Income Taxes, Property Values, and Migration*

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Abstract

Consider a jurisdiction in which some locations are more desirable than others, and which has both poor and rich residents. A tax imposed on the rich causes the price of desirable locations to fall. The fall in property values reduces the incentive of the rich to migrate, thereby allowing for more redistributive taxation than is predicted by standard models in public finance.

Key words: Taxes, property values, migration, redistribution

1 Introduction

A state or other jurisdiction which imposes high taxes on the rich may induce some of its residents to move away, and induce poor people to move into the state in search of the redistributive benefits. Such migration would appear to limit the ability of a state to redistribute income, or to finance generous social benefits. The problem may generate a "race to the bottom," with each state attempting to attract rich residents by taxing them at a lower rate than other states do, generating an equilibrium in which no state imposes redistributive taxes. Despite this theoretical possibility, we see governments engaged in large redistribution. Migration may be limited for several reasons: people may face a

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cost of moving; some people may prefer to live in one locality rather than another; property values may decline in response to higher taxes, thereby reducing the incentives to move.

This paper examines the last two effects. In particular, we suppose that good locations are scarce in any jurisdiction: people who want to live near the beach or on top of a mountain with a gorgeous view will find such locations limited.[1] We shall see that a small income tax imposed on the rich in a jurisdiction with heterogeneous locations reduces property values in desirable locations, reduces the utility of the rich, and increases the utility of the poor. Tax incidence, however, is complicated because a person's utility depends on three factors: his income, the rent he pays, and the location where he lives. The utility of each rich person falls, regardless of where he lives within the jurisdiction. The incomes of rich people, after paying the tax and after paying rents, fall, but by differing amounts. Property values also fall, hurting landlords. These results, which are related to the research tradition in urban economics, thus extend the conventional public finance view on taxes and migration.

2 Literature

We shall consider select aspects of housing and location. For an analysis of how owner-occupied housing with heterogeneous locations provides insurance against rent increases see Ortalo-Magne and Rady (2002).

Taxes and migration The effect of taxes on migration is a central topic in studies of international tax competition; see, for example, Wildasin (1994) and Sinn (1997). The Christiansen-Hagen-Sandmo model (1994) shows how differences in average income tax rates affect migration. Though migration is influenced by relative employment and earnings opportunities, they have been considered elsewhere, and we do not.[2]

Voting Several papers consider the tax rates that a majority of voters in a jurisdiction will adopt; see Westhoff (1977), Epple, Filimon, and Romer (1984), Epple and Romer (1991), and Goodspeed (1989). The models assume that individuals differ along a single dimension, typically income. In these models, an appropriately defined marginal rate of substitution is assumed to vary monotonically across households. Use of such a monotonicity condition on the marginal rate of substitution was first introduced by Ellickson (1971). Under this assumption, households will be perfectly stratified by income.

Epple and Platt (1998) develop a model of local jurisdictions in which households differ in both income and tastes, and can thus generate less stark income

[1] The scarcity of desirable locations may also make the property tax attractive. We focus, however, on an income tax imposed on rich persons, with labor supply inelastic.

[2] See Bover and Muellbauer (1989) for labor market aspects and Haavio and Kauppi (2002) for the effects of liquidity constraints. Cameron and Muellbauer (1998) consider commuting as an alternative to migration.

stratification. Hendricks (1999) considers how redistribution affects mobility, which in turn determines the identity of the voters and the levels of redistribution they favor.

Taxes and property values The effects of taxes on property values and on migration are studied by Epple and Romer (1991). They argue that though local redistribution induces sorting of the population, the induced changes in property value make redistribution feasible. A major difference between their analysis and ours is that in Epple and Romer (1991) land is homogeneous, whereas in our model some locations are preferred to others.[3]

Epple and Platt (1998) study equilibrium and redistribution in a system of local jurisdictions when households differ in their preferences and in their incomes. In most models, complete income stratification is a necessary condition for equilibrium. In our model rich people can live in all communities. We differ from the literature in one important way: other studies suppose that land is homogeneous in a jurisdiction. We assume it is heterogeneous.

3 Assumptions

Residents Each resident is either rich or poor. All have the same utility function. All rich people have the same (high) pre-tax income, y^R; all poor people have the same (low) pre-tax income y^P. The rich live in private houses; the poor live in public housing projects. Land differs in its location and the rental price. Location is indicated by e, the elevation at which a person resides, also determining the quality of housing. Elevation ranges from 0 to $H = 1$. Each elevation may accommodate a density of one resident. Therefore, if the whole hill is occupied, the population size on the hill is unity.

Housing in the low-quality valley, where $e = 0$, is in perfectly elastic supply. Land on the hill (with $e > 0$) is in inelastic supply, with the density of land at each elevation set to 1. Thus, we make the natural assumption that low-quality land is abundant, but that high-quality land is scarce. That is, the mass of rich people can exceed the mass of residence units on high-quality land. We do not restrict the relative number of the rich and the poor.

Individuals have preferences over consumption (x) and elevation (e), represented by the utility function

$$W = u(x) + v(e),$$

with standard concavities. (In some of the following we shall make the more specific assumption that $W = x + \gamma e$, with γ an exogenous constant.) People have no jurisdiction-specific preferences. Initially, the jurisdiction has n^R rich people; migration can change that number. The number of poor residents is fixed at n^P. The poor reside in public rental units in the valley, enjoying locational utility $v(0)$.

[3] Epple and Romer's model is, however more general than ours in their treatment of housing: unlike them we suppose that the size of a house and of a lot is fixed.

Government policy Government can impose a tax of τ on each rich resident. Below we shall make various assumptions over how the revenue is redistributed. One assumption is that the government redistributes all tax revenue from the rich to the poor. Another assumption is that government is a Leviathan, keeping the tax revenue for its own purposes. We take a more general view, allowing the government to keep a share α of its tax revenue for its own purposes and redistributing the rest to the poor. Let the number of rich people in the equilibrium with migration be $\widetilde{n}^R$; then the total tax revenue is $\widetilde{n}^R\tau$. Thus, the waste by the government is $\alpha\widetilde{n}^R\tau$. Let t be the transfer to each poor person, so that aggregate transfers are

$$n^P t = (1-\alpha)\widetilde{n}^R\tau.$$

We assume throughout that the tax is not confiscatory: the post-tax income of a rich person exceeds the post-transfer income of a poor person.

Migration The poor do not migrate either into or out of any jurisdiction. The rich can migrate. The reservation utility to a rich person outside the jurisdiction is $\overline{W}$: no rich person will live in the jurisdiction if his utility is less than that.

Land Housing (or land) is owned by absentee landlords. A rich person can choose to live in the valley or on the hill; the rent at elevation e is c_e. The rent in the valley is zero.

4 Closed economy

We begin our analysis by considering a single closed economy. Any individual can choose where to live in the jurisdiction, but he cannot avoid the tax by migration. For a rich person to be indifferent between living in the valley and on the hill he must enjoy the same utility at both:

$$u(y^R - c_e) + v(e) = u(y^R) + v(0).$$

This determines the equilibrium rental price c_e of housing at location e. A unique convex gradient (x, e) describes an equilibrium:

$$\frac{dx}{de} = -\frac{v'(e)}{u'(x)} < 0,$$

with $dx^2/de^2 = -v''(e)/u'(x) \geq 0$. Thus, $dc_e/de > 0$ and $dc_e^2/de^2 \leq 0$. That is, the marginal willingness to pay for a better location declines with the elevation.[4]

The rich on the hill enjoy the same utility as the rich in the valley. Continuity of the $u(.)$ function and of the $v(.)$ function imply that $\lim_{e\to 0} c_e = 0$. Thus, rents on the hill decline smoothly to zero when moving down to the valley.

[4] With more rich people in the jurisdiction than units on the hill, a poor person would not occupy any private rental unit on the hill even if those would be open to him, as his marginal utility from consumption is higher than that of the rich. Poor people would thus live in the valley. Equilibrium would thus be characterized by spatial segregation.

Lemma 1 *Rents decline smoothly to zero when moving down to the valley:* $\lim_{e\to 0}\ c_e = 0$.

Property owners exploit the whole surplus generated by a better location. The rich in the valley, however, enjoy higher utility than the poor in the valley: $u(y^R) + v(0) > u(y^P) + v(0)$.

4.1 Effect of tax

Consider a small tax at the level τ. The indifference condition for the rich about residence location is

$$u(y^R - \tau - c_e) + v(e) = u(y^R - \tau) + v(0).$$

Since the rich now have less income, the marginal willingness to pay for a desirable location falls. Hence, property values on the hill fall:

$$\frac{\partial c_e}{\partial \tau} = \frac{u'(y^R - \tau)}{u'(y^R - \tau - c_c)} - 1 < 0.$$

The rich in the valley suffer, as their consumption declines by the amount of the tax. The rich on the hill suffer the same utility loss, because in equilibrium their utility equals the utility of those rich living in the valley.

$$\frac{\partial (y^R - \tau - c_e)}{\partial \tau} = -\frac{u'(y^R - \tau)}{u'(y^R - \tau - p)} < 0.$$

Proposition 2 *A small income tax on the rich coupled with transfers to the poor, in a jurisdiction with heterogeneous locations, reduces property values in desirable locations, reduces the utility of the rich, and increases the utility of the poor.*

Tax incidence, however, is surprisingly complicated both among the rich people and among landlords.

Proposition 3 *The tax reduces the utility of the rich in the valley and on the hill by the same amount. Consumption by the rich in the valley declines by the amount of the tax. Consumption by the rich on the hill declines by less than by the amount of the tax; some of the tax burden is shifted to landlords because rents on the hill decline.*

Proof. With the rent in the valley equal to zero, landlords in the valley will not share the tax burden. Since post-tax utility of a rich person on the hill must be identical to that of a rich person in the valley, the tax must reduce the utility of a rich person in the valley and on the hill by the same amount. Decreasing marginal utility of consumption leads a tax on a rich person to reduce his willingness to pay for desirable locations, thereby reducing rents. ■

4.2 Optimal tax

To analyze what tax a government would choose, we allow government to have both kleptocratic and redistributive objectives. Let the fraction of tax revenue government can redistribute be $1-\alpha$. Let $1-\lambda$ be the weight government places on redistribution. Then social welfare is

$$SW = \gamma\alpha n^R\tau + (1-\lambda)(1-\alpha)n^R\tau.$$

Government maximizes the tax revenue from the rich, $n^R\tau$, subject to the constraint

$$\begin{aligned} y^R - \tau &\geq y^P + t \\ &= y^P + (1-\alpha)n^R\tau/n^P. \end{aligned}$$

Then, the optimal tax level in a closed economy is

$$\tau^* = \frac{y^R - y^P}{1 + (1-\alpha)n^R/n^P}.$$

The optimal tax therefore increases with the inequality in pre-tax income, with the government's ability to appropriate the tax revenue for non-redistributive purposes, and with the relative proportions of rich and poor in the population. If the share of the poor is large, the optimal tax is high. Note, however, that the optimal tax does not depend on the relative importance of the kleptocratic and redistributive objective. Below we mainly work with $\alpha = \lambda = 1$.

5 Migration

Consider next taxation when rich people can migrate from one country to another. Now a person can choose where to live in one country, and in which country to live. Tax competition thus arises between countries. Our main interest is to show that nevertheless the countries can impose positive taxes on the rich. The reason that taxes do not generate a " race to the bottom" is that an increase in taxes in one country reduces property values at desirable locations, thereby reducing the incentive to migrate. Or seen differently, since the number of desirable locations in any country is limited, people living at the most desirable locations in one country will not want to move to the other country even if taxes are higher in the home country.

Indeed, different countries often impose different tax rates. Moreover, different jurisdictions within a country have different tax rates. For example, within the European Union, Denmark imposes a higher income tax than does Ireland. This demonstrates the possibility of different taxes across countries even when mobility is present. We shall see that the resulting tax equilibria can be complicated. It is therefore helpful to start by sketching the fundamental mechanisms involved.

We consider two identical countries. This section consider only rich people; section (6) below extends the analysis to consider poor people. For the sake of argument, let the mass of people living on the hill equal one. In each country the mass of people in the valley is N^V. The total population is thus $1 + N^V$. Recall from above that in any country, a rich person living on the hill must enjoy the same utility as a rich person living in the valley. The equality is generated by the willingness of a person to pay for a desirable location, making for an equilibrium in the rental market on the hill with rental prices continuously increasing with elevation. People living in better locations thus consume less of the composite consumption good. A tax on rich people reduces the welfare of each rich person by the same amount, independent of his location.

When the countries engage in tax competition and people can migrate, the equilibrium tax rates may differ across countries. In the valley, rents are zero; hence they cannot accommodate the higher tax, and the rich emigrate from the valley. On the hill, rents will fall. As the rents at the bottom of the hill are low and hence may not allow for the adjustment by the full tax difference, rich people living at the bottom of the hill emigrate. Demand for locations on the hill declines, and rents fall.

We continue by working with the utility function $U = x + \gamma e$ with the consumption $x = y^R - \tau - c_e$. The valuation of location per unit of elevation at all occupied locations is thus given by γ. Define $\underline{e}^i$ as the lowest occupied location in country i in the post-tax equilibrium. The rent in the lowest occupied elevation is zero. Generally, the rent in country i at elevation e, with $1 \geq e \geq \underline{e}^i$, is

$$c_e^i = \gamma(e - \underline{e}^i).$$

5.1 Nash equilibrium

We shall consider in turn tax competition in a Nash equilibrium and in a Stackelberg solution.

Under some conditions, no Nash equilibrium in symmetric pure strategies exists. Suppose rich people initially live both in the valley and on the hill. People living in valley are highly sensitive to small tax differences and willing to change their location in the face of even small tax differences. Each country therefore gains from charging a tax infinitesimally smaller than the other country does. And no symmetric equilibrium can appear; because then the country with the lower tax would choose the tax rate to be infinitesimally less than the other country's. By continuity, the result holds even when some rich people live in the valley, however, few.

A Nash equilibrium in pure strategies can exist if the number of rich people is sufficiently small so that the all live on the hill. A country which charges an infinitesimally higher tax will then not lose all its residents because property values in the high tax country will fall.

To solve for a Nash equilibrium, we must determine the reaction functions of the two countries. Assume that initially both hills are occupied, while there no one lives in either valley. Therefore, population is one in both countries. Denote the countries by a and b. If the two countries would adopt different taxes, then the country with higher taxes would lose part of its population. By the expression of c_e, the rent at the lowest occupied location would drop to zero. Denote next the migration from country a to country b by m_{ab}. If $m_{ab} > 0$, then people migrate from a to b; if $m_{ab} < 0$, people migrate from b to a. Given the utility function adopted, $x + \gamma e$, an equilibrium condition for residents is that $Y^R - \tau_a + \gamma L_a = Y^R - \tau_b + \gamma L_b$. It therefore follows that

$$m_{ab} = \frac{\tau_a - \tau_b}{\gamma}.$$

The tax revenues of the two countries are

$$TR_a = \tau_a(1 - m_{ab})$$

$$TR_b = \tau_b(1 + m_{ab}).$$

We next solve for the reaction functions of the two countries by differentiating with respect to their decision variables. This yields

$$\frac{\partial TR_a}{\partial \tau_a} = 1 - \frac{\tau_a - \tau_b}{\gamma} - \frac{\tau_a}{\gamma} = 0$$
$$\frac{\partial TR_b}{\partial \tau_b} = 1 + \frac{\tau_a - \tau_b}{\gamma} - \frac{\tau_b}{\gamma} = 0.$$

These can be solved for

$$\tau_a(\tau_b) = \frac{\gamma + \tau_b}{2}$$

$$\tau_b(\tau_a) = \frac{\gamma + \tau_a}{2}.$$

In a symmetric equilibrium,

$$\tau_a = \tau_a = \gamma.$$

Notice that neither jurisdiction has an incentive to deviate. Since $\frac{\partial^2 TR_i}{\partial \tau_i^2} < 0$ for $i = a, b$, choosing either a lower or a higher tax rate would result in a lower tax revenue.

Proposition 4 *In a symmetric Nash equilibrium with rich people living only on the hill, the tax rate is positive and determined by the relative valuation of elevation, $\tau_a = \tau_a = \gamma$.*

Note that in such a tax equilibrium, although migration is feasible, it does not occur.

5.2 Stackelberg solution

Consider next the solution when the countries set taxes sequentially. We consider one country as a leader and the other as follower. The leader can be interpreted as a country which cannot quickly change policy, such as a country with two houses in the legislature rather than only one, or a country with a president who can veto tax bills passed by the legislature. We shall, however, also consider below the conditions under which the leader or the follower enjoy higher welfare. Let the tax set by the leader be τ_l; the tax set by the follower is τ_f. We consider both an unconstrained tax choice and a constrained choice, for example, that a rich person's after-tax income must exceed the income of a poor person.

Without a formal analysis and to gain some intuition, suppose rich people live in both the valley and on the hill. Suppose the follower may set its tax rate below that of the leader. We consider below formally both the case where the follower chooses an infinitesimally lower tax than the leader and where it chooses a lower tax by a discrete amount. A follower which would set the tax infinitesimally below that of the leader would attract all people from the valley of the leader. The benefit to the follower from cutting its tax more than infinitesimally below that of the leader is that it can thereby attract some tax payers from the leader's hill. Tax revenue from previously attracted tax payers of the follower, however, is reduced. The greater the number of people living in the valley, the larger the fall in revenue from initial residents, while the benefits in terms of additional tax payers do not change. To cope with both mechanisms, we turn to a formal analysis.

The follower sets its optimal tax after observing the tax set by the leader. The leader sets its tax optimally, anticipating the choice of the follower.

As before, let utility be separable and linear, $U = x+\gamma e$. Without migration, this implies that the rent per unit of elevation at all occupied locations is γ. Let the lowest occupied location in the leader-country in the post-tax equilibrium be $\underline{e}^l$; the rent there is zero. Generally, the rent in the leader country at elevation e, with $1 \geq e \geq \underline{e}^l$ is

$$c_e^l = \gamma(e - \underline{e}^l).$$

Then, the utility of the remaining residents in the leader country equals the utility of the person in the valley in the follower country. In the migration equilibrium with $\tau_l \geq \tau_f$, utilities across jurisdictions are equal in all locations, particularly at $\underline{e}^l$ at the leader country and in the valley of the follower country:[5]

$$(y^R - \tau_l) + \gamma\underline{e}^l = (y^R - \tau_f),$$

where the rent at $\underline{e}^l$ equals zero. Solving for $\underline{e}^l$ gives the number of people emigrating from the hill of the leader country,

$$\underline{e}^l(\tau_l, \tau_f) = \frac{\tau_l - \tau_f}{\gamma}.$$

[5] It is easiest to think that all migrants settle in the valley because they gain no utility from living on the hill. This follows because people aiming to live on the hill bid up the rents there until the whole surplus from living on the hill is captured by the landlords.

Suppose that country l initially has the higher tax.[6] Then with migration, an increase in tax in country l has two effects. It causes a reduction in rents in that country, raises the lowest location on the hill where people live, $\underline{e}^l$ and thus makes some people emigrate from the hill to the other country. The amount of migration is $\frac{\partial \underline{e}^l(\tau_l,\tau_f)}{\partial \tau_f} = \frac{1}{\gamma} > 0$. Rents uniformly capitalize the tax increase at all occupied locations on the hill where the rent initially exceeds the tax increase, $dc_e^l = -\gamma \frac{\partial \underline{e}^l}{\partial \tau_l} d\tau_l = -d\tau_l$. The same effects arise if the follower reduces its tax.

Recall that initially both countries are assumed to have population $1 + N^V$. With free migration and nationally chosen taxes with $\tau_l - \tau_f > 0$, the tax revenues are

$$\begin{aligned} TR_l &= (1 - \underline{e}^l)\tau_l \\ TR_f &= \tau_f(1 + \underline{e}^l + 2N^V). \end{aligned}$$

Thus, the leader loses all rich residents from its valley N^V; it also loses the rich residents on the hill who lived below $\underline{e}^l$. The share of the rich emigrating from the leader's hill increases with the tax differential.

5.2.1 Follower's reaction function

To determine the follower's optimal tax rate, we take now the leader's tax, τ_l as given. The follower has three alternative strategies: (1) set a tax which is lower than the leader' by a non-infinitesimal amount, (2) set a tax infinitesimally lower than the leader's, and (3) choose a higher tax. As it will turn out that in equilibrium, setting a higher tax is never optimal, we present here detailed analyses of only solutions (1) and (2); a detailed analysis of case (3) appears in Appendix A.

Discrete undercut by the follower With a discrete (non-infinitesimal) undercut, we insert $\underline{e}^l(\tau_l, \tau_f)$ into TR_f, differentiate with respect to τ_f and solve for τ_f (regarding τ_l as given). Thus, for the follower:

$$\begin{aligned} \frac{\partial TR_f}{\partial \tau_f} &= (1 + \underline{e}^l + 2N^V) + \tau_f \frac{\partial \underline{e}^l}{\partial \tau_f} \\ &= (1 + \frac{(\tau_l - \tau_f)}{\gamma} + 2N^V) - \tau_f \frac{1}{\gamma}. \end{aligned}$$

Setting this equal to zero gives

$$\tau_f = \frac{(1 + 2N^V)\gamma + \tau_l}{2}.$$

[6]The case where the follower initially has a higher tax can be analyzed in a similar way as below by switching indices l and f.

A discrete undercut must satisfy $\tau_f < \tau_l$, or

$$\begin{aligned} \frac{(1+2N^V)\gamma + \tau_l}{2} &< \tau_l \\ \tau_l &> (1+2N^V)\gamma. \end{aligned}$$

For a discrete undercutting to be a candidate for an optimal tax of the follower, the leader's tax rate must exceed $\tau_l > (1+2N^V)\gamma$. Furthermore, if the follower undercut the tax by more than γ, it would attract no additional rich residents from the leader country, and so has no incentive to do so. With undercutting by γ, only one person lives in country l; he lives at the top of the hill, as $\underline{e}^l(\tau_l, \tau_f) = 1$.Intuitively, living at the top with zero rent is just sufficient to compensate for the tax difference of $\tau_l - \tau_f = \gamma$.A smaller tax in the follower country would only result in loss of tax revenue with no additional tax payers. Therefore, the following individual rationality condition must hold:

$$\begin{aligned} \tau_l - \tau_f &\leq \gamma \\ \tau_l - \frac{(1+2N^V)\gamma + \tau_l}{2} &\leq \gamma \\ \tau_l &\leq (3+2N^V)\gamma. \end{aligned}$$

This in turn implies that an optimal tax with discrete undercutting of less than γ, $\tau_l - \tau_f < \gamma$ must satisfy the feasibility condition $(1+2N^V)\gamma < \tau_l < (3+2N^V)\gamma$. Thus,

Lemma 5 *The follower may choose a discrete undercut by less than γ if the leader set its tax in the range*

$$(1+2N^V)\gamma < \tau_l < (3+2N^V)\gamma.$$

Moreover,

Lemma 6 *If the leader chooses a higher tax, $\tau_l \geq (3+2N^V)\gamma$, and if it is optimal for the follower to choose a non-infinitesimal undercut, then the followers undercuts the leader by precisely γ, thereby attracting all tax payers from the leader's country.*

We will examine whether a discrete undercut is optimal after analyzing an infinitesimal undercut. The discrete undercut results in the follower having a tax revenue of

$$\begin{aligned} TR_f &= \frac{\left[(1+2N^V)\gamma + \tau_l\right]^2}{4\gamma}, if (1+2N^V)\gamma < \tau_l < (3+2N^V)\gamma \\ TR_f &= (\tau_l - \gamma)(2+2N^V), \text{ if } \tau_l \geq (3+2N^V)\gamma. \end{aligned}$$

The first tax revenue follows from inserting the tax rate $\tau_f = \frac{(1+2N^V)\gamma + \tau_l}{2}$ into the revenue $\tau_f(1 + \underline{e}^l + 2N^V)$ with $\underline{e}^l(\tau_l, \tau_f) = \frac{\tau_l - \tau_f}{\gamma}$. The last tax revenue, $(\tau_l - \gamma)(2+2N^V)$, reflects the ability of the follower to attract all tax payers from the leader country; it is optimal for the follower to do that if $\tau_l \geq (3+2N^V)\gamma$.

Infinitesimal undercut by the follower Consider next the case where the follower undercuts the leader's tax by an infinitesimal amount. Then

$$\tau_f = \tau_l - \varepsilon, \text{ where } \varepsilon \to 0.$$

In the limit

$$\begin{aligned} \lim_{\varepsilon \to 0} TR_l &= \tau_l \\ \lim_{\varepsilon \to 0} TR_f &= \tau_l(1 + 2N^V). \end{aligned}$$

This follows because with an infinitesimal undercut, the follower attracts people only from the valley. It remains to determine conditions under which the follower has no incentive to deviate, or no incentive to choose a discretely lower tax. We first summarize the follower's payoffs:

Lemma 7 *The follower's payoffs with the follower's optimal strategy are given by*
(i) $TR_f = (\tau_l - \gamma)(2 + 2N^V)$, *if the follower chooses a discrete undercut by* γ*;*
(ii) $TR_f = \frac{\left[(1+2N^V)\gamma+\tau_l\right]^2}{4\gamma}$, *if the follower chooses a discrete undercut by less than* γ
(iii) $TR_f = \tau^l(1 + 2N^V)$, *if the follower chooses an infinitesimal undercut.*

Proof. (i) Follows from the observation that by undercutting by a discrete amount γ, the follower attracts all rich people from the leader's country. For (ii) and (iii), see above. ■

We suggest that the follower's strategy satisfies:

Proposition 8 *The follower's optimal tax is*
(i) $\tau_f = \tau_l - \gamma$ *if* $\tau_l \geq (3 + 2N^V)\gamma$*;*
(ii) $\tau_f = \frac{(1+2N^V)\gamma+\tau_l}{2}$ *if* $(1 + 2N^V)\gamma < \tau_l < (3 + 2N^V)\gamma$*;*
(iii) $\tau_f = \lim_{\varepsilon \to 0}(\tau_l - \varepsilon)$ *if* $(1 + 4N^V)\gamma - \sqrt{(1 + 4N^V)^2\gamma^2 - \gamma^2} \leq \tau_l \leq (1 + 2N^V)\gamma$*;*
(iv) $\tau_f = \frac{\gamma+\tau_l}{2}$ *if* $\tau_l < (1 + 4N^V)\gamma - \sqrt{(1 + 4N^V)^2\gamma^2 - \gamma^2}$*.*

Proof. (i) If the leader sets a high tax, with $\tau_l \geq (3 + 2N^V)\gamma$, the follower's tax revenue under discrete undercutting with $\tau_f = \tau_l - \gamma$ exceeds the tax revenue with infinitesimal undercutting if $(\tau_l - \gamma)(2 + 2N^V) > \tau_l(1 + 2N^V)$. This is equivalent to $\tau_l > (2 + 2N^V)\gamma$, which thus holds.

(ii) If the leader sets the tax in the interval, $(1 + 2N^V)\gamma < \tau_l < (3 + 2N^V)\gamma$, it is optimal for the follower to undercut by a discrete amount if the tax revenues satisfy $\frac{\left[(1+2N^V)\gamma+\tau_l\right]^2}{4\gamma} > \tau_l(1 + 2N^V)$. This is equivalent to the condition that $[\tau_l - (1 + 2N^V)\gamma]^2 > 0$, which always holds.

(iii) If the leader sets a low tax rate, $\tau_l \leq (1 + 2N^V)\gamma$, the follower's optimal response may be to undercut by an infinitesimal amount but not by a discrete amount which is less than γ; see the Lemma above. However, undercutting by γ in this case cannot be optimal since the tax revenue under infinitesimal undercutting is greater, $\tau_l(1+2N^V) > (2+2N^V)(\tau_l - \gamma)$. The lower limit in (iii) is discussed in Appendix A.

(iv) is proved in Appendix A which shows, however, that this solution cannot be an equilibrium. ■

Consistency requires that the upper bound of (iii) exceeds the lower bound. This must hold as

$$
\begin{aligned}
(1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2 - \gamma^2} &\leq (1+2N^V)\gamma \\
2N^V &\leq \sqrt{(1+4N^V)^2 - 1} \\
4(N^V)^2 &\leq 8N^V + 16(N^V)^2 \\
0 &\leq 8N^V + 12(N^V)^2.
\end{aligned}
$$

5.2.2 Leader's optimal tax

Discrete undercut by the follower Recall that with $e = 1, \gamma e = \gamma$, and so γ measures the valuation of the highest location. If the leader chooses a high tax, $\tau_l \geq (3+2N^V)\gamma$, it anticipates that the follower finds it optimal to cut its tax by γ, attracting all tax payers from the leader's country, except the person at the top who is indifferent. Then the leader's payoff is zero. If the leader instead chooses $(1+2N^V)\gamma < \tau_l < (3+2N^V)\gamma$, the follower chooses a discrete cut which is less than γ and given by $\tau_f = \frac{(1+2N^V)\gamma+\tau_l}{2}$.The leader's tax revenue is then

$$
\begin{aligned}
TR_l &= \left(1 - \frac{(\tau_l - \tau_f)}{\gamma}\right)\tau_l \\
&= \left(\frac{2\gamma - \tau_l + (1+2N^V)\gamma}{2\gamma}\right)\tau_l.
\end{aligned}
$$

To determine the leader's optimal choice when $(1+2N^V)\gamma < \tau_l < (3+2N^V)\gamma$, we maximize its tax revenue function. The first-order condition is

$$
\begin{aligned}
2\gamma + (1+2N^V)\gamma &= 2\tau_l \\
\tau_l &= \frac{\gamma(3+2N^V)}{2}.
\end{aligned}
$$

As the second derivative is negative, this solution indeed gives the maximum (and positive) tax revenue in this region.

It remains to check that the tax rate for the leader in this solution exceeds the tax rate when $\tau_l \leq (1+2N^V)\gamma$. So the following inequality must hold

$$
\begin{aligned}
\frac{\gamma(3+2N^V)}{2} &> (1+2N^V)\gamma \\
3+2N^V &> 2+4N^V \\
1 &> 2N^V.
\end{aligned}
$$

This holds only for small population in the valley. The tax revenue of the leader in the interim solution $(1+2N^V)\gamma < \tau_l < (3+2N^V)\gamma$ is $\left(1 - \frac{(\tau_l - \tau_f)}{\gamma}\right)\tau_l$, to be developed into

$$
\begin{aligned}
TR_l &= \left(\frac{2\gamma - \frac{\gamma(3+2N^V)}{2} + (1+2N^V)\gamma}{2\gamma}\right)\frac{\gamma(3+2N^V)}{2} \\
TR_l &= \left(\frac{4\gamma - \gamma(3+2N^V) + 2(1+2N^V)\gamma}{4\gamma}\right)\frac{\gamma(3+2N^V)}{2} \\
&= \left(\frac{\gamma(3+2N^V)}{4\gamma}\right)\frac{\gamma(3+2N^V)}{2} \\
&= \frac{\gamma(3+2N^V)^2}{8}.
\end{aligned}
$$

We summarize

Lemma 9 *If the leader chooses $\tau_l > (1+2N^V)\gamma$ and $N^V < \frac{1}{2}$, then the leader's optimal tax is $\tau_l = \frac{\gamma(3+2N^V)}{2}$. This results in tax revenue to the leader of $TR_l = \frac{\gamma(3+2N^V)^2}{8}$.*

Proof. Follows from above. ■

Infinitesimal undercut by the follower Suppose now that the solution has the follower undercut by an infinitesimal amount. From the previous proposition we know that the follower will so behave if the leader chooses a low tax rate satisfying

$$(1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2 - \gamma^2} \le \tau_l \le (1+2N^V)\gamma.$$

Then the leader thus anticipates that the follower undercuts its tax by an infinitesimal amount. The leader's tax revenue is then

$$TR_l = \tau_l,$$

which is its tax rate τ_l times the tax base, equaling 1. In the analyzed region the leader's tax revenue thus equals its tax. Thus,

Lemma 10 *If the leader chooses $(1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2 - \gamma^2} \le \tau_l \le (1+2N^V)\gamma$, then it is optimal for it to choose $\tau_l = (1+2N^V)\gamma$. The leader's tax revenue is then $TR_l = (1+2N^V)\gamma$.*

Optimal tax of the leader In Appendix A we prove:

Lemma 11 *The leader will never choose a tax so low that the follower would find it optimal to choose a higher tax.*

This result excludes a tax rate $\tau_l < (1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2 - \gamma^2}$. Therefore, it remains to compare the strategies of a tax in the region $(1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2-\gamma^2} \leq \tau_l \leq (1+2N^V)\gamma$ and in the region $\tau^l > (1+2N^V)\gamma$.

Our results can be summarized with

Proposition 12 *(i) If $N^V < \frac{1}{2}$, the leader's optimal tax is $\tau_l = \frac{\gamma(3+2N^V)}{2}$; the follower then chooses $\tau_f = \frac{(5+6N^V)\gamma}{4}$. The leader's tax revenue is then $\frac{\gamma(3+2N^V)^2}{8}$.*
(ii) If $N^V \geq \frac{1}{2}$, the leader's optimal tax is $\tau_l = (1+2N^V)\gamma$; the follower then chooses an infinitesimally lower tax. The leader's tax revenue is $\tau_l = (1+2N^V)\gamma$.

Proof. The leader's choices follow from the above Lemmas. (i) The follower's tax with $N^V < \frac{1}{2}$ is given by inserting $\tau_l = \frac{\gamma(3+2N^V)}{2}$ into $\tau_f = \frac{(1+2N^V)\gamma+\tau_l}{2}$. The leader's tax revenue is larger with $\tau_l = \frac{\gamma(3+2N^V)}{2}$ than with $\tau_l = (1+2N^V)\gamma$ if and only if

$$\begin{aligned}
\frac{\gamma(3+2N^V)^2}{8} &> (1+2N^V)\gamma \\
(3+2N^V)^2 &> 8(1+2N^V) \\
9+12N^V+4(N^V)^2 &> 8+16N^V \\
(1-2N^V)^2 &> 0.
\end{aligned}$$

This always holds. (ii) We proved earlier that the follower would undercut the leader by an infinitesimal amount if $N^V > \frac{1}{2}$. Then the leader's optimal tax is $\tau_l = (1+2N^V)\gamma$. ■

5.2.3 Constrained taxation

Countries naturally face different types of constraints in their tax policies. The extreme constraint is that taxes cannot exceed income. We also suppose that taxation cannot reduce a rich person's post-tax income to below a poor person's income. In an open economy, the leader faces the additional constraint that it must anticipate the subsequent tax policy by the follower. As this was analyzed above, we consider here the implications of exogenously determined constraints.

Assume next that the tax the leader can set is constrained to lie below some level, $\tau^{\max}$. If $\tau^{\max} < (1+2N^V)\gamma$, then the leader chooses $\tau^{\max}$ and the follower $\tau^{\max} - \varepsilon$. If $(1+2N^V)\gamma < \tau^{\max} < \frac{(3+2N^V)\gamma}{2}$, and the leader would choose with unconstrained optimization a higher tax than $\tau^{\max}$, it remains to calculate whether the leader would find it optimal to choose $\tau^{\max}$ or $(1+2N^V)\gamma$. With $(1+2N^V)\gamma < \tau^{\max} < \frac{(3+2N^V)\gamma}{2}$, the leader's choice of $\tau^{\max}$ would result in the follower's choice of

$$\tau_f = \frac{(1+2N^V)\gamma + \tau^{\max}}{2}.$$

The leader's tax revenue is then, with a choice of $\tau^{\max}$,

$$TR_l = \left(1 - \frac{(\tau^{\max} - \frac{(1+2N^V)\gamma+\tau^{\max}}{2})}{\gamma}\right)\tau^{\max}$$

$$TR_l = \left(1 - \frac{(2\tau^{\max} - (1+2N^V)\gamma - \tau^{\max})}{2\gamma}\right)\tau^{\max}$$

$$TR_l = \left(\frac{(3+2N^V)\gamma - \tau^{\max}}{2\gamma}\right)\tau^{\max}$$

Notice that

$$\frac{\partial TR_l}{\partial \tau^{\max}} = \frac{(3+2N^V)\gamma}{2\gamma} - \frac{\tau^{\max}}{\gamma}.$$

As this is positive in the analyzed region, TR_l increases with $\tau^{\max}$. In the limit, the leader's tax revenue when he chooses $\tau^{\max}$ is

$$\begin{aligned} & \lim_{\tau^{\max}\to(1+2N^V)\gamma} \left(\frac{(3+2N^V)\gamma - \tau^{\max}}{2\gamma}\right)\tau^{\max} \\ = \quad & (1+2N^V)\gamma. \end{aligned}$$

We thus obtain

Proposition 13 *If the leader would choose a higher tax rate without constraint than the maximum tax rate $\tau^{\max}$ that is allowed with constraint, then the leader chooses with the constraint $\tau^{\max}$.*

Our result indicates that if the leader would choose a tax rate lower than what would result in a discrete undercut by the follower without a binding constraint on the maximum tax rate, then the leader chooses the maximum tax rate when the constraint is binding. The follower undercuts by an infinitesimal amount. If the leader would, without a binding constraint on taxation, choose such a high tax rate that the follower would maximally undercut, the leader chooses the maximum allowed tax rate with a binding constraint. If this maximum tax rate is at most $(1+2N^V)\gamma$, then the constraint makes the follower switch from discrete undercutting to infinitesimal undercutting. If the maximum tax rate is more than $(1+2N^V)\gamma$ then the follower chooses discrete undercutting also with the constraint.

6 Migration with both rich and poor people

So far we ignored the behavior of poor people. This section extends the model to consider them. We suppose that initially there are both rich and poor in each of the two countries. Suppose land on the hill is scarce, so that initially some rich people in each country live in the valley.

We know from the previous sections that a high domestic tax rate leads rich people to migrate from the valley, and from the lower parts of the hill. Not all rich people leave the hill, because the tax causes the rental rates on the hill to fall, making migration less attractive. In standard models of taxation, the migration decisions of the rich depend only on the tax they pay, not on the choices made by the poor. But in our model the behavior of the poor can increase migration by the rich.

A domestic tax which causes some rich people to migrate from the hill in one country to the other country initially create vacancies at the bottom of the hill, or at locations $(0, \underline{e}^l)$. If there are no poor people, then the rentals at these locations are zero. But if some poor people initially live in the valley, these locations are attractive to them; they will bid the rents there to positive levels. The increase in rents at locations $(0, \underline{e}^l)$ compared to what they would be in the absence of poor people increases rents at higher elevations. This increase in rents in turn induces additional rich people to migrate from the hill to the other country. In equilibrium, rich people live at locations above $\underline{e}^l_P$, where $\underline{e}^l_P > \underline{e}^l$. The magnitude of the difference between $\underline{e}^l_P$ and $\underline{e}^l$ depends on the willingness to pay by poor people. Suppose the willingness to pay by a poor person for an increase in elevation is $\gamma_P < \gamma_R$, where the willingness to pay by a rich person is γ_R. The greater is γ_P, the higher is $\underline{e}^l_P$. This gives us

Proposition 14 *The greater is the willingness by poor people to pay for living at a higher elevation, the smaller is the tax capitalization in property values and the stricter are the limits to redistribution.*

In the presence of both rich and poor people on the hill, rents depend on the willingness to pay by both. Rent at elevation e, with $0 < e < \underline{e}^l_P$, is $\gamma_P e$. Rent at elevation e above $\underline{e}^l_P$ is $\gamma_P \underline{e}^l_P + \gamma_R(e - \underline{e}^l_P)$. In the country with rich people living in both the valley and the hill (and so with no poor persons living on the hill), rents are given by $\gamma_R e$.

Assume next that the leader country has a higher tax. The utility of a rich person residing at elevation $\underline{e}^l_P$ is $\gamma_R \underline{e}^l_P$. The rent there is $\gamma_P \underline{e}^l_P$. The equilibrium with migration is now

$$(y^R - \tau_l - \gamma_P \underline{e}^l_P) + \gamma_R \underline{e}^l_P = (y^R - \tau_f),$$

Therefore

$$\underline{e}^l_P = \frac{\tau_l - \tau_f}{\gamma_R - \gamma_P}.$$

Observe that all of our results which applied with no poor people can be generalized to hold in their presence:

Proposition 15 *All the results derived in the absence of poor hold in their presence, when γ is replaced by $\gamma_R - \gamma_P$, and $\underline{e}^l$ is replaced by $\underline{e}^l_P$.*

Proof. Follows directly by reinterpreting variables in earlier analysis. ■

This proposition highlights the importance of different preferences for location. The government can raise revenue from taxing the rich only if the poor have a lower willingness to pay for desirable locations. It is the lower willingness to pay for desirable locations by the poor which makes rents on the hill decline as the tax on the rich increases. That makes redistributive taxation possible, even when migration is costless.

7 Appendix A

The follower would never choose an infinitesimal increase, as this would result in losing all tax payers from the valley, while little raising tax revenue from those staying on the hill. Consider next the case where the leader chooses such a low tax that the follower chooses a higher tax. Now the equilibrium with migration is

$$(y^R - \tau_l) = (y^R - \tau_f) + \gamma e^f,$$

where the rent at e^f equals zero. Solving for e^f gives the number of people emigrating from the hill of the follower country,

$$e^f(\tau_l, \tau_f) = \frac{(\tau_f - \tau_l)}{\gamma}.$$

The corresponding tax revenues are

$$\begin{aligned} TR_l &= (1 + e^f + 2N^V)\tau_l \\ TR_f &= \tau_f(1 - e^f). \end{aligned}$$

Therefore,

$$\begin{aligned} TR_f &= \tau_f(1 - \frac{(\tau_f - \tau_l)}{\gamma}) \\ &= \tau_f - \frac{\tau_f(\tau_f - \tau_l)}{\gamma} \end{aligned}$$

and the first-order condition is

$$\frac{\partial TR_f}{\partial \tau_f} = 1 - \frac{2\tau_f - \tau_l}{\gamma} = 0.$$

This results in

$$\tau_f = \frac{\gamma + \tau_l}{2}.$$

Therefore, the follower's tax revenue is

$$\begin{aligned} TR_f &= \tau_f(1 - \frac{(\tau_f - \tau_l)}{\gamma}) \\ &= \frac{\gamma + \tau_l}{2}(1 - \frac{(\frac{\gamma+\tau_l}{2} - \tau^l)}{\gamma}) \\ &= \frac{\gamma + \tau_l}{2}(1 - \frac{\gamma - \tau_l}{2\gamma}) \\ &= \frac{\gamma + \tau_l}{2}(\frac{\gamma + \tau_l}{2\gamma}) \\ &= \frac{(\gamma + \tau_l)^2}{4\gamma} \end{aligned}$$

A discrete increase occurs only if it results in a larger tax revenue than an infiniteseimal decrease, or only if

$$\frac{(\gamma+\tau_l)^2}{4\gamma} > (1+2N^V)\tau_l.$$

This is equivalent to

$$\begin{aligned}(\gamma+\tau_l)^2 &> 4\gamma(1+2N^V)\tau_l \\ \gamma^2+2\gamma\tau_l+(\tau_l)^2 &> 4\gamma(1+2N^V)\tau_l \\ \gamma^2-(2+8N^V)\gamma\tau_l+(\tau_l)^2 &> 0\end{aligned}$$

Setting this equal to zero and solving for τ_l yields

$$\tau_l = \frac{(2+8N^V)\gamma \pm \sqrt{(2+8N^V)^2\gamma^2-4\gamma^2}}{2}$$

Therefore, either

$$\tau_l > \frac{(2+8N^V)\gamma + \sqrt{(2+8N^V)^2\gamma^2-4\gamma^2}}{2}$$

or

$$\tau_l < \frac{(2+8N^V)\gamma - \sqrt{(2+8N^V)^2\gamma^2-4\gamma^2}}{2}.$$

The first root can be excluded as it is already in the region where undercutting is more attractive. Therefore, the follower would find it optimal to increase the tax only if

$$\tau_l < (1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2-\gamma^2}.$$

We must check that it is not optimal to choose so low a tax rate that the follower would choose a higher tax rate. If

$$\tau_l < (1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2-\gamma^2},$$

the follower would choose

$$\tau_f = \frac{\gamma+\tau_l}{2}.$$

The leader's problem is now to maximize

$$\begin{aligned}TR_l &= (1+e^f+2N^V)\tau_l \\ &= (1+\frac{(\tau_f-\tau_l)}{\gamma}+2N^V)\tau_l \\ &= (1+\frac{(\frac{\gamma+\tau_l}{2}-\tau_l)}{\gamma}+2N^V)\tau^l \\ &= (1+\frac{(\gamma+\tau_l-2\tau_l)}{2\gamma}+2N^V)\tau_l \\ &= (1+\frac{(\gamma-\tau_l)}{2\gamma}+2N^V)\tau_l.\end{aligned}$$

The first-order derivative is

$$\begin{aligned}\frac{\partial TR_l}{\partial \tau_l} &= (1 + \frac{(\gamma - \tau_l)}{2\gamma} + 2N^V) - \frac{\tau_l}{2\gamma} \\ &= \frac{1}{\gamma}\left[(1+2N^V)\gamma - \tau_l\right].\end{aligned}$$

By the constraint for τ_l, this is always positive. (Notice that in the region where the follower would choose a higher tax, $\tau_l < (1+2N^V)\gamma$). Therefore, the leader's tax revenue is maximized in the region with the follower choosing a higher tax by

$$\tau_l = (1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2 - \gamma^2}.$$

The tax revenue is then

$$\begin{aligned}TR_l &= (1 + \frac{(\gamma - \left[(1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2-\gamma^2}\right])}{2\gamma} + 2N^V)\left[(1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2-\gamma^2}\right] \\ &= (\frac{3\gamma + 4N^V\gamma - \left[(1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2-\gamma^2}\right]}{2\gamma})\left[(1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2-\gamma^2}\right] \\ &= (\frac{2\gamma + \sqrt{(1+4N^V)^2\gamma^2-\gamma^2}}{2\gamma})\left[(1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2-\gamma^2}\right] \\ &= \frac{1}{2\gamma}\left(2\gamma + \sqrt{(1+4N^V)^2\gamma^2-\gamma^2}\right)\left[(1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2-\gamma^2}\right] \\ &= \frac{1}{2\gamma}\left[2\gamma(1+4N^V)\gamma + (-1+4N^V)\gamma\sqrt{(1+4N^V)^2\gamma^2-\gamma^2} - (1+4N^V)^2\gamma^2 + \gamma^2\right].\end{aligned}$$

Next, divide the analysis in two parts: (i) $(1+4N^V)\gamma < 2\gamma$ (that is, $N^V < 0.25$) and (ii) $(1+4N^V)\gamma \geq 2\gamma$. If $(1+4N^V)\gamma < 2\gamma$, the tax revenue is less than

$$\frac{1}{2\gamma}\left[2\gamma(1+4N^V)\gamma - (1+4N^V)^2\gamma^2 + \gamma^2\right].$$

This expression results from removing the negative term from the last line. The solution with infinitesimal undercutting by the follower results in a larger tax revenue if

$$\begin{aligned}(1+2N^V)\gamma &> \frac{1}{2\gamma}\left[2\gamma(1+4N^V)\gamma - (1+4N^V)^2\gamma^2 + \gamma^2\right] \\ 2(1+2N^V) &> 2(1+4N^V) - (1+4N^V)^2 + 1 \\ 0 &> -4N^V(1+4N^V)\end{aligned}$$

Therefore, it is not optimal for the leader to choose such a low tax that the follower would undercut it if $N^V < 0.25$.

(ii) If $(1+4N^V)\gamma > 2\gamma$, that is, $N^V > 0.25$, the tax revenue is less than

$$\begin{aligned}
TR_l &= \left(\frac{(1+4N^V)\gamma + \sqrt{(1+4N^V)^2\gamma^2 - \gamma^2}}{2\gamma}\right)\left[(1+4N^V)\gamma - \sqrt{(1+4N^V)^2\gamma^2 - \gamma^2}\right] \\
&= \frac{\gamma^2}{2\gamma} \\
&= \frac{\gamma}{2}.
\end{aligned}$$

Compare this with the tax revenue when the follower chooses an infinitesimal undercut, namely $\frac{\gamma(3+2N^V)^2}{8}$. Clearly, the equilibrium with infinitestimal undercutting by the follower results in a higher tax revenue for the leader.

References

[1] Cassidy, Glenn, Dennis Epple, and Thomas Romer (1989) "Redistribution by local governments in a monocentric urban area." *Regional Science and Urban Economics*, 19: 421-454.

[2] Christiansen, V., K.P.Hagen, and A. Sandmo (1994), "The scope for taxation and public expenditures in an open economy." *Scandinavian Journal of Economics*, 96(3): 289-309.

[3] De Donder, P. and J. Hindriks (1998) "Un modele simple de concurrence fiscale ou la mobilite des pauvres favorise la redistribution." *Actualite Economique*, 75(1-2-3):

[4] Ellickson, Bryan (1971) "Jurisdictional fragmentation and residential choice." *American Economic Review*, 61(2): 334-339.

[5] Epple, Dennis, R. Filimon and Thomas Romer (1984) "Equilibrium among local jurisdictions: Towards an integrated treatment of voting and residential choice." *Journal of Public Economics*, 24: 281-308.

[6] Epple, D. and T. Romer (1991) "Mobility and redistribution." *Journal of Political Economy*, 99: 828-858.

[7] Epple, Dennis and Glenn J. Platt (1998) "Equilibrium and local redistribution in an urban economy when households differ in both preferences and incomes." *Journal of Urban Economics*, 43: 23-51.

[8] Goodspeed, Timothy J. (1989) "A re-examination of the use of ability to pay taxes by local governments." *Journal of Public Economics*, 38: 319-342.

[9] Hindriks, Jean (2001) "Mobility and redistributive politics." *Journal of Public Economic Theory*, 3: 95-120.

[10] Mansoorian, A., and G.M. Myers (1997) "On the consequences of government objectives for economies with mobile populations." *Journal of Public Economics*, 63: 265-281.

[11] Sinn, H-.W. "The selection principle and market failure in systems competition." *Journal of Public Economics*, 66: 247-274.

[12] Westhoff, F. (1977) "Existence of equilibria in economies with a local public good." *Journal of Economic Theory*, 14: 84-112.

[13] Wildasin, D.E. (1994) "Income redistribution and migration." *Canadian Journal of Economics*, 27: 637-656.

8 Notation

c_e Rent at elevation i

L_i Lowest elevation at which a rich person lives in country i

n^R Number of rich people

TR Tax revenue

$\overline{W}$ Reservation utility of a rich person

x Consumption

y^R Pre-tax income of a rich person

τ Tax on each rich person