac{1}{1-\alpha}\right\}.$$

An immediate consequence is

**Corollary 1**

*The optimal contract in period 2 has the following comparative statics properties:*

*(i)* $\dfrac{\partial\, y_2^0}{\partial(w+y_1)} > 0,$

(ii) $\dfrac{\partial e_2^0}{\partial(w+y_1)} < 0,$

(iii) $\dfrac{\partial \Pi_2^0}{\partial(w+y_1)} < 0.$

The corollary exhibits the wealth ratchet effect. The higher the worker's initial wealth in period 2, the lower his marginal utility of income earned from effort, and therefore the lower his effort in period 2 and the lower the firm's profits under the optimal contract. Consequently, the firm must pay higher wages to induce a given effort by the worker.

Note that when $w + y_1$ is sufficiently large, the worker's optimal effort, his remuneration, and the profits of the firm all become zero. Therefore high wealth destroys any profitable transaction between the firm and the worker. In contrast, if the worker's wealth is zero ($w + y_1 = 0$), the marginal utility of wealth becomes infinite, the disutility of work is zero, and the firm can induce the worker to exert maximal efforts at zero remuneration. We summarize these observations in the following corollary.

**Corollary 2**

(i) *Suppose* $w + y_1 \geq \dfrac{\alpha T b_2}{1-\alpha}$. *Then,* $e_2^0 = 0$, $\Pi_2^0 = y_2^0 = 0$.

(ii) *Suppose* $w + y_1 = 0$. *Then,* $e_2^0 = T$, $y_2^0 = 0$, $\Pi_2^0 = T b_2$.

## 4.2 Period 1

We now turn to period 1. Note that under incomplete monopsony, in period 1 the firm must offer the worker at least his reservation utility in that period. The firm's objective in that period is to

$$\max_{y_1 = C_1(b_1 e_1)} \left\{ \Pi = \Pi_1 + \Pi_2^0 \right\}$$
$$\text{s.t. } (w+y_1)^{\alpha}(T-e_1)^{1-\alpha} \geq w^{\alpha} T^{1-\alpha},\ IC$$

In period 1, the firm must be careful about the incentive constraint of the worker, as outlined below. In the Appendix we show:

**Proposition 2**

*(i) The firm offers the incentive contract*

$$y_1^0 = C_1(b_1 e_1) = \begin{cases} \overline{y}_1 & \textit{if} \quad b_1 e_1 > b_1 e_1^0 \\ w\, T^{\frac{1-\alpha}{\alpha}} (T - e_1)^{\frac{\alpha-1}{\alpha}} - w & \textit{if} \quad b_1 e_1 \leq b_1 e_1^0 \end{cases}$$

*with*

$$e_1^0 = T - \left( b_1^{-1}\, w^\alpha\, T^{2\,(1-\alpha)}\, b_2^{1-\alpha} \left(\frac{1-\alpha}{\alpha}\right)^\alpha \right)^{\frac{1}{2-\alpha}} \quad (4)$$

$$\overline{y}_1 = w\, T^{\frac{1-\alpha}{\alpha}} \left(T - e_1^0\right)^{\frac{\alpha-1}{\alpha}} - w. \quad (5)$$

*(ii) The worker chooses $e_1^0$.*

*(iii) The firm's profits are*

$$\Pi^0 = b_1\, e_1^0 - \overline{y}_1 + \Pi_2^0(\overline{y}_1).$$

The proposition shows that the firm offers incentive contracts with caps on the worker's earnings in period 1. The intuition runs as follows. At $(e_1^0, y_1^0 = \overline{y}_1)$ the firm maximizes its profits over the two periods subject to the worker's participation constraint. The lifetime utility of the worker when he chooses a particular effort level $e_1$ is

$$U\left(w + y_1^0, T - e_1\right) + U\left(w + y_1^0, T\right) = \begin{cases} U(w + \overline{y}_1, T - e_1) + U(w + \overline{y}_1, T) & \text{if } e_1 > e_1^0 \\ U(w, T) + U(w + y_1^0(e_1),\, T) & \text{if } e_1 \leq e_1^0 \end{cases}$$

Since the worker enjoys utility $U(w, T)$ in period 1 and, since $\partial y_1^0 / \partial e_1 > 0$ for $e_1 < e_1^0$, the wealth ratchet effect allows the worker to obtain the highest possible utility in period 2, and the worker strictly prefers $e_1^0$ over any value $e_1 < e_1^0$. The income cap at $\overline{y}_1$ means that a worker who increases effort beyond $e_1^0$ earns no additional income in period 1, and benefits from no ratchet effect in period 2. For the firm, the wealth ratchet effect embodied in Proposition 2 implies

**Corollary 3**

*Optimal contracts in period 1 have the following comparative statics properties.*

*(i)* $\dfrac{\partial\, e_1^0}{\partial\, b_2} < 0, \; \dfrac{\partial\, e_1^0}{\partial\, b_1} > 0$

*(ii)* $\dfrac{\partial\, \overline{y}_1}{\partial\, b_2} < 0, \; \dfrac{\partial \overline{y}_1}{\partial\, b_1} > 0$

Increased worker productivity in the future induces the firm to reduce the worker's pay and effort in period 1, with the aim of reducing the cost of high-powered incentives in period 2.

To illustrate the distortion induced by the ratchet effect, suppose that in period 2 the firm would replace the worker of period 1 by a new worker with wealth $w$ who has the same productivity. Then the firm would choose in both periods the same incentive contract that we described for the period 2 problem above. Denote the period 1 contract under this scenario by $\hat{y}_1$ and optimal effort levels by $\hat{e}_1$. Optimal choices are:

$$\hat{y}_1 = w\,T^{\frac{1-\alpha}{\alpha}}(T-e_1)^{\frac{\alpha-1}{\alpha}} - w \tag{6}$$

$$\hat{e}_1 = T - w^{\alpha}\,b_1^{-\alpha}\,T^{1-\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \tag{7}$$

We can express $e_1^0$ as

$$e_1^0 = T - \left(w^{\alpha}\,b_1^{-\alpha}\,T^{2-2\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\left(\frac{b_2}{b_1}\right)^{1-\alpha}\right)^{\frac{1}{2-\alpha}}$$

and obtain

$$e_1^0 = T - \left(\big(T-\hat{e}_1\big)\left(\frac{b_2}{b_1}\right)^{1-\alpha} T^{1-\alpha}\right)^{\frac{1}{2-\alpha}}.$$

For $b_2 \geq b_1$ we have

$$e_1^0 < T - \big(T-\hat{e}_1\big)^{\frac{1}{2-\alpha}}\,T^{\frac{1-\alpha}{2-\alpha}} < T - \big(T-\hat{e}_1\big) = \hat{e}_1.$$

Hence, we obtain

**Corollary 4**

*(i) If $b_2 \geq b_1$, then $e_1^0 < \hat{e}_1$*

*(ii)* $\dfrac{\partial(\hat{e}_1 - e_1^0)}{\partial\, b_2} > 0.$

The wealth ratchet effect induces firms to lower incentives in period 1. The distortion increases with the worker's productivity in period 2.

We saw that a firm which recognizes the wealth ratchet effect caps incomes. Without caps the worker would choose higher effort than is optimal for the firm. Indeed, we immediately obtain:

**Corollary 5**

*Suppose the firm offered a worker employed over two periods the same payment schedule in the two periods. That is, in period 1 the firm offers* $\hat{y}_1 = wT^{\frac{1-\alpha}{\alpha}}(T-e_1)^{\frac{\alpha-1}{\alpha}} - w$ *without caps; in period 2 the worker faces* $y_2^0 = (w+y_1)\,T^{\frac{1-\alpha}{\alpha}}(T-e_2)^{\frac{\alpha-1}{\alpha}} - w - y_1$. *The worker then chooses* $\hat{e}_1 = T$ *and the firm would need to pay an infinite amount.*

Our result shows a disadvantage of high-powered incentives created, for example, by stock options. A firm which allows its workers to earn high incomes from high effort or from large capital gains will face very wealthy workers who demand even more income in the future to work as hard as desired. Our analysis can provide a new argument why short-term stock options can backfire (see e.g. *Economist* 2002).

# 5 Complete monopsony power

Consider next a firm with complete monopsony power: it offers a pay schedule in period 1 which the worker anticipates will generate his reservation utility over two periods. Since the problem is complex, to simplify we set $\alpha = 1/2$. We consider a subgame perfect solution: in period 2 the firm offers the worker a pay schedule that induces effort and yields the worker his reservation utility in period 2. But in period 1, the worker is willing to accept utility lower than $w^{1/2}\,T^{1/2}$ because he knows that increased pay in period 1 leads to higher pay and utility in period 2, namely $(w+y_1)^{1/2}\,T^{1/2}$.

Working backwards, the contractual problem of the firm in period 2 is the same as under incomplete monopsony power, namely

$$\max_{C_2(b_2 e_2)} \{\Pi_2 = b_2 e_2 - y_2\}$$
$$\text{s.t. } (w+y_1+y_2)^{1/2}(T-e_2)^{1/2} \geq (w+y_1)^{1/2}\,T^{1/2}.$$

The solution is again characterized by $(y_2^0(e_2),\, e_2^0,\, \Pi_2^0)$. In period 1 the firm must offer compensation which yields the worker his reservation utility. The participation constraint is thus

$$(w+y_1)^{1/2}(T-e_1)^{1/2} + (w+y_1+y_2)^{1/2}(T-e_2)^{1/2} \geq 2w^{1/2}T^{1/2}. \tag{8}$$

The firm's objective in period 1 is to

$$\max_{C_1(b_1 e_1)} \left\{\Pi = \Pi_1 + \Pi_2^0\right\}$$
$$\text{s.t. } (w+y_1)^{1/2}(T-e_1)^{1/2} + (w+y_1+y_2)^{1/2}(T-e_2)^{1/2} \geq 2w^{1/2}T^{1/2}, IC,$$

where $y_2^0$ and $e_2^0$ are the optimal functions derived in proposition 1. In the Appendix we show:

**Proposition 3**

*(i) The firm offers the incentive contract*

$$y_1^* = C_1(b_1 e_1) = \begin{cases} \tilde{y}_1 & \text{if } b_1 e_1 > b_1 e_1^* \\ \dfrac{4wT}{\left((T-e_1)^{1/2}+T^{1/2}\right)^2} - w & \text{if } b_1 e_1 \leq b_1 e_1^* \end{cases}$$

*with*

$$e_1^* = T - \left(\frac{-2\sqrt{T}}{3} + \left(-\frac{q}{2} + D^{1/2}\right)^{1/3} + \left(-\frac{q}{2} + D^{1/2}\right)^{1/3}\right)^2$$
$$q = c - \frac{2}{27}T^{3/2},\ D = \frac{c^2}{4} - \frac{c}{27}T^{3/2},\ c = -2T\,w^{1/2}\,\frac{b_2}{b_1}^{\frac{1}{2}}$$
$$\tilde{y}_1 = \frac{4\,w\,T}{\left((T-e_1^*)^{1/2}+T^{1/2}\right)^2} - w$$

*(ii) The worker chooses $e_1^*$*

*(iii) The firm's profits are*

$$\Pi^* = b_1\,e_1^* - \tilde{y}_1 + \Pi_2^0(\tilde{y}_1)$$

As proposition 3 indicates, the incentive schedule under complete monopsony power is complex. Again the firm caps income. But because the worker has no strict incentive to exert more effort when caps are absent, the cap is less crucial than under incomplete monopsony power.

For further insight, we first make a general comparison of effort levels and then we discuss some examples. Under complete monopsony power, the wealth ratchet effect will again generate inefficiencies compared to a situation where the firm could hire a new worker in period 2. A firm which could hire a new worker in period 2 would offer in period 1 the incentive schedule $\hat{y}_1$ as derived in subsection 4.2, with associated effort level $\hat{e}_1$. Now the relationship between $\hat{e}_1$ and $e_1^*$ is ambiguous, as we discuss in the following.

We start with the following proposition whose proof is given in the Appendix:

**Proposition 4**

*(i) If $b_2$ is sufficiently larger than $b_1$ then $\hat{e}_1 > e_1^*$*

*(ii)* $\dfrac{\partial(\hat{e}_1 - e_1^*)}{\partial b_2} > 0$

We next illustrate the relationship between $\hat{e}_1$ and $e_1^*$ by examples. We first discuss an example with excessive effort.

**Example 1:**

Suppose $b_1 = b_2$ and $\frac{2}{27}T^{1/2} = w^{1/2}\frac{b_2}{b_1}^{1/2} = w^{1/2}\, b_1^{-1/2}$. Then

$$\begin{aligned}
\hat{e}_1 &= \frac{25}{27}T \\
e_1^0 &= T\left(1 - \frac{\sqrt[3]{4}}{9}\right) \\
e_1^* &= T\left(1 - \frac{1}{9}\left(-2 + (3+2\sqrt{2})^{1/3} + (3-2\sqrt{2})^{1/3}\right)^2\right) \approx 0,99T
\end{aligned}$$

The example shows that the prospect of high wealth in period 2 1 induces excessive effort in period 1, despite the low-powered incentive schedule.

The next example illustrates the opposite case when $b_2$ is sufficiently larger than $b_1$.

**Example 2**:

Suppose $b_2 = b_1 \cdot \left(\frac{26}{9}\right)^2$, $w^{1/2}\, b_1^{-1/2} = \frac{13}{81}T^{1/2}$. Then

$$\begin{aligned}
\hat{e}_1 &= \frac{68}{81}T \\
e_1^0 &= T\left(1 - \frac{13\sqrt[3]{52}}{81}\right) \approx \frac{32}{81}T \\
e_1^* &= \frac{65}{81}T
\end{aligned}$$

Now effort $e_1^*$ in period 1 is less than effort in the single-period case.

# 6 Applications

This section applies the results from the preceding sections to explain particular phenomena, and explores how public policy could consider the consequences of the wealth ratchet effect.

## 6.1 Excessive labor turnover and firing

Under the assumption of incomplete monopsony we will show that the wealth ratchet effect can induce excessive firing and turnover of workers.

Suppose that learning by doing in period 1 increases productivity from $b_1$ to $b_2$; to benefit from this potential gain the firm must rehire the worker in period 2. In period 2 the firm can hire a new worker; he would have wealth $w$ and productivity $b_1$. We investigate circumstances which induce the firm to fire a worker at the end of period 1. We assume that $b_2 > b_1$ and set $\alpha = 1/2$. The firm's long-term profits if it fires the worker at the end of period 1 are $\Pi^f$; its profits if it rehires the worker in period 2 are $\Pi^{nf}$. When the worker is fired

$$
\begin{aligned}
e_1^f &= e_2^f = T - \sqrt{\frac{wT}{b_1}} \\
y_1^f &= y_2^f = wT\sqrt{\frac{b_1}{wT}} - w \\
\Pi_1 &= \Pi_2 = \left(\sqrt{Tb_1} - \sqrt{w}\right)^2 .
\end{aligned}
$$

Accordingly,

$$\Pi^f = 2\left(\sqrt{Tb_1} - \sqrt{w}\right)^2 = 2\,Tb_1 - 4\sqrt{Tb_1}\sqrt{w} + 2w$$

When the worker in period 1 is rehired

$$\begin{aligned}
e_1^{nf} &= T - \Big(\frac{T}{b_1}\sqrt{b_2\,w}\Big)^{\frac{2}{3}} \\
y_1^{nf} &= wT\left(\frac{b_1}{T\sqrt{b_2 w}}\right)^{\frac{2}{3}} - w \\
\Pi_1^{nf} &= b_1\,e_1^{nf} - y_1^{nf} \\
e_2^{nf} &= T - \sqrt{\frac{T(w+y_1^{nf})}{b_2}} \\
y_2^{nf} &= \left(w+y_1^{nf}\right)T\left(\frac{b_2}{T(w+y_1^{nf})}\right)^{\frac{1}{2}} - w - y_1^{nf} \\
\Pi_2^{nf} &= Tb_2 + w + y_1^{nf} - 2\sqrt{\left(w+y_1^{nf}\right)b_2\,T} \\
\Pi^{nf} &= b_1\,e_1^{nf} + Tb_2 + w - 2\sqrt{\left(w+y_1^{nf}\right)b_2\,T} \\
&= T(b_1+b_2) + w - b_1\left(\frac{T}{b_1}\sqrt{b_2\,w}\right)^{\frac{2}{3}} - 2\left(T^2\,b_2 w\,b_1\right)^{\frac{1}{3}} \\
&= T(b_1+b_2) + w - 3(T^2\,b_1 b_2\,w)^{\frac{1}{3}}.
\end{aligned}$$

In the Appendix we show:

**Proposition 5**

*A critical value $b_2^*$ exists, with $b_2^* > b_1$, such that the firm fires the worker in period 1 if $b_1 \leq b_2 < b_2^*$.*

Proposition 5 indicates that the firm fires the worker at the end of period 1 even though he will be more productive in period 2. Because the wealth ratchet effect increases the pay necessary to motivate the worker, potential productivity gains arising from experience or from learning-by-doing are unrealized. Clearly, from a social point of view, in the range $b_1 < b_2 < b_2^*$ firing and labor turnover are excessive. The result of Proposition 5 can explain why older and wealthy workers may find it difficult to obtain jobs that guarantee their reservation utility although they are more experienced and more productive than younger workers.

## 6.2 Immediate consumption

Our model can explain why firms may offer pay packages which induce workers to consume more in the first period, thereby reducing future wealth. This is particularly obvious when monopsony power is incomplete.

## 6.3 Savings

From our model, firms would prefer that workers consume their wealth rather than save or buy durable goods. Tax provisions which encourage home-buying may increase the wealth ratchet effect, whereas rules which ease access to credit cards will reduce the wealth ratchet effect. Indeed, under our view, the low savings rates in the US may contribute to the high rate of labor force participation, high level of hours worked, and extensive use of incentive payments in the US.

## 6.4 Piece rates

An extension of the model is to consider a pay schedule which is linear in effort. An increase in the worker's wealth will then improve the worker's terms of trade, at the expense of the employer. The firm will then want to avoid worsening the terms of trade by not paying much in period 1.

Linear compensation may arise for several reasons:

1. Rules against discriminating across people.

2. If compensation is non-linear, then the worker or firm will rent-seek to allocate more of the output in one period.

3. With many different types of workers, the firm will find it difficult to determine optimal non-linear prices.

4. If payment is non-linear, workers can enter into side deals to have one worker get credit for all the output.

We shall look first at the socially optimal solution. Let the worker be paid $b$ per unit of output. In period 1, let the firm charge the worker a fixed sum $F$ to keep him at his reservation utility, so that his expected utility over two periods is

$$\sqrt{24 - e_1}\sqrt{-F + w + be_1}. \tag{9}$$

The first-order condition is

$$\left[\ e_1 = \tfrac{F+24b-w}{2b}\ \right]. \tag{10}$$

The corresponding utility is

$$\frac{\sqrt{-F+24b+w}\sqrt{\frac{-(F-24b-w)}{b}}}{2}$$

To determine $F$, we set this equal to reservation utility:

$$\frac{\sqrt{-F+24b+w}\sqrt{\frac{-(F-24b-w)}{b}}}{2} = \sqrt{24}\sqrt{w}.$$

# 7 Conclusion

Our model is consistent with the existence of rising wage profiles. Of course there are other explanations. A worker's marginal product may increase with his experience, and in a competitive labor market so will his wage. Or, as Lazear (1979) explains and as is consistent with models of efficiency wages, the prospect of a rising wage may induce effort in the current period. But none of these models predicts a hysteresis effect, whereby an increase in income in some period causes all future incomes to rise. Thus, our model could explain why a surging stock market which caused an executive's pay to soar in some year could make his future pay high.

# 8 Appendix

**Proof of proposition 1:**

The contract $C_2(b_2 e_2)$ must obey the participation constraint that implies

$$y_2 = (w + y_1)\, T^{\frac{1-\alpha}{\alpha}}\, (T - e_2)^{\frac{\alpha-1}{\alpha}} - w - y_1 \tag{11}$$

Maximizing $\Pi_2$ with respect to $e_2$ and using $y_2$ from the PC yields the first-order condition:

$$\frac{\partial \Pi_2}{\partial e_2} = b_2 - (w + y_1)\, T^{\frac{1-\alpha}{\alpha}} (T - e_2)^{-\frac{1}{\alpha}} \frac{1-\alpha}{\alpha} = 0$$

Solving yields

$$b_2 (T - e_2)^{\frac{1}{\alpha}} = (w + y_1)\, T^{\frac{1-\alpha}{\alpha}}\, \frac{1-\alpha}{\alpha}$$

$$e_2^0 = T - \left(\frac{w + y_1}{b_2}\right)^{\alpha} T^{1-\alpha} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}$$

The firm therefore wants to achieve $e_2^0$. Using our tie-breaking rule the schedule

$$y_2^0 = C_2(b_2 e_2) = (w + y_1)\, T^{\frac{1-\alpha}{\alpha}}\, (T - e_2)^{\frac{\alpha-1}{\alpha}} - w - y_1$$

is sufficient to achieve this objective: the worker is indifferent between different effort levels and will choose $e_2^0$, and so the IC is also satisfied. Note that

$$\frac{\partial\, e_2^0}{\partial (w + y_1)} < 0$$

$$\frac{\partial\, y_2^0}{\partial (w + y_1)} = \left(\frac{T}{T - e_2}\right)^{\frac{1-\alpha}{\alpha}} - 1 > 0$$

Finally, equilibrium profits are given by:

$$\begin{aligned}
\Pi_2^0 &= b_2 e_2^0 - y_2^0 \\
&= T b_2 - b_2 \left(\frac{w + y_1}{b_2}\right)^{\alpha} T^{1-\alpha} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \\
&- (w + y_1)\, T^{\frac{1-\alpha}{\alpha}} \left\{ \left(\frac{w + y_1}{b_2}\right)^{\alpha} T^{1-\alpha} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \right\}^{\frac{\alpha-1}{\alpha}} + w + y_1 \\
&= T b_2 + w + y_1 - (w + y_1)^{\alpha}\, b_2{}^{1-\alpha}\, T^{1-\alpha} \left\{ \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \frac{1}{1-\alpha} \right\}
\end{aligned}$$

$\blacksquare$

**Proof of proposition 2:**

From the participation constraint we obtain:

$$(w + y_1)^\alpha = w^\alpha T^{1-\alpha}(T - e_1)^{\alpha-1}$$

We proceed in two steps. We first maximize profits of the firm subject to the participation constraint above. In the second step, we consider the incentive constraint of the worker. In the first step, we have:

$$\max_{e_1} \left\{\Pi = \Pi_1 + \Pi_2^0\right\}$$
$$\text{s.t.}(w + y_1)^\alpha = w^\alpha T^{1-\alpha}(T - e_1)^{\alpha-1}$$

We have

$$\Pi = b_1 e_1 - y_1 + T b_2 + w + y_1 - (w + y_1)^\alpha\, b_2^{1-\alpha}\, T^{1-\alpha} \left\{ \left(\frac{1-\alpha}{\alpha}\right)^\alpha \frac{1}{1-\alpha}\right\}$$

Using the participation constraint yields:

$$\Pi = b_1 e_1 + T b_2 + w - w^\alpha\; T^{2-2\alpha}\, b_2^{1-\alpha}(T - e_1)^{\alpha-1} \left\{ \left(\frac{1-\alpha}{\alpha}\right)^\alpha \frac{1}{1-\alpha}\right\}$$

The first-order condition is

$$\frac{\partial\, \Pi}{\partial\, e_1} = b_1 - w^\alpha\, T^{2-2\alpha}\, b_2^{1-\alpha}\, (T - e_1)^{\alpha-2} \left\{\left(\frac{1-\alpha}{\alpha}\right)^\alpha\right\} = 0 \qquad (12)$$

yielding

$$e_1^0 = T - \left(b_1^{-1}\, w^\alpha\, T^{2-2\alpha}\, b_2^{1-\alpha} \left(\frac{1-\alpha}{\alpha}\right)^\alpha\right)^{\frac{1}{2-\alpha}}$$

If the firm needed to take into account only the PC, it would want the worker to exert effort $e_1^0$. The associated wage according to the PC, called $\overline{y}_1$, is given by

$$\overline{y}_1 = w\, T^{\frac{1-\alpha}{\alpha}}\, (T - e_1^0)^{\frac{\alpha-1}{\alpha}} - w$$

In the second step, we investigate how the firm can induce the worker to choose exactly $e_1^0$. Recalling that the worker's utility in period 2 will be $U(w+y_1,T)$, the two-period utility of the worker for a particular choice $e_1$ under the proposed incentive schedule is

$$U(w+y_1,T-e_1)+U(w+y_1,T) = \begin{cases} U\left(w+\overline{y}_1, T-e_1\right)+U(w+\overline{y}_1,T) & \text{if } e_1 > e_1^0 \\ U(w,T)+U\left(w\,T^{\frac{1-\alpha}{\alpha}}(T-e_1)^{\frac{\alpha-1}{\alpha}},T\right) & \text{if } e_1 \leq e_1^0 \end{cases}$$

Accordingly,

$$\frac{\partial\{U(w+y_1,T-e_1)+U(w+y_1,T)\}}{\partial\, e_1} \begin{cases} <0 & \text{if } e_1 > e_1^0 \\ >0 & \text{if } e_1 < e_1^0 \end{cases}$$

The worker maximizes utility by choosing $e_1^0$. By imposing caps on income at $\overline{y}_1$, the firm ensures that the worker's effort does not exceed $e_1^0$, and so he does not exert excessive effort with the aim of benefiting from the wealth ratchet effect in the future.

Lastly, the overall profits of the firm are given as:

$$\Pi^0 = b_1\, e_1^0 - \overline{y}_1 + \Pi_2^0(\overline{y}_1)$$

■

**Proof of proposition 3:**

From the PC we obtain:

$$(w+y_1)^{\frac{1}{2}} = \frac{2w^{\frac{1}{2}}\,T^{\frac{1}{2}}}{(T-e_1)^{\frac{1}{2}}+T^{\frac{1}{2}}}$$

Again, we proceed in two steps. We first maximize profits subject only to the PC. Using the PC, profits of the firm amount to:

$$\begin{aligned} \Pi &= b_1\, e_1 + \Pi_2^0 - y_1 \\ &= b_1\, e_1 + Tb_2 + w - (w+y_1)^{\frac{1}{2}}\; 2b_2^{\frac{1}{2}}\; T^{\frac{1}{2}} \\ &= b_1\, e_1 + Tb_2 + w - \frac{4T\, w^{\frac{1}{2}}\; b_2^{\frac{1}{2}}}{(T-e_1)^{\frac{1}{2}}\; + T^{\frac{1}{2}}} \end{aligned}$$

The first-order condition is

$$\frac{\partial \Pi}{\partial e_1} = b_1 - 4T\, w^{\frac{1}{2}}\, b_2^{\frac{1}{2}}\, \frac{\frac{1}{2}(T-e_1)^{-\frac{1}{2}}}{\left\{(T-e_1)^{\frac{1}{2}} + T^{\frac{1}{2}}\right\}^2} = 0$$

By setting $x = (T-e_1)^{\frac{1}{2}}$ we obtain:

$$b_1\{x + T^{\frac{1}{2}}\}^2 - 2T\, w^{\frac{1}{2}}\; b_2^{\frac{1}{2}}\; \frac{1}{x} = 0$$

or

$$x^3 + ax^2 + dx + c = 0$$

with $a = 2T^{\frac{1}{2}}$, $d = T$, $\quad c = -2T\, w^{\frac{1}{2}}\, \frac{b_2^{\frac{1}{2}}}{b_1}$.

The solution of this cubic equation is given by standard formulas:

$$\begin{aligned} p &= \frac{3b - d^2}{3} = -\frac{T}{3} \\ q &= c + \frac{2d^3}{27} - \frac{db}{3} = c - \frac{2}{27}\, T^{\frac{3}{2}} \\ D &= \left(\frac{P}{3}\right)^3 + \left(\frac{q}{2}\right)^2 = \frac{c^2}{4} - \frac{c}{27}\, T^{\frac{3}{2}} \end{aligned}$$

Since $c < 0$ we have $D > 0$ and thus one real solution exists, given by:

$$x_1^* = -\frac{a}{3} + \left(-\frac{q}{2} + D^{\frac{1}{2}}\right)^{\frac{1}{3}} + \left(-\frac{q}{2} - D^{\frac{1}{2}}\right)^{\frac{1}{3}}$$

which yields

$$e_1^* = T - \left(-\frac{2T^{\frac{1}{2}}}{3} + \left(-\frac{q}{2} + D^{\frac{1}{2}}\right)^{\frac{1}{3}} + \left(-\frac{q}{2} - D^{\frac{1}{2}}\right)^{\frac{1}{3}}\right)^2$$

The associated income level is determined by $\tilde{y}_1$.

In the second step, we observe that the worker is indifferent between different effort levels if he is offered the compensation schedule satisfying the PC over the lifetime $y_1 = \frac{4w\,T}{\left((T-e_1)^{\frac{1}{2}} + T^{\frac{1}{2}}\right)^2} - w$.

Future higher utility from the ratchet effect is offset by lower income in the first period. To ensure that the worker doesn't go beyond $e_1^*$, the firm can cap income at $\tilde{y}_1$ which we incorporate into the compensation schedule. An indifferent worker who acts in the interest of the firm would choose $e_1^*$ even with no upper bounds on income, so the cap is not strictly necessary.

■

**Proof of proposition 4:**

The comparison between $\hat{e}_1$ and $e_1^*$ yields: $\hat{e}_1 > e_1^*$ is equivalent to

$$w^{\frac{1}{2}}\, b_1^{-\frac{1}{2}}\, T^{\frac{1}{2}} < \left( -\frac{2T^{\frac{1}{2}}}{3} + \left(-\frac{q}{2} + D^{\frac{1}{2}}\right)^{\frac{1}{3}} + \left(-\frac{q}{2} - D^{\frac{1}{2}}\right)^{\frac{1}{3}} \right)^2$$

We next observe that for $c < 0$

$$\left(-\frac{q}{2} + D^{\frac{1}{2}}\right)^{\frac{1}{3}} + \left(-\frac{q}{2} - D^{\frac{1}{2}}\right)^{\frac{1}{3}} > \frac{2T^{\frac{1}{2}}}{3}$$

since the left side monotonically declines with $c$ and equal to $\dfrac{2T^{\frac{1}{2}}}{3}$ for $c = 0$. We next observe that $e_1^*$ is monotonically decreasing in $b_2$. For $b_2 = 0$ we have $c = 0$ and $e_1^* = T$. For $b_2$ sufficiently large we obtain $e_1^* = 0$. Since $\hat{e}_1$ is independent of $b_2$, the first and second assertion of the proposition follow.

■

**Proof of proposition 5:**

We start by showing that

$$\Pi^{nf}(b_1, b_2) < \Pi^{f}(b_1) \text{ for } b_1 = b_2.$$

$$\begin{aligned} \Pi^{f}(b_1) - \Pi^{nf}(b_1, b_2) &= w - 4\sqrt{Tbw} + 3(T^2\, b_2 b_1\, w)^{\frac{1}{3}} \\ &= \frac{1}{Tb}\left\{ \frac{w}{Tb} - 4\sqrt{\frac{4}{Tb}} + 3\left(\frac{w}{Tb}\right)^{\frac{1}{3}} \right\} \\ &= \frac{1}{Tb}\left\{ x - 4\, x^{\frac{1}{2}} + 3\, x^{\frac{1}{3}} \right\} \end{aligned}$$

when we set $x = w/Tb$). Since $w < Tb$ we have $0 < x < 1$. It remains to show that $\Delta \equiv x - 4\, x^{1/2} + 3\, x^{1/3} > 0$. Setting $y = x^{1/6}$ yields

$$\Delta = y^6 - 4y^3 + 3y^2 = y^2(y-1)^2(y^2 + 2y + 3) > 0.$$

Hence, $\Pi^{nf} < \Pi^{f}$ for $b_1 = b_2$. We next calculate

$$\begin{aligned}\frac{\partial\,\Pi^{nf}}{\partial\,b_2} &= T - T^{\frac{2}{3}}\,b_1^{\frac{1}{3}}\,b_2^{-\frac{2}{3}}\,w^{\frac{1}{3}} \\ &= T\left(1 - \left(\frac{w}{Tb_1}\right)^{\frac{1}{3}}\left(\frac{b_1}{b_2}\right)^{\frac{2}{3}}\right)\end{aligned}$$

Since $\frac{w}{Tb_1} < 1$ and $\frac{b_1}{b_2} < 1$ for $b_2 \geq b_1$ we obtain $\frac{\partial\,\Pi^{nf}}{\partial\,b_2} > 0$. Lastly, we have

$$\lim_{b_2 \to \infty} \Pi^{nf} = \infty.$$

Hence, by the mean value theorem, the assertion follows.

■

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$B_i$ Output in period $i$

$b_i$ Productivity factor in peirod $i$, or $B_i/e_i$

$C_i(\cdot)$ Compensation contract offered by firm for output in period $i$

$e_i$ Worker's effort in period $i$

$e_1^0$ Worker's optimal effort in period 1 under firm's contract with incomplete monopsony power and worker rehired in period 2

$e_1^f$ Worker's optimal effort in period 1 under firm's contract with incomplete monopsony power, worker is fired in period 2, and $\alpha = 1/2$

$e_1^{nf}$ Worker's optimal effort in period 1 under firm's contract with incomplete monopsony power, worker is rehired in period 2, and $\alpha = 1/2$

$e_1^*$ Worker's optimal effort in period 1 under firm's contract with perfect monopsony power and worker rehired in period 2

$e_2^0$ Worker's optimal effort in period 2 under firm's contract with incomplete monopsony power and worker rehired in period 2

$\hat{e}_1$ Worker's optimal effort in period 1 under firm's contract with incomplete monopsony power and worker replaced in period 2

$T$ Upper bound on the worker's effort

$U$ Utility function of worker

$w$ Initial wealth of worker

$y_i$ Income in period $i$

$y_2^0$ Worker's income in period 2 under firm's contract with imperfect monopsony power and worker rehired in period 2

$\hat{e}_1$ Worker's income in period 1 under firm's contract with imperfect monopsony power and worker replaced in period 2

$y_1^*$ Worker's income in period 1 under firm's contract with perfect monopsony power and worker rehired in period 2

$\alpha$ Parameter in Cobb-Douglas utility function

$\Pi_t$ Firm's profits in period $t$

$\Pi$ Sum of firm's profits over two periods